Universität Leipzig
Fakultät für Physik und Geowissenschaften
Jürgen Vollmer

Sommersemester 2021
Fr, 13. 4. 2021

Pattern Formation and Nonlinear Dynamics

Blatt 1. Means of Qualitative Analysis

1. Dimensional Analysis of Flight Trajectories

(a) How does the initial velocity v_0 impact the distance W of a thrown object (stone, ball, or shot) or a jump?

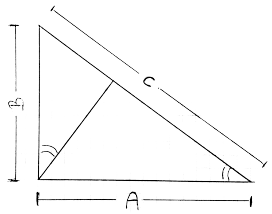
(b) How does the initial velocity v_0 depend on
the force F acting by the responsible muscle
the accelerated mass M, and
the distance L of the path where the acceleration is performed?

(c) Estimate the maximum distance
of throwing a stone of mass $m = 200\,\mathrm{g}$,
of a standing jump for a human and a grass hopper.

(d) Make an explicit analysis of standing jumps by exploring how their distance scales with the ratio of characteristic sizes (i.e., body length) of the jumper.

2. Pythagoras' Theorem

Have a look at the sketch to the right. The indicated angle will be denoted as β.



(a) Show that the area $\mathcal{F}_C$ of the full triangle may be written as

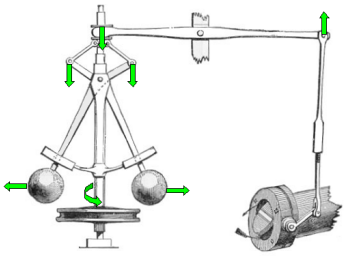
$$\mathcal{F}_c = C^\nu\, f(\beta)$$

Here, $f(\beta)$ is a dimensionless function of the angle β. Determine only ν, not $f(\beta)$.

(b) The indicated height of the triangle divides its total area into two areas. They are right-angled and similar to the original one, except that their hypotenuses are of length A and B. What can you say about the areas $\mathcal{F}_A$ and $\mathcal{F}_B$ of these triangles?

(c) Give a proof of Pythagoras' theorem!

3. Bifurkation Analysis of the Centrifugal Governor



Centrifugal governors are used in steam engines to control to rotation frequency of wheels driven by the engine. It comprises two metal balls that are attached to arms of length L. For small frequencies Ω the heavy steel balls are hanging straight down on their arms. When Ω exceeds a critical frequency ω_c the arms move outwards, and this opens a valve that is controlling the pressure in the engine.

(a) Assume that ω_c only depends on the mass M of the balls, the length L of the arms, and the gravitational acceleration g. Determine ω_c by dimensional analysis.

(b) Employ the Lagrange formalism to determine the equations of motion of the balls. Sketch the dynamics in phase space for $\Omega < \omega_c$ and $\Omega > \omega_c$.

(c) How does the critical angle change, when one takes into account the fact that the governor has two balls with radius R?

(d) In (b) we derived the equations of motion for the deflection $\theta(t)$ of the balls, when there is no friction. What changes when there is a damping? Determine a dimensionless parameter δ that characterizes the damping.

(e) Revisit the trajectories of the governor in phase space $(\theta, \dot{\theta})$. How does the diagram look like for different values of δ? Are there new bifurcations points? If yes: what do they refer to?