Робашик Д., Ульманн А. E-2771

Вычисление коэффициентов массовых формул

В работе вычисляются коэффициенты массовых формул SU(3) и SU(6) для ортогональных систем операторов и рассматриваются некоторые отношения между ними.

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Calculation of the Coefficients of the Mass Operators

For normed orthogonal systems of operators the coefficients for SU(3) and SU(6) mass formulas are calculated and relations between these coefficients are considered.

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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

D. Robaschik, A. Uhlmann

CALCULATION OF THE COEFFICIENTS OF THE MASS OPERATORS

1966

1. Using the experimental data (A.H. Rosenfeld et al.[1]) we calculate the coefficients of the SU(6) mass formulas (Beg and Singh[2]) for the 35- and 56-plet. The coefficients for the corresponding SU(3) subrepresentations have been obtained too. The mass operator, as it is a tensor operator acting on the representation n, consists of the $I = J = Y = 0$ terms of the self abjoint irreducible representations contained in the direct product $n \times n^*$.

$$m = \sum_{\substack{J= \; J=Y=0 \\ C n \times n^*}} a_i m_i$$

To calculate the coefficients a_i we use for the tensor operators the invariant scalar product

$$(t_1, t_2) = \frac{Tr\,(t_1\; t_2)}{n}$$

which allows us to obtain

$$a_i = \frac{Tr(m_{exp}\; m_i)}{n}$$

In tables we collect:

Table I Tensoroperators for SU(3) mass operators
Table II Coefficients of the SU(3) mass operators
Table III Tensoroperators for SU(6) mass operators
Table IV Coefficients of the SU(6) mass operators
Table V Connection between SU(3) and SU(6) coefficients (56-plet)
Table VI Connection between SU(3) and SU(6) coefficients (35-plet)

2. The table II of the SU(3) coefficients (J. Ginibre[3]) shows that for both, the linear and squared masses, the 27-contributions are small and vary by going from one octet to another quite arbitrarely. Therefore the magnitude of the 27-coefficients seems to be not a criterion for the preference of either linear or squared mass formulas. However, the squared meson masses show the nice wellknown regularity

$$a_8(0^-) = -184\sqrt{\frac{2}{5}}\,, \qquad a_8(1^-) = -183\sqrt{\frac{2}{5}}\,, \qquad a_8(2^+) = -185\sqrt{\frac{2}{5}} \qquad (10^3(\mathrm{MeV})^2)$$

In ref.[4] it is assumed that the mesons $\frac{A_1 + \sqrt{2}\,B}{\sqrt{3}}$ and D could be the π and η particle of a new 1^+ octet. Assuming the a_{27} to be small also here, we may conclude from the well satisfied rule (squared masses)

$$2f' + f + A_1 + 2B = 3D + 3A_2$$

that we have additionally

$$a_8(1^+) \simeq -184\sqrt{\frac{2}{5}}\,.$$

It is therefore not unreasonable to assume a universal octet contribution a_8 for the squared meson masses.

Dealing with the 0^-, 1^- and 2^+ mesons only the coefficient a_1 may be represented fairly well by

$$a_1 = 168 + 287\,J(J+1),\ (m^2) \qquad (10^3(\mathrm{MeV})^2)$$

which allows to write down a mass formula including these mesons (Barut[5]).

For the meson octet we have calculated the relations between the linear mass and squared mass coefficients (ℓ_1 and s_1)

$$s_1 = \ell_1^2 + \ell_8^2 + \ell_{27}^2$$
$$s_8 = 2\ell_1\ell_8 + \frac{3}{10}\sqrt{\frac{8}{5}}\,\ell_8^2 - \frac{4}{5}\sqrt{\frac{8}{5}}\,\ell_{27}^2 + \frac{6}{5}\sqrt{\frac{3}{5}}\,\ell_8\ell_{27}$$
$$s_{27} = 2\ell_1\ell_{27} - \frac{8}{5}\sqrt{\frac{8}{5}}\,\ell_8\ell_{27} - \frac{3}{5}\sqrt{\frac{3}{5}}\,\ell_8^2 - \frac{26}{15}\sqrt{\frac{3}{5}}\,\ell_{27}^2$$

These relations point out that the nonvanishing of the 27-plet coefficient and the negative sign of the octet coefficient allows sum rules both in m and m^2.

3. The SU(6) coefficients (Harari and Rashid[6], Bisiacchi and Fronsdal[7]) are collected in Table IV. It turns out that for the mesons the squared mass formula and for baryons the linear mass formula seems to be more preferable. The physical meson states are assumed to be given by the u-chain. This is reflected by the relation

$$5a_{189_1} + 2\sqrt{2}\,a_{189_8} - 3\sqrt{3}\,a_{189_{27}} = -\sqrt{35}\,a_{405_1} - 4a_{405_8} + 3a_{405_{27}}$$

From the condition $a_8(0^-) = a_8(1^-)$ follows

$$\sqrt{2}\,a_{405_8} = a_{189_8} \quad (m^2)$$

which is not very well satisfied. Building up the coefficients a_{189_8} and a_{405_8} (Table VI) one has to subtract two large quantities in a different

manner and for this reason the small deviations of the octet coefficients (1%) becomes important. Bisiacchi and Fronsdal have given mass formulas for m^{-2} and derived the relations (in our notation)

$$\sqrt{\frac{7}{5}}\,a_{405_1} = a_{189_1} \qquad \sqrt{2}\,a_{405_8} = a_{189_8} \qquad \sqrt{3}\,a_{405_{27}} = a_{189_{27}} \qquad (m^{-2})$$

which are very well satisfied by the experimental data. The corresponding relations for the coefficients of the m^2-formula are unfortunately very complicated.

References

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2. M.A.B.Beg and V.Singh., Phys. Rev. Letters, 13, 418 (1 964).
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5. A.O. Barut. Trieste lecture, 1965.
6. H. Harari and M.A. Rashid. Phys. Rev., 143 , 1354 (1966).
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Table I

Tensor operators for SU(3) mass operators

$$t_1 = 1$$

$$t_{8a} = Y$$

$$t_{8s} = J(J+1) - \frac{Y^2}{4} - \frac{1}{6} C_2^{(3)}$$

$$t_{27} = \frac{4}{9} J(J+1) + Y^2 - \frac{1}{6} C_2^{(3)}$$

$$t_{64} = \frac{2}{3}\left(\frac{1}{3} C_3^{(3)} - \frac{1}{2} C_2^{(3)}\right) + 5Y^2(1+Y^2) + \frac{5}{3} Y^2\left(J(J+1) - \frac{3}{4}Y^2 - C_2^{(3)}\right)$$

To get normed operators with $(m_i, m_i) = 1$ we form

$$m_i = \alpha_i t_i$$

where α_i

	α_i octet	α_i decuplet
t_1	1	1
t_{8a}	$\sqrt{2}$	1
t_{8s}	$2\sqrt{\frac{2}{5}}$	–
t_{27}	$3\sqrt{\frac{3}{5}}$	$\frac{3}{2}\sqrt{\frac{3}{7}}$
t_{64}	–	$\sqrt{\frac{1}{14}}$

Table II

Coefficients of the SU(3) mass formula

$$m = a_1 m_1 + a_{8a} m_{8a} + a_{8s} m_{8s} + a_{27} m_{27} + a_{64} m_{64}$$

	particles	a_1	a_{8s}	a_{8a}	a_{27}	a_{64}
linear	0^- mesons	368,3	$-281{,}6\sqrt{\frac{2}{5}}$	–	$12{,}3\sqrt{\frac{3}{5}}$	–
	1^- mesons	850	$-107\sqrt{\frac{2}{5}}$	–	$-2\sqrt{\frac{3}{5}}$	–
	2^+ mesons	1376	$-64\sqrt{\frac{2}{5}}$	–	$5\sqrt{\frac{3}{5}}$	–
square	0^- mesons	167,7	$-184\sqrt{\frac{2}{5}}$	–	$7{,}6\sqrt{\frac{3}{5}}$	–
	1^- mesons	729	$-183\sqrt{\frac{2}{5}}$	–	$-13\sqrt{\frac{3}{5}}$	–
	2^+ mesons	1904	$-185\sqrt{\frac{2}{5}}$	–	$16\sqrt{\frac{3}{5}}$	–
linear	$\frac{1}{2}^+$ baryons	1150,2	$51{,}8\sqrt{\frac{2}{5}}$	$-94{,}5\sqrt{2}$	$-3{,}3\sqrt{\frac{3}{5}}$	–
	$\frac{3}{2}^+$ baryons	1383	–	-147	$0{,}6\sqrt{\frac{3}{7}}$	$0{,}6\sqrt{\frac{1}{14}}$
square	$\frac{1}{2}^+$ baryons	1762	$103\sqrt{\frac{2}{5}}$	$-214\sqrt{2}$	$11\sqrt{\frac{3}{5}}$	–
	$\frac{3}{2}^+$ baryons	1934	–	-418	$29\sqrt{\frac{3}{7}}$	$-4\sqrt{\frac{1}{14}}$

units: MeV
10^3 (MeV)^2

Table III

Tensor operators for SU(6) mass operators

$$t_1 = 1$$

$$t_{35_a} = Y$$

$$t_{35_s} = \frac{1}{6} C_2^{(6)} + \frac{1}{2}\left(2S(S+1) - C_2^{(4)} + \frac{Y^2}{4}\right)$$

$$t_{189_1} = -\frac{1}{5} C_2^{(6)} - \left(2J(J+1) - C_2^{(3)}\right)$$

$$t_{189_8} = \frac{1}{24} C_2^{(6)} - \frac{1}{6}\left(2J(J+1) - C_2^{(3)}\right) - \left(J(J+1) - \frac{Y^2}{4} - N(N+1) - S(S+1)\right) + \frac{1}{8}\left(2S(S+1) - C_2^{(4)} + \frac{Y^2}{4}\right)$$

$$t_{189_{27}} = \frac{3}{10} C_2^{(6)} + \frac{3}{10}\left(2J(J+1) - C_2^{(3)}\right) + \frac{4}{5}\left(J(J+1) - \frac{Y^2}{4} - N(N+1) - S(S+1)\right) + \frac{4}{10}\left(2S(S+1) - C_2^{(4)} + \frac{Y^2}{4}\right) - 4S(S+1) + Y^2$$

$$t_{405_1} = -\frac{5}{4} C_2^{(6)} + \left(2J(J+1) + C_2^{(3)}\right)$$

$$t_{405_8} = -\frac{7}{48} C_2^{(6)} + \frac{1}{6}\left(2J(J+1) + C_2^{(3)}\right) - \left(J(J+1) - \frac{Y^2}{4} + N(N+1) - S(S+1)\right) - \frac{7}{16}\left(2S(S+1) - C_2^{(4)} + \frac{Y^2}{4}\right)$$

$$t_{405_{27}} = -\frac{3}{10} C_2^{(6)} - \frac{3}{10}\left(2J(J+1) + C_2^{(3)}\right) + \frac{4}{5}\left(J(J+1) + N(N+1) - S(S+1) - \frac{Y^2}{4}\right) - \frac{4}{10}\left(2S(S+1) - C_2^{(4)} + \frac{Y^2}{4}\right) + 4S(S+1) + 3Y^2$$

$$\left.\begin{aligned} t_{2695_8} &= \frac{1}{5}\sqrt{\frac{7}{3}}\left[\left(2J(J+1) - \frac{13}{2}\right)Y + 2\left(J(J+1) - \frac{15}{4}\right)\left(J(J+1) - \frac{Y^2}{4} - 1\right)\right]\cancel{\sqrt{\frac{7}{3}}\ \frac{1}{5}} \\ t_{2695_{27}} &= \frac{1}{40}\left(60J(J+1) - 213\right)\left(\frac{4}{9}J(J+1) + Y^2 - \frac{1}{6}C_2^{(3)}\right) \\ t_{2695_{64}} &= \frac{1}{6\sqrt{35}}\left(J(J+1) - \frac{3}{4}\right)\left[4 + 5Y^2(1+Y^2) + \frac{5}{3}Y^2\left(J(J+1) - \frac{3}{4}Y^2 - C_2^{(3)}\right)\right] \end{aligned}\right\}$$

these operators are already simplified and normed for the 56-plet

Table IV

Normed operators $m_i = \alpha_i\, t_i$

α for	t_1	t_{35_a}	t_{35_s}	t_{189_1}	t_{189_8}	$t_{189_{27}}$	t_{405_1}	t_{405_8}	$t_{405_{27}}$
35-plet	1		$\frac{\sqrt{70}}{16}$	$\frac{5}{24}\sqrt{\frac{7}{2}}$	$\frac{\sqrt{7}}{2}$	$\frac{5}{8}\sqrt{\frac{14}{3}}$	$\frac{7}{24}\sqrt{\frac{5}{2}}$	$\sqrt{\frac{7}{2}}$	$\frac{5}{24}\sqrt{14}$
56-plet	1	$\frac{1}{2}\sqrt{\frac{14}{3}}$	-	-	-	-	$\frac{7}{24}\sqrt{\frac{2}{5}}$	$\frac{1}{5}\sqrt{14}$	$\frac{1}{2}\sqrt{\frac{7}{17}}$

Coefficients of the SU(6) mass formula

$$m = \sum a_i\, m_i$$

	mesons lin.	mesons squar.	baryons lin.	baryons squar.
a_1	741	603	1316	1765
a_{35a}	-	-	-142,1	-389
a_{35s}	100	138	-	-
a_{189_1}	-157	-186	-	—
a_{189_8}	41,8	1,5	-	-
$a_{189_{27}}$	-3,5	-8,6	-	-
a_{405_1}	128,8	147,6	104,3	267
a_{405_8}	-26,7	0,8	-22,8	-24,4
$a_{405_{27}}$	3,3	-1,7	0	17
a_{2695_8}	-	-	-4,3	-20,5
$a_{2695_{27}}$	-	-	-1,4	-0,1
$a_{2695_{64}}$	-	-	-0,1	-0,8

units : MeV or 10^3 (MeV)^2

Table V

Connection between SU(3) and SU(6) coefficients (56-plet)

$$a_1 = \frac{1}{7}\left(2b_1 + 5c_1\right)$$

$$a_{35} = \frac{1}{\sqrt{12}}\left(\sqrt{12}\, b_{8a} + 5\, c_{8a}\right)$$

$$a_{405_1} = \sqrt{\frac{10}{7}}\left(-b_1 + c_1\right)$$

$$a_{405_8} = \frac{1}{5\sqrt{14}}\left(-4\sqrt{\frac{5}{2}}\, b_{8s} + \frac{10}{\sqrt{12}}\, b_{8a} - 5\, c_{8a}\right)$$

$$a_{405_{27}} = \frac{6}{5}\,\frac{1}{7\sqrt{14}}\left(\sqrt{\frac{5}{3}}\, b_{27} + \sqrt{\frac{7}{3}}\, 5\, c_{27}\right)$$

$$a_{2695_8} = \frac{1}{5\sqrt{21}}\left(-6\sqrt{\frac{5}{2}}\, b_{8s} - \frac{10}{\sqrt{12}}\, b_{8a} + 5\, c_{8a}\right)$$

$$a_{2695_{27}} = -\frac{2}{5}\sqrt{\frac{5}{3}}\, b_{27} + \frac{1}{7}\sqrt{\frac{7}{3}}\, c_{27}$$

$$a_{2695_{64}} = \frac{5}{7}\sqrt{\frac{14}{10}}\, c_{64}$$

Notation :

a 56-plet of SU(6)

b $1/2^+$ octet of SU(3)

c $3/2^+$ decoplet of SU(3)

Table VI

Connection between SU(3) and SU(6) coefficients (35-plet)

$$a_1 = \frac{1}{35}\left(8b_1 + 24c_1 + 3d_1\right)$$

$$a_{35_8} = \frac{1}{\sqrt{70}}\left(-\sqrt{\frac{5}{2}}\left(b_8 + 3c_8\right) + d\right)$$

$$a_{189_1} = \frac{2}{5\sqrt{14}}\left(3b_1 - c_1 - 2d_1\right)$$

$$a_{189_8} = \frac{1}{5\sqrt{7}}\left(-\sqrt{\frac{5}{2}}\left(3b_8 + c_8\right) - \frac{5}{3}d\right)$$

$$a_{189_{27}} = \sqrt{\frac{2}{35}}\left(-b_{27} + 3c_{27}\right)$$

$$a_{405_1} = \frac{2}{7\sqrt{10}}\left(-3b_1 + 5c_1 - 2d_1\right)$$

$$a_{405_8} = \frac{1}{5\sqrt{14}}\left(\sqrt{\frac{5}{2}}\left(3b_8 - 7c_8\right) - \frac{5}{3}d\right)$$

$$a_{405_{27}} = \sqrt{\frac{6}{35}}\left(b_{27} + c_{27}\right)$$

Notation :

a 35-plet of SU(6)

b 0^- octet of SU(3)

c 1^- octet of SU(3)

$$d_1 = \frac{2\omega + \phi}{3}$$

$$d = \phi - \omega$$