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Pattern Formation and Nonlinear Dynamics

Blatt 1. Means of Qualitative Analysis

1. Dimensional Analysis of Flight Trajectories

(a) How does the initial velocity v_0 impact the distance W of a thrown oject (stone, ball, or shot) or a jump?

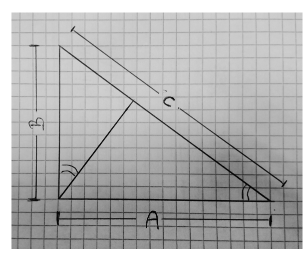
(b) How does the initial velocity v_0 depend on
the force F acting by the responsible muscle
the accelerated mass M, and
the distance L of the path where the acceleration is performed?

(c) Estimate the maximum distance
of throwing a stone of mass $m = 200\,\mathrm{g}$,
of a standing jump for a human and a grass hopper.

(d) Make an explicit analysis of standing jumps by exploring how their distance scales with the ratio of characteristic sizes (i.e., body length) of the jumper.

2. Pythagoras' Theorem

Have a look at the sketch to the right. The indicated angle will be denoted as β.



(a) We suggest that the area $\mathcal{F}_C$ of the full triangle may be written as

$$\mathcal{F}_c = C^\nu \, f(\beta)$$

Here, $f(\beta)$ is a dimensionless function of the angle β.

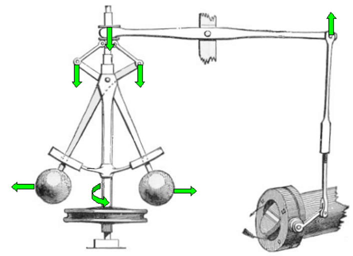
(b) The indicated height of the triangle divides its total area into two areas. They are right-angled and similar to the original one, except that their hytetenuses are of length A and B. What can you say about the areas $\mathcal{F}_A$ and $\mathcal{F}_B$ of these triangles?

(c) Give a proof of Pythagoras' theorem!

3. Bifurkation Analysis of the Rotational Governor

In the lecture we discussed the rotational governor:

We determined the bifurcation diagram, showing that a single heavy ball that can go left and right leaves the equilibrium position at the bottom and starts to rise when the rotation frequency exceeds a critical value $\omega_c = \sqrt{g/L}$. Here g is the gravitational acceleration, and L the length of the arm.



(a) How does the critical angle change, when one takes into account the fact that the governor has two balls with radius R?

(b) We derived the equations of motion for the deflection $\theta(t)$ of the balls, when there is no friction acting. What does change when there is a damping? Determine a dimensionless parameter δ that characterizes the damping.

(c) We had a look at the trajectories of the governor in phase space $(\theta, \dot{\theta})$. How does the diagram look like for different values of δ? Are there new bifurcations points? If yes: what do they refer to?