P2 - 4050

ВОПРОСЫ ТЕОРИИ ЭЛЕМЕНТАРНЫХ ЧАСТИЦ

Труды Международного семинара
по теории элементарных частиц
(Варна, Болгария, 6 - 19 мая 1968 г.)

TOPICS IN ELEMENTARY PARTICLE THEORY

Proceedings of the International Seminar
on Elementary Particle Theory

(Varna, Bulgaria, May 6 - 19, 1968)

SPECTRAL SUM RULES FOR TWO-POINT-FUNCTIONS AND FORMFACTORS OF TENSOR CURRENTS

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1. In many cases matrix elements of vector (axialvector) currents are dominated by 1 (1 , 0) mesons. To each current corresponds therefore one or a pair of mesons. In the case of the tensor mesons we consider a similar situation The nonet of 2^+- mesons is well known. Now the question arises : exists a SU(3)-plet of tensor currents which is simply related to the 2^+- mesons ? This problem has been discussed by several authors [1,2,3].

In the case I=0, Y=0 the tensor curfent is known . It is the energy stress tensor $T_{\mu\nu}$. The momentum operator is

$$P_\mu = \int T_{\mu 0}\, d^3x \qquad (I)$$

The discussion of mass formulas shows that the mass operator has in a good approximation the decomposition

$$m = \overset{(0)}{m} + \overset{(8)}{m} . \qquad (2)$$

Taking into account this property of the mass operator the energy tensor may have the following structure :

a) Only the SU(3) singlet $\overset{(0)}{T}_{\mu\nu}$ has the physical properties of the energy tensor . The octet term in eq. (2) is a consequence of the symmetry breaking in the mass multiplets.

b) The energy tensor has in general the structure

$$T_{\mu\nu} = \overset{(0)}{T}_{\mu\nu} + \overset{(8)}{T}_{\mu\nu\, I=0, Y=0} \qquad (3)$$

i.e. it consists of a SU(3) singlet and a SU(3) octet term with I=0, Y=0 .

In the first case the structure of the tensor octet (if it exists at all) is quite unclear. In the second case it should have the same structure as the energy tensor. Essential properties of the energy tensor are

$$T^{\mu\nu}{}_{,\nu} = 0 \qquad T^{\mu}_{\mu} \neq 0 . \qquad (4)$$

We postulate that these relations are valid for the full octet of tensor currents.
In the next two sections we discuss mainly the consequences of the assumption b) for possible spectral sum rules for two point functions and formfactors.

2. Following the derivation of spectral sum rules for the two point functions in the vector case (Das, Mathur, Okubo [4]) we obtain similar sum rules in the tensor case . The kinematical decomposition of the two point function

$$\Delta_{\mu\nu\alpha\beta}(q^2) = \int e^{iqx} T \langle 0 | T_{\mu\nu}(x) T_{\alpha\beta}(0) | 0 \rangle \tag{5}$$

is

$$\begin{aligned} \Delta_{\mu\nu\alpha\beta}(q^2) &= \sum_{i=1}^{5} f_i(q^2) I^i_{\mu\nu\alpha\beta} \\ &= \int \frac{dm^2}{m^2-q^2} \left[\rho_1 I_1 + \rho_2 I_2 + \frac{\rho_3}{m^2} I_3 + \frac{\rho_4}{m^2} I_4 + \frac{\rho_5}{m^4} \right]_{\mu\nu\alpha\beta} \end{aligned} \tag{6}$$

with the invariants

$$\begin{aligned} I_{1\,\mu\nu\alpha\beta} &= g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} \,, \quad I_{2\,\mu\nu\alpha\beta} = g_{\mu\nu} g_{\alpha\beta} \,, \\ I_{3\,\mu\nu\alpha\beta} &= (q_\mu q_\alpha g_{\nu\beta} + q_\nu q_\beta g_{\mu\alpha} + q_\nu q_\alpha g_{\mu\beta} + q_\mu q_\beta g_{\nu\alpha}) \,, \\ I_{4\,\mu\nu\alpha\beta} &= (q_\mu q_\nu g_{\alpha\beta} + q_\alpha q_\beta g_{\mu\nu}) \,, \quad I_{5\,\mu\nu\alpha\beta} = q_\mu q_\nu q_\alpha q_\beta \,, \end{aligned} \tag{7}$$

(we do not consider Schwinger terms [5]). The corresponding dispersion relations are

$$\begin{aligned} f_1 &= \int \frac{\rho_1}{m^2-q^2} dm^2 \,, \qquad f_2 = \int \frac{\rho_2}{m^2-q^2} dm^2 \,, \\ f_3 &= \int \frac{\rho_3}{m^2(m^2-q^2)} dm^2 \,, \qquad f_4 = \int \frac{\rho_4}{m^2(m^2-q^2)} dm^2 \,, \\ f_5 &= \int \frac{\rho_5}{m^4(m^2-q^2)} dm^2 \,. \end{aligned} \tag{8}$$

The condition $T^{\mu\nu}{}_{,\nu}=0$ gives relations between the spectral functions

$$\rho_1+\rho_3=0, \quad \rho_2+\rho_4=0, \quad 2\rho_3+\rho_4+\rho_5=0 \tag{9}$$

Now we are able to put in the physical assumptions. In the case a) we can not say anything about the structure of the SU(3)-plet of tensor currents so that we get here no result. In the other case we have assumed that the tensor currents form an octet . Using this symmetry asymptotically we demand for

$$F^{a\,b}_{\mu\nu\,\alpha\beta} = \Delta^{a}_{\mu\nu\alpha\beta} - U\Delta^{a}_{\mu\nu\alpha\beta} \tag{10}$$

$U|a\rangle = |b\rangle$ U: $SU(3)$ transformation , a: $SU(3)$ index

superconvergent sum rules

$$\int (\rho_i^a - \tilde{\rho}_i^a)\, dm^2 = 0 \qquad i = 1 \ldots 5 \tag{11a}$$

$$\int (\rho_i^a - \tilde{\rho}_i^a)\, m^{-2} dm^2 = 0 \qquad i = 3 \ldots 5 \tag{11b}$$

$$\int (\rho_i^a - \tilde{\rho}_i^a)\, m^{-4} dm^2 = 0 \qquad i = 5 \tag{11c}$$

Taking into account the coupling constants

$$\langle 0|\overset{(8)}{T}{}^{b}_{\alpha\beta}|0^+\rangle = \frac{1}{\sqrt{2\omega}}\, g_b \frac{1}{3}\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{m^2}\right) \tag{12}$$

$$\langle 0|\overset{(8)}{T}{}^{b}_{\alpha\beta}|2^+\rangle = \frac{1}{\sqrt{2\omega}}\, G_b\, \varepsilon_{\alpha\beta}$$

the saturation of these sum rules with 0^+- and 2^+-mesons gives

$$g_a^2 = g_b^2 \quad , \qquad G_a^2 = G_b^2 \quad , \tag{13a}$$

$$\frac{g_a^2}{m_a^2} = \frac{g_b^2}{m_b^2} \quad , \qquad \frac{G_a^2}{M_a^2} = \frac{G_b^2}{M_b^2} \quad , \tag{13b}$$

$$\frac{G_a^2}{M_a^4} - \frac{G_b^2}{M_b^4} = -\frac{1}{18}\left(\frac{g_a^2}{m_a^4} - \frac{g_b^2}{m_b^4}\right) . \tag{13c}$$

The solution of this system of equations is the symmetry case

$$g_a = g_b \quad , \quad G_a = G_b \quad , \quad m_a = m_b \quad , \quad M_a = M_b \tag{14}$$

To get better solutions we have to omit badly convergent sum rules (Eq. (9a))or to combine these sum rules to take symmetry breaking effects of first order into account (Das, Mathur, Okubo). So we obtain again good sum rules . Therefore we use

$$\int dm^2 \left(\rho_i^3 + 3\rho_i^2 - 4\rho_i^4 \right) = 0 \qquad i = 1 \ldots 5 \tag{I5}$$

or

$$\begin{aligned} G_\varepsilon^2 + G_{\varepsilon'}^2 &= \tfrac{1}{3}\left(4 G_{K^{**}}^2 - G_{A_2}^2\right), \\ g_\eta^2 + g_{\eta'}^2 &= \tfrac{1}{3}\left(4 g_K^2 - g_\pi^2\right) \end{aligned} \tag{I6}$$

instead of (11a) and (13a) . The general solution is

$$G_{K^{**}}^2 = \frac{M_{K^{**}}^2}{M_{A_2}^2} G_{A_2}^2 \quad , \quad g_K^2 = \frac{m_K^2}{m_\pi^2} g_\pi^2 \tag{I7}$$

$$G_\varepsilon^2 = \frac{M_\varepsilon^2 (3M_{\varepsilon'}^2 - 4M_{K^{**}}^2 + M_{A_2}^2)}{3M_{A_2}^2 (M_{\varepsilon'}^2 - M_\varepsilon^2)} G_{A_2}^2 \; , \; g_\eta^2 = \frac{m_\eta^2 (3m_{\eta'}^2 - 4m_K^2 + m_\pi^2)}{3m_\pi^2 (m_{\eta'}^2 - m_\eta^2)} g_\pi^2 \; ,$$

$$G_{\varepsilon'}^2 = \frac{M_{\varepsilon'}^2 (3M_\varepsilon^2 - 4M_{K^{**}}^2 + M_{A_2}^2)}{3M_{A_2}^2 (M_\varepsilon^2 - M_{\varepsilon'}^2)} G_{A_2}^2 \; , \; g_{\eta'}^2 = \frac{m_{\eta'}^2 (3m_\eta^2 - 4m_K^2 + m_\pi^2)}{3m_\pi^2 (m_\eta^2 - m_{\eta'}^2)} g_\pi^2 \; ,$$

$$\left(\frac{1}{M_{A_2}^4} - \frac{1}{M_{A_2}^2 M_{K^{**}}^2} \right) G_{A_2}^2 = -\frac{1}{18} \left(\frac{1}{m_\pi^4} - \frac{1}{m_\pi^2 m_K^2} \right) g_\pi^2 \; , \tag{I8}$$

$$\frac{\frac{1}{m_\pi^2} - \frac{1}{m_K^2}}{\frac{1}{m_\pi^2} - \frac{1}{m_\eta^2 m_{\eta'}^2}\left(m_\eta^2 + m_{\eta'}^2 - \frac{1}{3}(4m_K^2 - m_\pi^2)\right)} = \frac{\frac{1}{M_{A_2}^2} - \frac{1}{M_{K^{**}}^2}}{\frac{1}{M_{A_2}^2} - \frac{1}{M_\varepsilon^2}\frac{1}{M_{\varepsilon'}^2}\left(M_\varepsilon^2 + M_{\varepsilon'}^2 - \frac{1}{3}(4M_{K^{**}}^2 - M_{A_2}^2)\right)} .$$

The results are relations between coupling constants and a mass formula. Because of the very unclear situation of the 0^+- mesons it is useless to discuss this formula further. We remark, that it is quite arbitrary to introduce two 0^+- mesons with I=0, Y=0 and not of principal importance for the derivation of a mass formula in this manner . A similar treatment is given by Joshi and Pande [2] . These authors make different assumptions about the structure of the tensor current octet (they assume $T^{\mu}_{\mu} = 0$, $T^{\mu\nu}{}_{,\nu} \neq 0$). So they saturate the sum rules with 0^+ , 1^- , 2^+ mesons (these 0^+- mesons belong to the divergence part of $T^{\mu\nu}$) and obtain a mass formula connecting these mesons.

3. In the dispersion relations for formfactors we make the approximations

This means , the spectral functions are saturated with 0^+ and 2^+ - mesons only . Of special interest are formfactors between identical particles because in this case their value at vanishing momentum transfer can be connected with the experimentally observed masses.
For matrix elements between 0^- -mesons and 0^+ - mesons (we choose these mesons for simplicity) the kinematical decompositions are

(19)

0^-- mesons:

$$\langle k_2 - | T_{\mu\nu} | k_1 - \rangle = \frac{1}{\sqrt{4\omega_1\omega_2}} \left[(k_1+k_2)_\mu (k_1+k_2)_\nu a_- + \left(g_{\mu\nu} - \frac{(k_1-k_2)_\nu (k_1-k_2)_\mu}{(k_1-k_2)^2} \right) b_- \right]$$

0^+- mesons: (20)

$$\langle k_2 + | T_{\mu\nu} | k_1 + \rangle = \frac{1}{\sqrt{4\omega_1\omega_2}} \left[(k_1+k_2)_\mu (k_1+k_2)_\nu a_+ + \left(g_{\mu\nu} - \frac{(k_1-k_2)_\nu (k_1-k_2)_\mu}{(k_1-k_2)^2} \right) b_+ \right]$$

The assumption

$$\langle k_1 | T_{\mu 0}(0) | k_1 \rangle = \lambda k_{1\mu} \tag{21}$$

leads to

$$a(0) = \frac{\lambda}{2} , \quad b(0) = 0 . \tag{22}$$

With the help of the SU(3) symmetry properties of the tensor currents and the particles in the in-and out-states of the matrix elements we may construct asymptotically better convergent amplitudes .

In the case a) (the energy tensor is a SU(3) singlet)
we consider for the amplitude a of the difference of suitable matrix elements unsubtracted or superconvergent dispersion relations . As result we get the usual SU(3) mixing theory. Unfortunately the mixing angle appears as function of unknown coupling constants.

In the case b) the energy tensor is $T_{\mu\nu} = \overset{(0)}{T}_{\mu\nu} + \overset{(8)}{T}_{\mu\nu}\,_{I=0,Y=0}$
Therefore the corresponding values of the invariant amplitudes at vanishing momentum transfer are

$$\overset{(0)}{a}_- = \frac{1}{2}\frac{\overset{(0)}{\mu}}{\mu}\ , \quad \overset{(8)}{a}_- = \frac{1}{2}\frac{\overset{(8)}{\mu}}{\mu}\ , \quad b_\pm = 0\ , \quad \mu\text{ - mass of the } 0^- \text{mesons}$$
$$\overset{(0)}{a}_+ = \frac{1}{2}\frac{\overset{(0)}{m}}{m}\ , \quad \overset{(8)}{a}_+ = \frac{1}{2}\frac{\overset{(8)}{m}}{m} \tag{23}$$

Superconvergent dispersion relations for $\langle\pi|\overset{(0)}{T}_{\mu\nu}|\pi\rangle - U\langle\pi|\overset{(0)}{T}_{\mu\nu}|\pi\rangle$
$\langle\pi|\overset{(8)}{T}_{\mu\nu}|\pi\rangle - U\langle\pi|\overset{(8)}{T}_{\mu\nu}|\pi\rangle$, $\langle 0|\overset{(0)}{T}_{\mu\nu}\overset{(8)}{T}_{\mu\nu}|0\rangle$ lead us to the SU(3) mixing theory with $M_f = M_{f'}$. For this reason we use simple dispersion relations . From $\langle k_2 a-|\overset{(0)}{T}_{\mu\nu}|k_1 a-\rangle - U\langle k_2 a-|\overset{(0)}{T}_{\mu\nu}|k_1 a-\rangle$ for 0^- - mesons we get

$$\overset{(0)}{\mu}\left(\frac{1}{\mu_a} - \frac{1}{\mu_b}\right) = (g_{aaf} - g_{bbf})\frac{\sigma_f}{M_f^2} + (g_{aaf'} - g_{bbf'})\frac{\sigma_{f'}}{M_{f'}^2} \tag{24}$$

$$U|a\rangle = |b\rangle \tag{25}$$

The definition of coupling constants are

$$\langle k a-|j_{a^-}|f\rangle = \frac{1}{\sqrt{2\omega}}\frac{1}{\sqrt{2\omega'}}\epsilon^{\mu\nu}k_\mu k_\nu g_{aaf}\ , \quad \langle k a-|j_{a^-}|\eta\rangle = \frac{1}{\sqrt{2\omega 2\omega'}}g_{aa\eta}\ ,$$

$$\langle f|\overset{(0)}{T}_{\mu\nu}|0\rangle = \epsilon_{\mu\nu}\sigma_f\frac{1}{\sqrt{2\omega}}\ , \quad \langle\eta|\overset{(0)}{T}_{\mu\nu}|0\rangle = \frac{1}{\sqrt{2\omega}}\frac{1}{3}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\sigma_\eta\ .$$

The considerations for 0^+ - mesons give

$$\overset{(0)}{m}\left(\frac{1}{m_a} - \frac{1}{m_b}\right) = (\gamma_{aaf} - \gamma_{bbf})\frac{\sigma_f}{M_f^2} + (\gamma_{aaf'} - \gamma_{bbf'})\frac{\sigma_{f'}}{M_{f'}^2} \tag{26}$$

with similar defined coupling constants.

Conditions for the existence of a solution for σ_f and $\sigma_{f'}$ of the equations (24) and (26) are

$$\begin{vmatrix} \mu^{(o)}\left(\frac{1}{\mu_a}-\frac{1}{\mu_b}\right) & g_{aaf}-g_{bbf} & g_{aaf'}-g_{bbf'} \\ \mu^{(o)}\left(\frac{1}{\mu_a}-\frac{1}{\mu_c}\right) & g_{aaf}-g_{ccf} & g_{aaf'}-g_{ccf'} \\ m^{(o)}\left(\frac{1}{m_a}-\frac{1}{m_d}\right) & \gamma_{aaf}-\gamma_{ddf} & \gamma_{aaf'}-\gamma_{ddf'} \end{vmatrix} = 0 \tag{27}$$

Further relations can be obtained if one considers $\langle a|T^{(8)}_{\mu\nu}|a\rangle - u\langle a|T^{(8)}_{\mu\nu}|a\rangle$ but all relations are corrections to the symmetry case involving many coupling constants, so that it is hard to prove it experimentally.

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