Homework Exercises 14

These problems will be solved in the seminars on February 3 and 4.

Self-Test Problems

Problem 14.1. Flight of a Bola.

A Bola is made of two or more weights that are connected by strings. They are whirled over the head and then released into free flight. During the flight the weights spin around their center of mass. Bolas are used by South American gauchos for hunting and catching cattle, similar to the use of lassos by North American cowboys. Inuit use them to hunt bird. Kung Fu fighters and jugglers denote them as meteors.

Pearson Scott Foresman
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In the following we consider a Bola consisting of two weights of the same mass m that are connected by an elastic string. The relative position of the masses is denoted as $\vec{R}$. For a distance $R = |\vec{R}|$ smaller than the length of the band L there is only gravity acting on the masses. For distances $R > L$ there is an additional force with modulus $k\,(R - L)$ acting in the direction of the respective other weight.

a) Determine the Lagrange function.
Evaluate the Euler-Lagrange functions to determine the equation of motion for the center of mass $\vec{Q}$ of the Bola, and the relative position $\vec{R}$ of the two weights.

b) Solve the equation of motion for $\vec{Q}$ for the case where the Bola is released with initial velocity $\vec{v}_0$ at the position $\vec{Q}_0$.

c) Show that the energy and the angular momentum is conserved for the relative motion $\vec{R}$.
Hint: It is sufficient to quote a pertinent theorem, rather than providing an explicit proof.

d) We will now write the relative distance as $\vec{R} = R(\cos\theta, \sin\theta)$. Why do we need only two components? How can one determine the third direction in space? What is the meaning of the angle θ?

e) Show that distance R between the weights obeys the following equation of motion

$$\ddot{R} = \begin{cases} a\, R^{-3} & \text{für} \quad 0 < R < L\,, \\ a\, R^{-3} - \omega^2\,(R - L) & \text{für} \quad L \leq R\,. \end{cases}$$

Specify how a and ω in this equation depend on the parameters of the problem.

f) Sketch the solutions in phase space.

Problem 14.2. Two masses hanging at a rubber band

Two weights of the same mass m are attached on opposite ends of a rubber band (as for the Bola in Problem 14.1). Here, however, the band is hanging over a roll. The weights are at height h_1 and h_2. They move only vertically, either one up and one down at a fixed length of the band, or stretching the band, or releasing tension on the band. We assume that friction and the mass of the band are negligible.

a) Sketch the problem, and indicate the relevant parameters and coordinates.

b) We describe the problem by adopting the coordinates $H = h_1 + h_2$ and $D = h_1 - h_2$. Verify that the Lagrange function will then take the following form

$$\mathcal{L}(H, D, \dot{H}, \dot{D}) = \frac{\mu}{2}\left(\dot{H}^2 + \dot{D}^2\right) - mg\,H - \frac{k}{2}\,H^2$$

Here k is the elastic module of the rubber band, and μ is an effective mass. How is μ related to m?

The expression for the $\mathcal{L}$ adopts a particular choice of the origin used to specify h_1 and h_2. Which choice has been used?

c) Determine the equations of motion for H and D.

d) Solve the equations of motion and interpret the result. For which values of H and D will you trust the result?