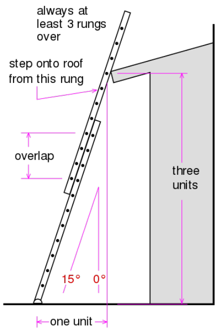
Homework Exercises 5

Your solution to the problems 5.4–5.6 should be handed in/presented
either during a seminar on Monday, Nov 18, at 11:00,
or in my mail box at ITP, room 105b, by Monday, Nov 18, 13:00.
Please submit only Problem 5.4(e) and Problem 5.6 when you participate in the seminar.

Warm-up

Problem 5.1. Torques acting on a ladder



Original: Bradley – Vector: Sarang
[Public domain from wikimedia]

This is again the sketch to the ladder leaning to the roof of a hut. The angle from the downwards vertical to the ladder is denoted θ. There is a gravitational force of magnitude Mg acting of a ladder. At the point where it leans to the roof there is a normal force of magnitude F_r. At the ladder feet there is a normal force to the ground of magnitude F_g, and a tangential friction force of magnitude F_f.

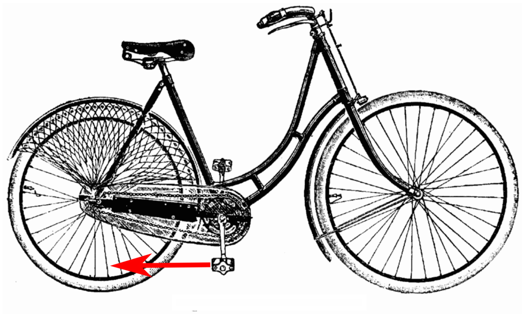
a) Determine the torque acting on the ladder. Does it matter whether you consider the torque with respect to the contact point to the roof, the center of mass, or the foot of the ladder?

b) The ladder will slide when the modulus of the friction force F_f exceeds a maximum value $\mu_S F_g$ where μ is the static friction coefficient for of the ladder feet on the ground. For metal feet on a wooden ground it takes a value of $\mu_S \simeq 2$. What does that tell about the angels wher the ladder starts to slide?

c) Why does a ladder commonly starts sliding when when a man has climbed to the top? Is there anything one can do against it? Is that even true, or just an urban legend?

Problem 5.2. Where will the bike go?

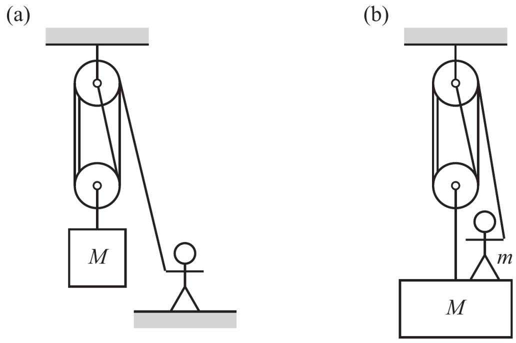
Consider the picture of the bicycle to the left. The red arrow indicates a force that is acting on the paddle in backward direction.
Will the bicycle move forwards or backwards?
Take a bike and do the experiment!



adapted from original: Damenfahrrad (Otto Lueger, 1904) [Public domain, wikimedia]

Problem 5.3. Tackling tackles and pulling pulleys

The sketch to the right shows to tackles where a person of mass m is hauling a weight of mass M. We don not consider effects of friction and the mass of the rope.



a) Which forces are required to hold the balance in sketch (a) and (b)?

b) Let the sketched person and the weight have masses of 75 kg and 300 kg, respectively. Which power is required when to haul the line at a speed of 1 m/s. Hint: The power is defined here as the change of (a) $M g\, z(t)$ and (b) $(M + m)\, g\, z(t)$, per unit time, respectively.

Homework Problems

Problem 5.4. Determining the volume, the mass, and the center of mass

Determine the mass M, the area or volume V, and the the center of mass $\vec{Q}$ of bodies with the following mass density and shape.

a) A triangle in two dimensions with constant mass density $\rho = 1\,\mathrm{kg/m^2}$ and side length 6 cm, 8 cm, and 10 cm.
Hint: Determine first the angles at the corners of the triangle. Decide then about a convenient choice of the coordinate system (position of the origin and direction of the coordinate axes).

b) A circle in two dimensions with center at position (a, b), radius R, and constant mass density $\rho = 1\,\mathrm{kg/m^2}$.
Hint: How do M, V and $\vec{Q}$ depend on the choice of the origin of the coordinate system?

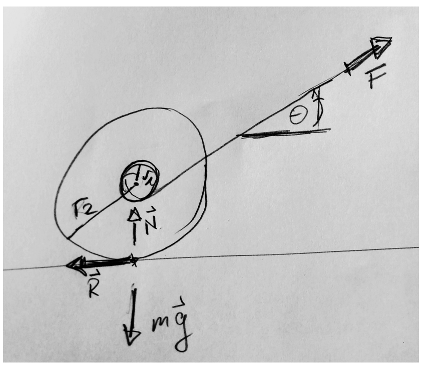
c) A rectangle in two dimensions, parameterized by coordinates $0 \leq x \leq W$ and $0 \leq y \leq B$, and a mass density $\rho(x, y) = \alpha\, x$.
What is the dimension of α in this case?

d) A three-dimensional wedge with constant mass density $\rho = 1\,\mathrm{kg/m^3}$ that is parameterized by $0 \leq x \leq W$, $0 \leq y \leq B$, and $0 \leq z \leq H - Hx/W$.
Discuss the relation to the result of part b).

e) A cube with edge length L. When its axes are aligned parallel to the axes $\hat{x}$, $\hat{y}$, $\hat{z}$, it density takes the form $\rho(x, y, z) = \beta\, z$.
What is the dimension of β in this case?

Problem 5.5. Walking a yoyo

The sketch to the right shows a yoyo of mass m standing on the ground. It is held at a chord that extends to the top right. There are four forces acting on the yoyo: gravity $m\vec{g}$, a normal force $\vec{N}$ from the ground, a friction force $\vec{R}$ at the contact to the ground, and the force $\vec{F}$ due to the chord. The chord is wrapped around an axle of radius r_1. The outer radius of the yoyo is r_2.



a) Which conditions must hold such that there is no net force acting on the center of mass of the yoyo?

b) For which angle θ will the torque vanish?

c) Perform an experiment: What happens for larger and for smaller angels θ? How does the yoyo respond when fix the height where you keep the chord and pull continuously?

Problem 5.6. Gradients and equipotential lines

Determine the derivatives of the following functions.

a) Equipotentail lines in the (x, y)-plane are lines $y(x)$ or $y(x)$ where a functions $f(x, y)$ takes a constant value. Sketch the equipotential lines of the functions

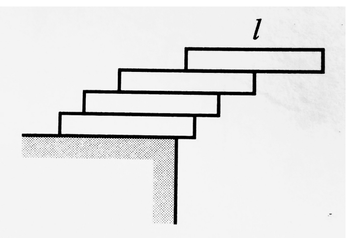
$$f_1(x, y) = (x^2 + y^2)^{-1} \qquad \text{and} \qquad f_2(x, y) = -x^2\, y^2$$

b) Determine the gradients $\nabla f_1(x, y)$ and $\nabla f_2(x, y)$.
Hint: The gradient $\nabla f_i(x, y)$ with $i \in \{1, 2\}$ is a vector $(\partial_x f_i(x, y), \partial_y f_i(x, y))$ that contains the two partial derivatives of the (scalar) function $f_i(x, y)$.

c) Indicate the direction and magnitude of the gradient by appropriate arrows in the sketch showing the equipotential lines. In which direction is the gradient pointing?

Bonus Problem

Problem 5.7. Piling bricks



Christmas is approaching, and Germans consume enormous amounts of chocolate. If you happen to come across a considerable pile of chocolate bars (or beer mats, or books, or anything else of that form) I recommend the following experiment:

a) We consider N bars of length l piled on a table. What is the maximum amount that the topmost bar can reach beyond the edge of the table.

b) The sketch above shows the special case $N = 4$.
However, what about the limit $N \to \infty$?