Homework Exercises 1

Your solution to the problems 1.1–1.3 should be handed in
either during a seminar on Monday, Oct 21, at 11:00,
or in my mail box at ITP, room 105b, by Monday, Oct 21, 13:00.
When you show up in the seminar and indicate that you are willing to present problem 1.1 or 1.3 on the blackboard, you need not hand in that problem.

Problems

Problem 1.1. Water waves
The speed of waves on the ocean depends only on their wave length L and the gravitational acceleration $g \simeq 10\,\mathrm{m/s^2}$.

a) How does the speed of the waves depend on L and g?

b) Unless it is surfing, the speed of a yacht is limited by its hull speed, i.e. the speed of a wave with wave length identical to the length of the yacht. Estimate the top speed of a 30 ft yacht.

c) Close to the beach the water depth H become a more important parameter than the wave length. How does the speed of the crest and the trough of the wave differ? What does this imply about the form of the wave?

Problem 1.2. Oscillation period of a particle attached to a spring
In a gravitational field with acceleration $g_{\mathrm{Moon}} = 1.6\,\mathrm{m/s^2}$ a particle of mass $M = 100\,\mathrm{g}$ is hanging at a spring with spring constant $k = 1.6\,\mathrm{kg/s^2}$. It oscillates with period T when it is slightly pulled downwards and released. We describe the oscillation by the distance $x(t)$ from its rest position.

a) Determine the dimensionless distance $\xi(t)$,
and the associated dimensionless velocity $\zeta(t)$.

b) Provide an order-of-magnitude guess of the oscillation period T.

Problem 1.3. Earth orbit around the sun

a) Light travels with a speed of $c \approx 3 \times 10^8\,\mathrm{m\,s^{-1}}$. It takes 8 minutes and 19 seconds to travel from Sun to Earth. What is the distance D of Earth and Sun in meters?

b) The period of the trajectory of the Earth around the Sun depends on D, on the mass $M = 2 \times 10^{30}\,\mathrm{kg}$ of the sun, and on the gravitational constant $G = 6.7 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$. Estimate, based on this information, how long it takes for the Earth to travel once around the sun.

c) Express your estimate in terms of years. The estimate of (b) is of order one, but still off by a considerable factor. Do you recognize the numerical value of this factor?

d) Upon discussing the trajectory $\vec{x}(t)$ of planets around the sun later on in this course, we will introduce dimensionless positions of the planets $\vec{\xi}(t) = \vec{x}(t)/L = (x_1(t)/L, x_2(t)/L, x_3(t)/L)$. How would you define the associated dimensionless velocities?

Bonus Problem

Problem 1.4. Printing the output of Phantom cameras

With a set of three phantom cameras one can simultaneously follow the motion of 100 particles in a violent 3d turbulent flow. Data analysis of the images provides particle positions with a resolution of 25,000 frames per second. You follow the evolution for 20 minute, print it double paged with 8 coordinates per line and 70 lines per page. A bookbinder makes 12 cm thick books from every 1000 pages. You put these books into bookshelves with seven boards in each shelf. How many meters of bookshelves will you need to store your data on paper?