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Dissipative Motion in State Spaces

Dissipative motions in state spaces of commutative C*- and W*-Algebras are examined. A theory of the transformation of states by stochastic (linear) maps is developed and discussed. The applications include the characterisation of certain kinds of evolutionary processes in the state spaces corresponding to systems of classical statistical physics.

Es werden dissipative Bewegungen in Zustandsräumen kommutativer C*- und W*-Algebren untersucht. Dazu wird systematisch eine Transformationstheorie von Zuständen unter stochastischen (linearen) Abbildungen entwickelt und diskutiert. Die dabei gewonnenen Ergebnisse finden unter anderem Anwendung bei der Charakterisierung bestimmter Typen von Zeitentwicklungen im Zustandsraum von Systemen der klassischen Statistischen Physik.

On étudie ici les mouvements dissipatifs dans les espaces d'états des C*- et des W*-algèbres commutatives. On développe ensuite systematiquement une théorie transformationnelle des états relativement à des applications stochastiques (linéaires) que l'on soumet à discussion. Les résultats ainsi obtenus s'appliquent, entre autres, à la charactérisation de développements temporels de types déterminés dans les espaces d'états des systemes des la physique statistique classique.

Иследуются трансформации пространств состояний коммутативных C*- и W*-алгебр. Результаты используются для описание определённых типов временной эволюции в пространствах состояний классической статистической физики.

P R E F A C E

Let us consider a linear transformation of the n-dimensional real vector space with a distinguished base into itself, mapping each probability vector onto a probability vector. Then the transformation and the matrix of the transformation is said to be <u>stochastic</u>.

One could try to generalize this setting: Given a linear space, L , and a convex set, K , contained in that space, ask for the linear maps $T : L \longrightarrow L$ which transform K into K .
This, obviously, is much too general for obtaining any remarkable result. In asking for a good choice we look for convex sets, K , which, eventually, may serve as a space of <u>states</u> for a <u>physical</u> system, i.e. a set of states a la GIBBS and VON NEUMANN, i.e. mixtures, mixed states (in the terminology of many textbooks).
Then a stochastic map may be identified with (the result of) a <u>motion</u> of these states respecting its <u>convex</u> structure, i.e., respecting the performing of new GIBBSian mixtures out of GIBBSian mixtures.

Stochastic mappings of this kind may be viewed as describing the change of the states, the physical system admits, in the course of time. This change will be <u>dissipative</u> generally. It involves some elements of <u>irreversibility</u> if not, by chance, the motion is an automorphism of K . The motion's deviation from an automorphism, its irreversible or dissipative character, will be "measured" by "entropy-like" functionals.
We then shall pose an inverse problem. Given a path, a trajectory, in the state space, is there a semigroup of stochastic mappings generating it.

But how to decide with respect of what convex sets, K , we have good reasons for considering them candidates of state spaces of physical systems? There is, of course, no answer once and forever. But the "algebraic approach" to Quantum Theory and Statistics selects out and points toward state spaces of C^*- and W^*-algebras. (Clearly, there are further essential and unavoidable examples in the state spaces of certain op*-algebras of unbounded nature. The insight into the geometry of these state spaces, however, is not yet sufficient for our purposes.)

At this place we like to stress that about the first half of our text does <u>not</u> require knowledge of C^*-theory.

Choosing for K the state space, $S_{\underline{A}}$, of a C^*-algebra $\underline{A}$, every affine map of the state space is induced by a linear map. Thus we fall back to the study of linear transformations, $T : \underline{A}^* \longrightarrow \underline{A}^*$, mapping the state space into the state space.

Having reached this point we have to distinguish rather sharply commutative and arbitrary algebras: What appears trivial for the former unfolds a rich structure in the non-commutative case, and what is almost untractable at present in the latter situation can be solved for commutative algebras.

What we are knowing concerning the non-commutative case we have collected in our book "Stochasticity and Partial Order" [9] .
The state of affairs as seen by us concerning state spaces of commutative C^*-algebras we try to explain on the following pages.

As to "technical remarks" we mention: Within every § we start again in marking equations by (1), (2), ... We refer to equation (10) of § 1.2 by (1.2-10) . "Theorem 1.8.4." , as an example, refers to the 4th theorem within § 1.8. References are given either by their number of appearance in the reference table or by that together with the author's name. Sometimes we use the symbol $\forall$ abbreviating "for every ... it is ... " . In writing := instead of = we like to stress that we are defining the left-hand side by the right-hand side. The symbols ST and St are used simultaneously for sets of stochastic maps.

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C o n t e n t s