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Pattern Formation and Nonlinear Dynamics

Blatt 4. Symbolic Dynamics

1. Variations of the Logistic Map.

Consider the map

$$L(x) = K - a\,x^2 \qquad \text{with } a, K \in \mathbb{R}\,.$$

How can ist dynamics be mapped to the one of the logistic map $\mu x(1-x)$?

2. Finite Subshifts.

We consider the map

$$f(x) = \begin{cases} (\eta - 1) + \eta\, x & \text{for} \quad x < 0 \\ (1 - \eta) + \eta\, x & \text{for} \quad x \geq 0 \end{cases}$$

that we also explored in the previous exercise and the lectures. We consider parameter values where η is increased form 1 to 2.

(a) Find the parameter value η_2 where 0^- lies on a period-two orbit.
Its symbol sequence will be $\overline{01}$.
Determine the Markov graph for the corresponding finite subshift.
Verify that all symbol sequences $\{\sigma\}$ that start with 0 fulfill the consistency condition $\{\sigma\} \leq \overline{01}$.

(b) Find a parameter value η_3 where 0^- lies on a period-three orbit.
What is the related symbol sequence?
Determine the Markov graph for the corresponding finite subshift.
Verify that all symbol sequences $\{\sigma\}$ that start with 0 fulfill the consistency condition $\{\sigma\} \leq \overline{01}$.

(c) Repeat the same steps for the generalized tent map

$$t(x) = \begin{cases} \alpha\, x & \text{for} \quad 0 \leq x < 1/2\,, \\ \alpha\,(1-x) & \text{for} \quad 1/2 < x \leq 1\,. \end{cases} \tag{1}$$

3. Periodic Orbits.

Find the number of *distinct* periodic orbits of length 3, 4, 5, and 6.
Hint: For instance there is only one period-two orbit. The Bernoulli shift maps $\overline{01} \to \overline{01}$, and vice versa.

4. Sequence of Emergence of Periodic Orbits.

Consider again the generalized tent map, Eq. (1).

(a) Write a Sage-program that analyzes how the symbol sequences of the maximum $x_c = 1/2$ evolve upon increasing α. For instanced you can increment α in very small steps, and figure out when the seuqence changes.

(b) Identify values α_i where the sequence changes, and verify that this corresponds to values where x_c lies on a periodich orbit.

(c) What are the symbol sequences corresponding to the periodic orbits found in (b)? Determine the order of appearance of the sequences of period one to six. How could you have guessed this order?