Spectral Triples of Holonomy Loops

- towards a semi-classical analysis

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Content

- **Motivation**
  - Noncommutative Geometry and the Standard Model of Particle Physics (Alain Connes).
  - Does the quantization procedure translate into NCG?
  - Ashtekar and loop variables; Loop Quantum Gravity.

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- **The Model**
  - A spectral triple over a space of connections.
  - The triple is based on a ordered system of finite graphs.
- **Physical Interpretation**
  - Space of connections.
  - The Poisson structure of General Relativity.
  - Semi-classical states and a classical Dirac operator.

Noncommutative Geometry

- **A Spectral Triple** is a collection (B, H, D):
  a $*$-algebra B represented as operator in the Hilbert space H; a self-adjoint, unbounded Dirac operator D with compact resolvent, acting in H such that $[D, b]$ is bounded $\forall b \in B$.

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- The Standard Model of Particle Physics (SM):
[Connes, Lott, Chamseddine, Marcolli, ...]
  - $B = C^\infty(M) \otimes B_F$, *almost commutative algebra*
  $B_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$,
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  - Spectral action principle $\rightarrow$ classical action of SM + GR.

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- Formulation of the Standard Model coupled to General Relativity as a single gravitational theory. The Standard Model emerges from a modification of space-time geometry:

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Our goal

To construct a framework which combines Noncommutative Geometry with elements of Quantum Gravity.

Gravity and Ashtekar variables

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- Poisson brackets

$$\{A^i_j(x), E^k_l(y)\} = \delta^i_l \delta^k_j \delta(x - y)$$

- $+$ Constraints related to the symmetries of GR (spatial diffeomorphism, Hamilton, Gauss)

- Shift focus from connections to holonomy and flux variables

$$h_L(A) = \mathrm{Hol}(L, A)$$

L loop on Σ

$$F^a_S(E) = \int_S \epsilon^i{}_{jk} E^a_i dx^j dx^k$$

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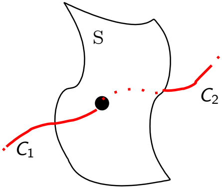
$$\{F^a_S(E), h_C(A)\} = \pm h_{C_1}(A) \tau^a h_{C_2}(A)$$



τ^a generator of $\mathfrak{su}(2)$, $C = C_1 C_2$ are curves in Σ.

Our Project

- **Aim:** To construct a spectral triple that involves an algebra of holonomy loops, i.e. functions on the space of smooth connections, denoted $\mathcal{A}$:

$$L : \nabla \to \text{Hol}(\nabla, L) \in M_n(\mathbb{C})$$

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- This project is inspired by Loop Quantum Gravity (LQG) - the construction of the Hilbert space [Ashtekar-Lewandowski].

The construction

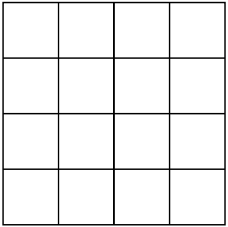
A single lattice

- Let Γ be a finite 3-dim finite lattice with edges $\{\epsilon_i\}$ and vertices $\{v_i\}$

$$\epsilon_j : \{0,1\} \to \{v_i\}$$

- Assign to each edge ϵ_i a group element $g_i \in G$.

$$\nabla : \epsilon_i \to g_i$$



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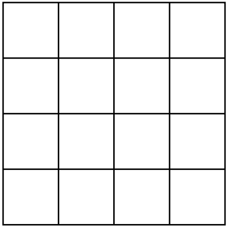

G is a compact Lie-group. The space of such maps is denoted $\mathcal{A}_\Gamma$. Notice:

$$\mathcal{A}_\Gamma \simeq G^n \quad \text{because} \quad \mathcal{A}_\Gamma \ni \nabla \rightarrow (\nabla(\epsilon_1), \ldots, \nabla(\epsilon_n)) \in G^n$$

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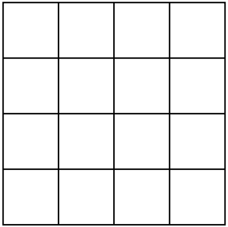

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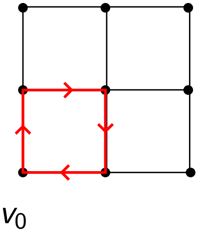
- The space $\mathcal{A}_\Gamma$ is a coarse-grained approximation of $\mathcal{A}$.

- **Algebra:** A loop L is a finite sequence of edges $L = \{\epsilon_{i_1}, \epsilon_{i_2}, \ldots, \epsilon_{i_n}\}$ running in Γ (choose basepoint v_0). Discard trivial backtracking.



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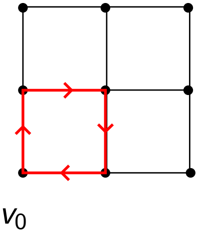

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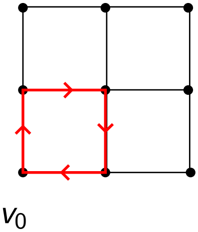

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  - Inversion: $L^* = \{\epsilon^*_{i_n}, \ldots, \epsilon^*_{i_j}, \ldots, \epsilon^*_{i_1}\}$

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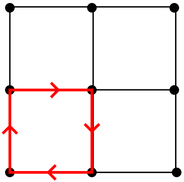

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    with $\quad \epsilon^*_j(\tau) = \epsilon_j(1-\tau) \ , \quad \tau \in \{0,1\}$
  - Consider formal, finite series of loops

$$a = \sum_i a_i L_i \ , \quad a_i \in \mathbb{C}$$

- The product between two elements a and b is defined

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- These elements have a natural norm

$$\|a\| = \sup_{\nabla \in \mathcal{A}_\Gamma} \| \sum a_i \nabla(L_i) \|_G$$

where the norm on the rhs is the matrix norm in G. The closure of the $\star$-algebra of loops with respect to this norm is a $C^\star$-algebra. We denote this loop algebra by $\mathcal{B}$.

- **Hilbert space:** There is the (somewhat) natural Hilbert space

$$\mathcal{H} = L^2(G^n, Cl(T^*G^n) \otimes M_l(\mathbb{C}))$$

involving the Clifford bundle over G^n (l size of rep. of G). L^2 is with respect to the Haar measure on G^n.

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  - The two factors in $\mathcal{H}$ (Clifford bundle, matrix factor) are needed:

    - to define a Dirac type operator,

    - to have a representation of the algebra of holonomy loops.
  - The loop algebra $\mathcal{B}$ has a natural representation on $\mathcal{H}$

$$f_L \cdot \psi(\nabla) = (1 \otimes \nabla(L)) \cdot \psi(\nabla)\,, \quad \psi \in \mathcal{H}$$

where the first factor acts on the Clifford-part of the Hilbert space and the second factor acts by matrix multiplication on the matrix part of the Hilbert space. Also

$$\nabla(L) = \nabla(\epsilon_{i_1}) \cdot \nabla(\epsilon_{i_2}) \cdot \ldots \cdot \nabla(\epsilon_{i_n})$$

with $L = \{\epsilon_{i_1}, \epsilon_{i_2}, \ldots, \epsilon_{i_n}\}$.

- **Dirac operator:** Choose a Dirac operator D on G^n (choose a metric on G and use Levi-Civita) and obtain

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- a candidate for a **spectral triple**

$$(\mathcal{B}, D, \mathcal{H})_\Gamma ,$$

on the level of the lattice Γ.

A family of lattices

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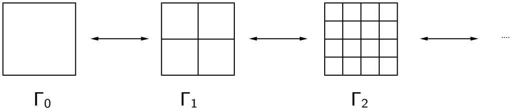
- Consider a system of nested lattices

$$\Gamma_0 \longrightarrow \Gamma_1 \longrightarrow \Gamma_2 \longrightarrow \ldots$$

with Γ_i a subdivision of Γ_{i-1}



On the level of the associated manifolds $\mathcal{A}_{\Gamma_i}$ this gives rise to projections

$$G^{n_0} \xleftarrow{P_{10}} G^{n_1} \xleftarrow{P_{21}} G^{n_2} \xleftarrow{P_{32}} G^{n_3} \xleftarrow{P_{43}} \ldots$$

- Consider next a corresponding system of spectral triples

$$(\mathcal{B}, D, \mathcal{H})_{\Gamma_0} \leftrightarrow (\mathcal{B}, D, \mathcal{H})_{\Gamma_1} \leftrightarrow (\mathcal{B}, D, \mathcal{H})_{\Gamma_2} \leftrightarrow \dots$$

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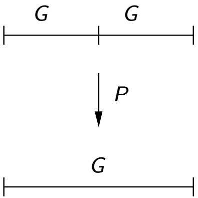
  with the requirement that the spectral triples are compatible with the projections/embeddings between graphs and Hilbert spaces.
- For the Hilbert space compatibility is easily obtained and compatibility for the algebra is clear.

- For the Dirac operator, the problem boils down to the simple case



$$P : G^2 \to G \ , \quad (g_1, g_2) \to g_1 \cdot g_2$$

corresponding to the compatibility condition

$$P^*(D_1 v)(g_1, g_2) = D_2(P^* v)(g_1, g_2) \ , \quad v \in L^2(G, Cl(T^*G))$$

where D_1 is a Dirac operator on G and D_2 is a Dirac operator on G^2.

- We have found several ways to solve this consistency problem. One solution is to consider the change of variables:

$$(g_1, g_2) \to (g_1 \cdot g_2, g_2) \equiv (g_1', g_2')$$

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- A Dirac operator compatible with this structure map is of the form

$$D = D_1 + aD_2$$

where a is a real parameter and D_1, D_2 are Dirac operators on G

$$D_j(\xi) = \sum_i e_i \cdot d_{e_i}(\xi) \qquad \xi \in L^2(G, Cl(TG))$$

where e_i are left-translated vectorfields.

- After repeated subdivisions this gives rise to a series of free parameters $\{a_k\}$.

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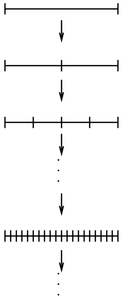
- By solving the $G^2 \to G$ problem repeatedly we end up with a Dirac type operator on the level of Γ_i



$$D = \sum_k a_k D_k$$

where D_k is a Dirac type operator corresponding to the k'th level.

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- Result: For a compact Lie-group G the triple $(\mathcal{B}, D, \mathcal{H})_{\overline{\mathcal{A}}}$ is a semi-finite⋆ spectral triple:
  - D's resolvent $(1 + D^2)^{-1}$ is compact (wrt. trace) and
  - the commutator $[D, a]$ is bounded

  Provided the sequence $\{a_i\}$ approaches ∞ sufficiently fast.

⋆*semi-finite: everything works up to a symmetry group with a trace (CAR algebra)* [Carey, Phillips, Sukochev].

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It appears that the spectral triple construction lies somewhere between Hamiltonian lattice gauge theory and LQG.

Spaces of connections

- Denote

$$\overline{\mathcal{A}} := \lim_{\overleftarrow{\Gamma}} \mathcal{A}_\Gamma$$

or roughly:

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- **Result:**

$$\mathcal{A} \hookrightarrow \overline{\mathcal{A}}$$

  which means that $\overline{\mathcal{A}}$ is a space of generalized connections.
- This result mirrors a similar result from LQG, based on the system of piece-wise analytic graphs. Here: it is possible to capture the full information of $\mathcal{A}$ with a countable system of graphs.

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- **Interpretation:** nonperturbative quantum field theory.

D interacting with the algebra

- First, for a single group element g corresponding to the $i'th$ copy of G in G^n we find

$$[D, g] = \sum_k \left(\pm g\sigma^a \right) \cdot e_i^a \qquad (a_i \equiv 1)$$

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- Next, the commutator between D and the loop L is

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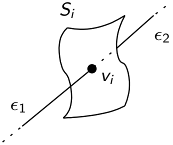
- This resembles the Poisson structure between loop and flux variables: A Lie-group generator is inserted into a loop in an intersection point.

- In fact, the left-invariant vector fields in D corresponds to flux-operators sitting at the vertices in the graphs.

- This means that D can be interpreted as a sum of flux operators, one for each copy of G.



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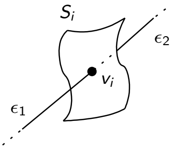

- The corresponding surfaces are 'dummy' in the sense that only the intersection points play any role in the following.

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    - The holonomy loops builds the algebra.
    - The flux operators are stored in the Dirac type operator.
    - These objects are build on a "dense" system of graphs.

Semi-classical states

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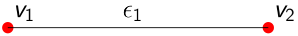

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\langle \bar{\phi}(g)|f_L|\phi(g)\rangle &= Hol(L,A) + \mathcal{O}(\hbar) \\
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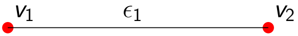

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- These are the same states which Thomas Thiemann has used to construct semi-classical states in a LQG-setup.

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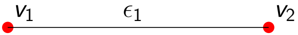

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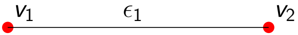

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$$\Psi(g) = \big(g\psi(v_2) + \mathrm{i}e_1^a\sigma^a\psi(v_1)\big)\phi(g)$$

gives, to lowest order, the expectation value of D

$$\begin{aligned}\langle\bar{\Psi}(g)|D|\Psi(g)\rangle &= a_n\big(- \bar{\psi}(v_1)\sigma^a E_a^1(\psi(v_2) - \psi(v_1))\\ &\quad +(\bar{\psi}(v_2) - \bar{\psi}(v_1))\sigma^a E_a^1\psi(v_1)\\ &\quad +\bar{\psi}(v_1)\{\epsilon A, \sigma^a E_a^1\}\psi(v_1)\big) + \mathcal{O}(\hbar)\end{aligned}$$

where we used $g \sim 1 + \epsilon A$, with $\epsilon = 2^{-n}$. Here "1" denotes the direction of ϵ_1.

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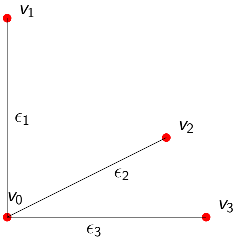
- This looks like a self-adjoint Dirac operator in 3-dimensions - in one point and in one direction.
- A clear interpretation of the sequence $\{a_n\}$: In the semi-classical limit, the parameters a_n are the inverse infinitesimal line elements:

$$a_n \sim \frac{1}{\Delta x}$$

Three copies of G



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$$\Psi(g_1, g_2, g_3) = \Big(g_1\sigma^a e_2^a \sigma^b e_3^b \psi(v_1) + g_2\sigma^a e_1^a \sigma^b e_3^b \psi(v_2) + g_3\sigma^a e_1^a \sigma^b e_2^b \psi(v_3) + \mathrm{i}\sigma^a e_1^a \sigma^b e_2^b \sigma^c e_3^c \psi(v_0)\Big)\phi_1(g_1)\phi_2(g_2)\phi_3(g_3)$$

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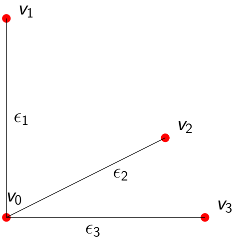

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gives the expectation value

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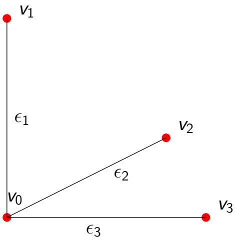

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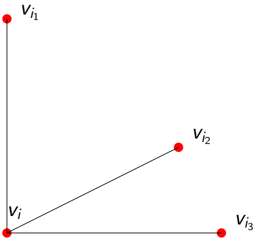
- Recall that E_a^m is the *densitised* triad field. It involves $e = det(e_a^m)$, where e_a^m is a spatial triad field.

Semiclassical states on $\overline{\mathcal{A}}$



At the n'th level, the state

$$\Psi_n(\overline{\mathcal{A}}) = \frac{1}{\mathcal{N}} \left(\sum_i \Psi_{v_i} \right) \Phi_n(\overline{\mathcal{A}}) \in \mathcal{H} \ ,$$

where $\mathcal{N} = \sqrt{(\#\text{new boxes})}$ and where

$$\begin{aligned} \Psi_{v_i}(g_{i_1}, g_{i_2}, g_{i_3}) \quad = \quad & g_{i_1}\sigma^a e_2^a \sigma^b e_3^b \psi(v_{i_1}) + g_{i_2}\sigma^a e_1^a \sigma^b e_3^b \psi(v_{i_2}) \\ & + g_{i_3}\sigma^a e_1^a \sigma^b e_2^b \psi(v_{i_3}) + \mathrm{i}\sigma^a e_1^a \sigma^b e_2^b \sigma^c e_3^c \psi(v_i) \end{aligned}$$

and

$$\Phi_n(\overline{\mathcal{A}}) = \prod_i \phi_i(g_i)$$

gives the expectation value of D

$$\lim_{n\to\infty} \langle \bar{\Psi}_n(\overline{\mathcal{A}}) | D | \Psi_n(\overline{\mathcal{A}}) \rangle = \int_\Sigma d^3x \sqrt{g}\, \bar{\psi}(x) (\sigma^a e_a^m \nabla_m + \nabla_m \sigma^a e_a^m) \psi(x) + \mathcal{O}(\hbar)$$

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- The spectral triple construction works for many systems of ordered graphs. This semi-classical analysis singles out lattices.
- Also, there are several ways to solve the consistency conditions for the Dirac operator. This semi-classical analysis singles out one of these solutions as natural.
- In the semi-classical limit all dependency on any finite part of the lattices vanish:

$$\begin{aligned} \psi(x+\epsilon)-\psi(x) &\rightarrow 0 \\ a_n &\rightarrow \infty\,, \quad \text{only in the continuum limit} \\ & \qquad\quad \text{of "infinitesimal" edges.} \end{aligned}$$

Comments and questions

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It is an interesting question if the algebra $C^\infty(\Sigma)$ can be obtained in a semi-classical limit. Also, will this algebra be commutative?

$C^\infty(\Sigma) \to C^\infty(\Sigma) \otimes M_n(\mathbb{C})$, - additional matrix factor?

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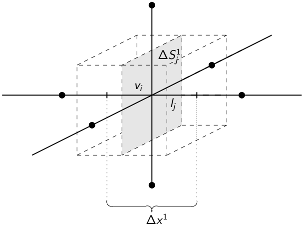
- Thus, we can rewrite the volume in terms of inifitesimal flux variables.

- Next, write the integral as a limit of cubes

$$Vol(\Sigma) = \lim_{\delta \to 0} \sum \Delta x_i F_{\Delta S_i}$$



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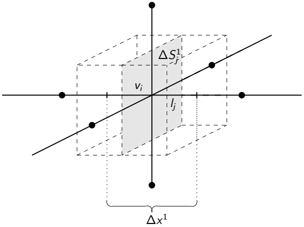

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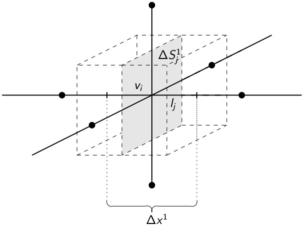

- This formulation of $Vol(\Sigma)$ is very similar to the Dirac operator D:
  - it is written in terms of an inductive system of lattices.
  - it is a sum of flux variables.
- To get from $Vol(\Sigma)$ to D :
  1. Quantize according to Poisson structure:

$$F^a_{\Delta S^m_j} \longrightarrow d_{\mathbf{e}^a_j}$$

  2. exchange dx^a with a corresponding element in the Clifford algebra $Cl(T^*G^n)$

$$dx^a \longrightarrow \mathbf{e^a_j}$$

- The Dirac operator D splits the infinitesimal constituents of the Riemannian integral, quantizes them and throws them into an infinite dimensional Clifford bundle.

Spectral action functional

- The spectral action functional (trace of heat-kernel) resembles a Feynman integral

$$Tr \exp(-s(D)^2) \sim \int_{\overline{\mathcal{A}}^{\Delta}} [d\nabla] \exp\left(-s(D)^2\right) \delta_{\nabla}(\nabla)$$

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where D^2 plays the role of an action or an energy.
- This object is finite.
- Perhaps the Hamiltonian of GR should be extracted from this object?

Connes Distance Formula

- **Connes distance formula:** Given a spectral triple $(\mathcal{A}, D, \mathcal{H})$ over a manifold $\mathcal{M}$ the distance formula reads

$$d(\xi_x, \xi_y) = \sup_{a \in \mathcal{A}} \left\{ |\xi_x(a) - \xi_y(a)| \big| \|[D, a]\| \leq 1 \right\}$$

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- If two configurations differ on a large scale, then the distance between them will be 'large' (difference weighted with small a's - large distance)
- If they differ only on short scales, then the distance will be 'small' (difference weighted with large a's - small distance).

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