Technische Universität Kaiserslautern

Student Seminar

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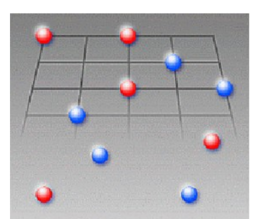
Electronic properties of a harmonically confined 1D Hubbard model



Outline

Electronic properties of a harmonically confined 1D Hubbard model

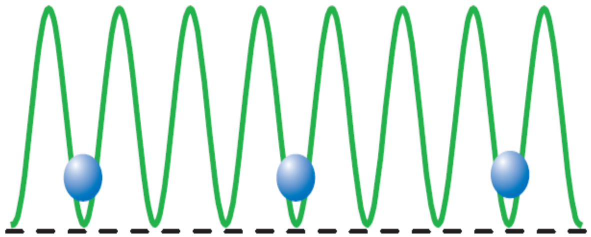
- Cold gases and the Hubbard model
- Results: Homogeneous part
- Results: Oscillations

Cold gases

- Aim: Study the behavior of correlated electrons in low dimensional condensed matter systems

➔ Artificial *crystal* of light: *Optical lattice*

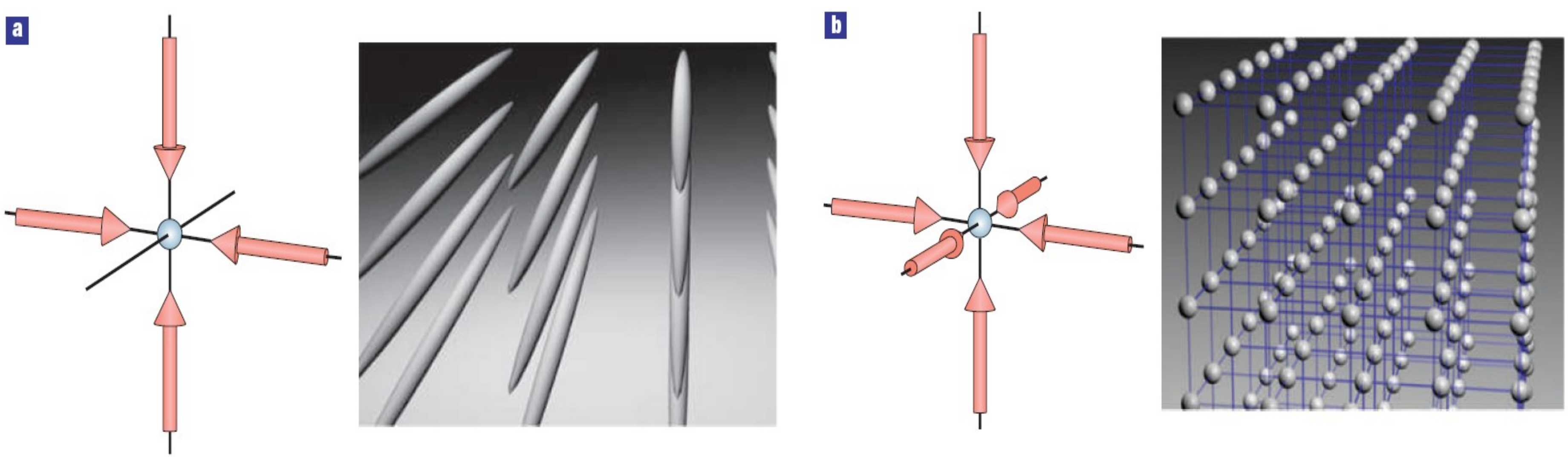
➔ Load with *ultracold atoms*



Cold gases

- Tune hopping and interaction as needed

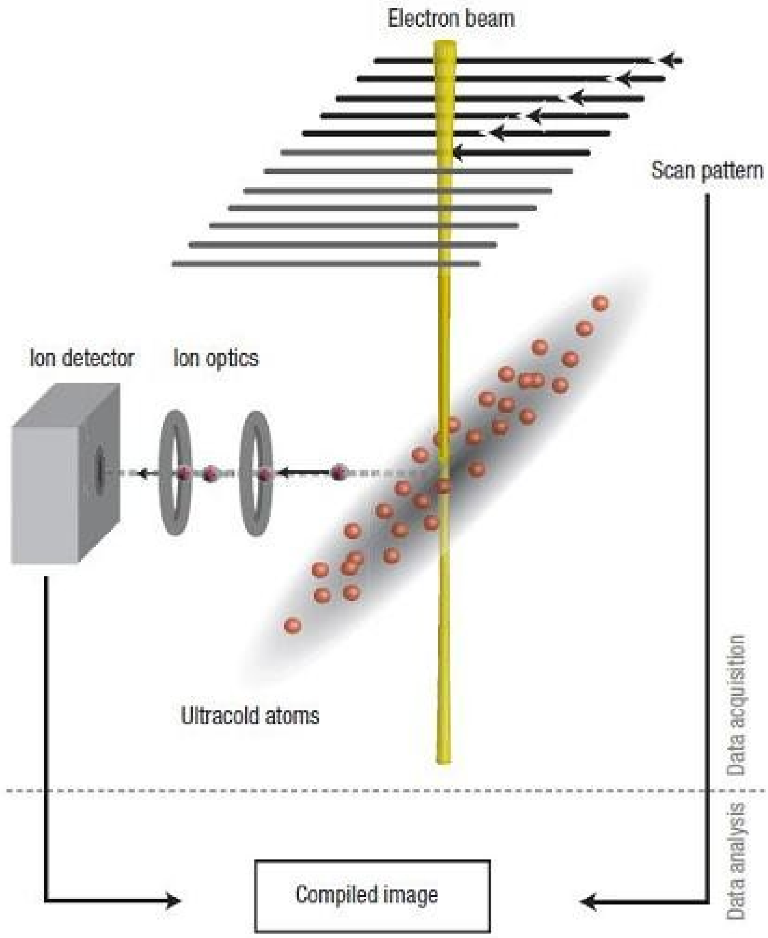
- Choose lattive geometry and dimension



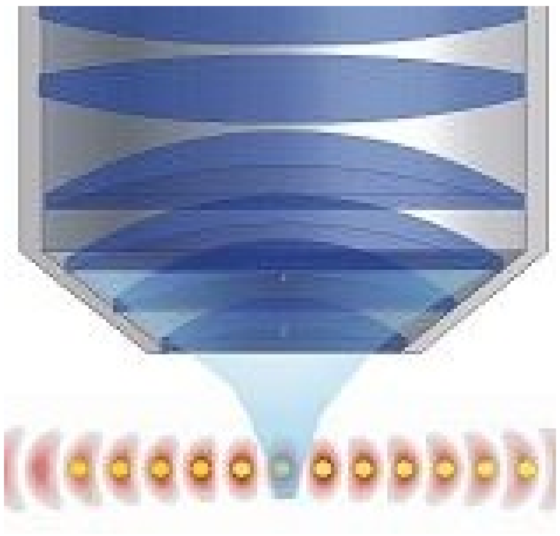
I. Bloch, *Nature Physics* **1**, 23-30 (2005)

Cold gases

- Detection of particles:



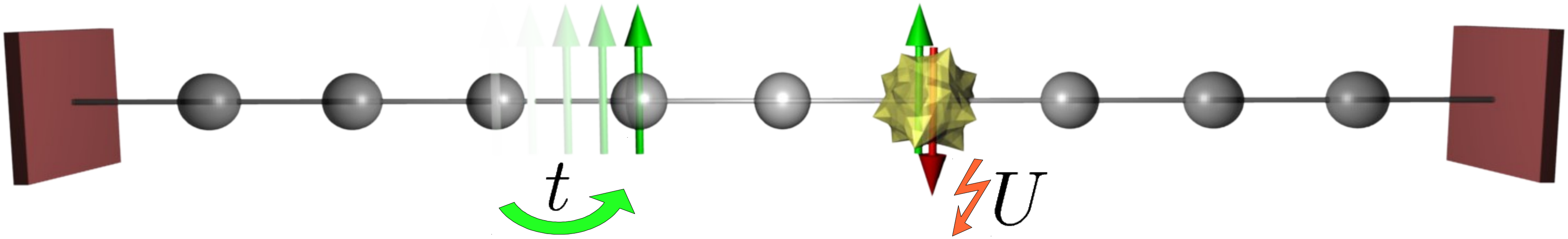
www.physik.uni-kl.de/ott



www.quantum-munich.de

1D Hubbard model

- 1D system of interacting fermions; *quantum wire*

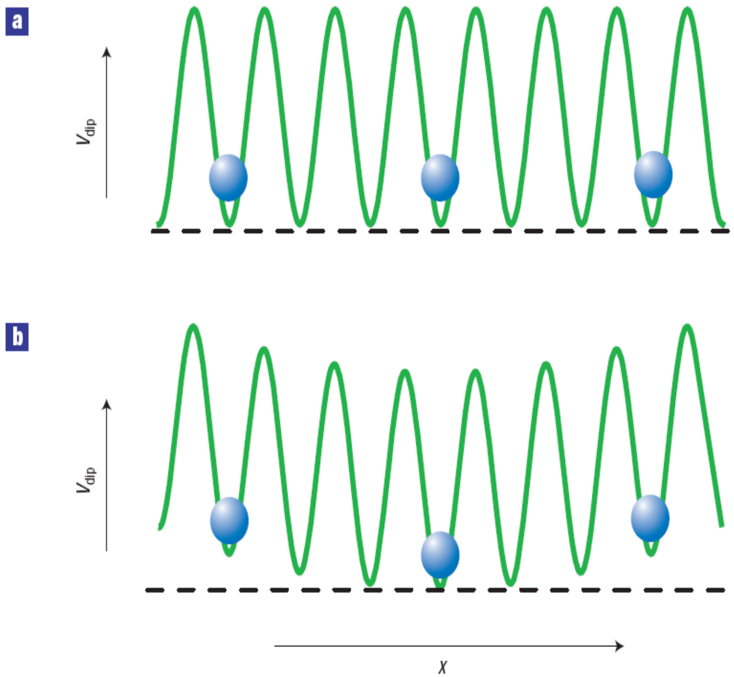


$$H = -t \sum_{j,\sigma} \left(\psi^\dagger_{j,\sigma} \psi_{j+1,\sigma} + \psi^\dagger_{j+1,\sigma} \psi_{j,\sigma} \right) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

- Kinetic energy: Hopping parameter t
- Coulomb repulsion: On-site interaction U
- Fixed band filling n

Cold gases

- But: Experiments are carried out in a trap:



$$H = -t \sum_{j,\sigma} \left(\psi^\dagger_{j,\sigma} \psi_{j+1,\sigma} + \psi^\dagger_{j+1,\sigma} \psi_{j,\sigma} \right) + U \sum_j n_{j,\uparrow} n_{j,\downarrow} + \sum_j \left(j - \frac{L+1}{2} \right)^2 n_j$$

  - which effects has the additional potential?

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Electronic properties of a harmonically confined 1D Hubbard model

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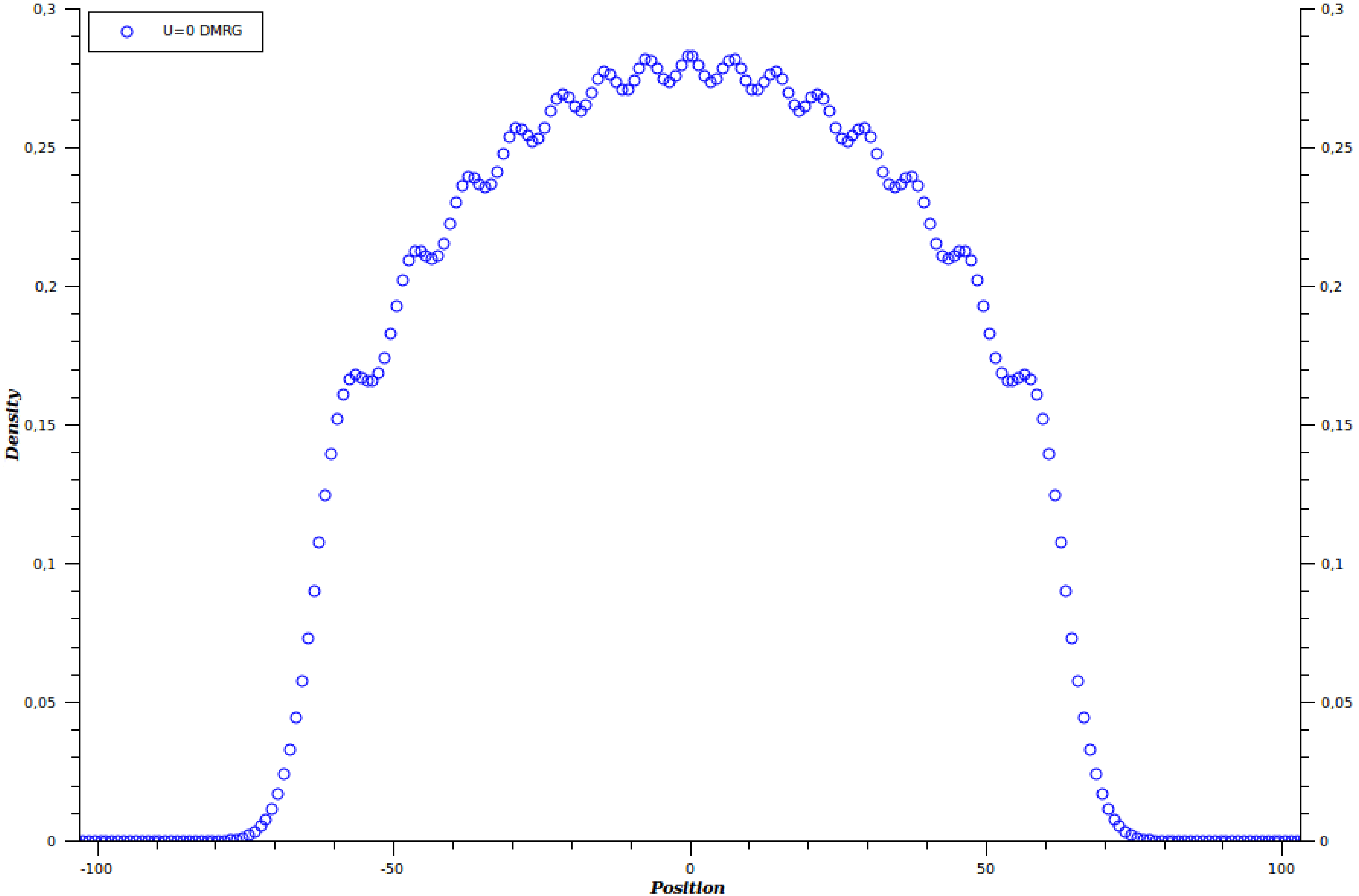
Density in the trap

- *Observable:* Electronic density



Density in the trap

- *Reference*: Numerical calculation:
  DMRG (Density Matrix Renormalization Group)
  - ✔ Quasi exact (controllable error)
  - ✔ Arbitrary potential
  - ✔ Arbitrary interaction

Density in the trap

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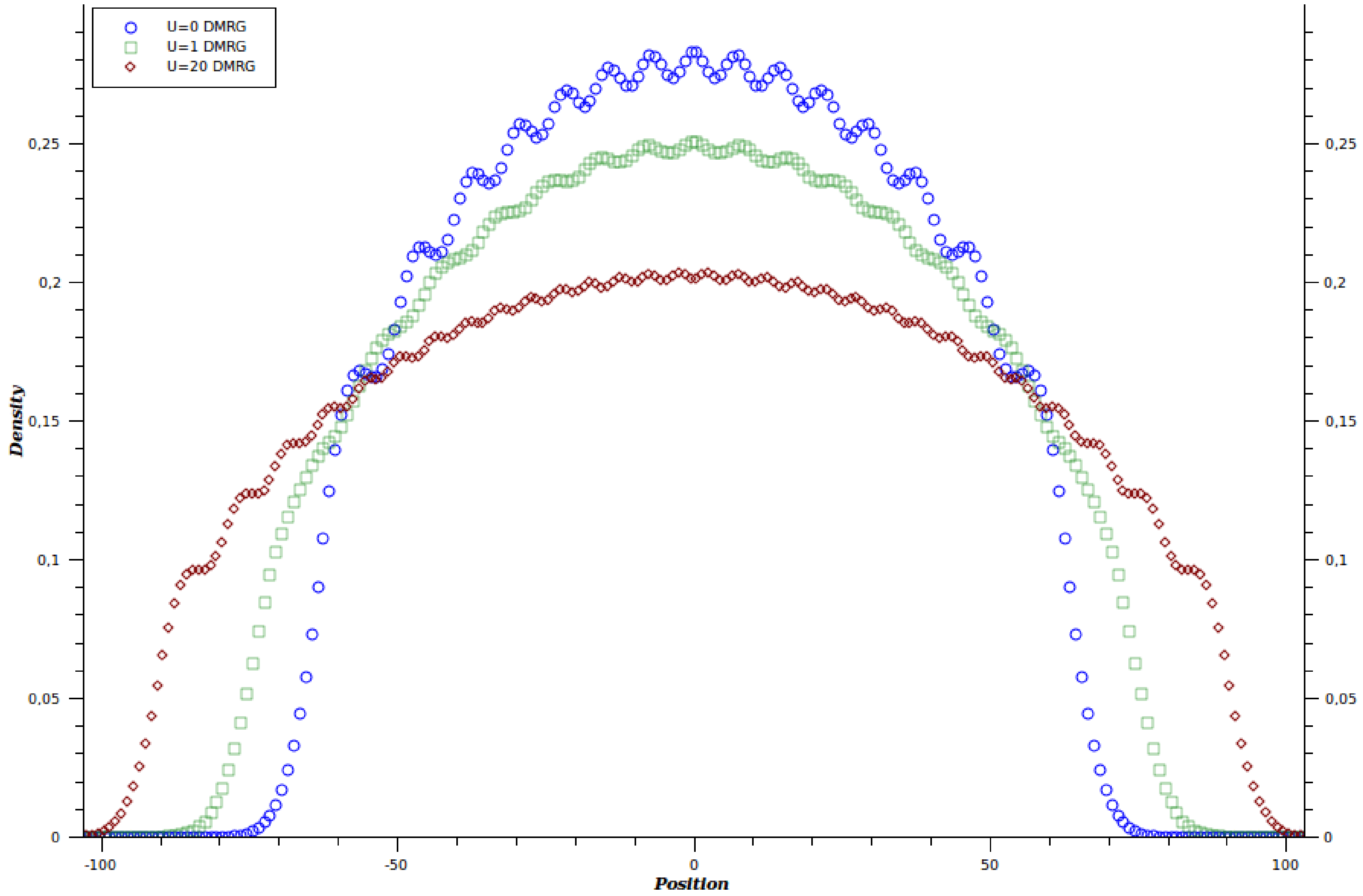

Density in the trap

- *Reference*: Numerical calculation:
  DMRG (Density Matrix Renormalization Group)
  - ✓ Quasi exact (controllable error)
  - ✓ Arbitrary potential
  - ✓ Arbitrary interaction
  - ✗ Computationally expensive for large systems
- Aim: Analytical expression for the density

Density in the trap

- $U=0$: Thomas-Fermi approach

$$\frac{\partial E}{\partial n_0} = \mu$$

at each point, with $\mu = \mu(x)$

Density in the trap

- U=0: Thomas-Fermi approach

$$\frac{\partial E}{\partial n_0} = \mu$$

at each point, with $\mu = \mu(x)$

- $\Rightarrow n_0(x) = \frac{k_F^{(0)}}{\pi} \sqrt{1 - (x/L_F)^2}$

with classical turning points $L_F = \sqrt{\frac{2N-1}{\omega}}$

Density in the trap

- $U > 0$: Repulsive interaction:
  - broadening
  - deformation

- Adjust the profile

$$n_0(x) \propto \left[1 - (x/L_F)^2\right]^{\alpha}$$

with fit parameters $L_F,\ \alpha \approx 0.5$

Density in the trap

- Can we determine the broadening and deformation of the cloud directly?

➔ Make use of exact solution: *Bethe Ansatz*

  - ✓ Analytical result
  - ✓ For arbitrary interaction

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Density in the trap

- Can we determine the broadening and deformation of the cloud directly?

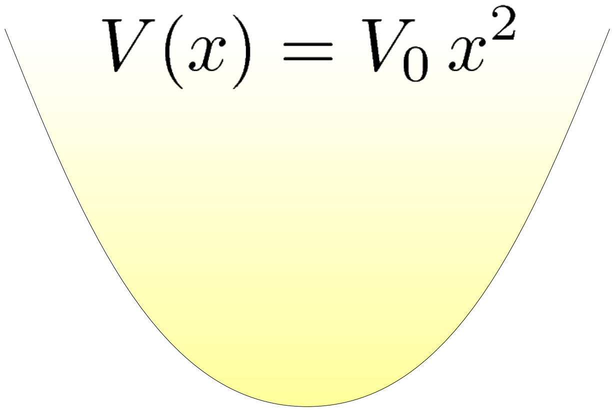
➔ Make use of exact solution: *Bethe Ansatz*

  - ✔ Analytical result
  - ✔ For arbitrary interaction
  - ✗ Coupled integral equations
  - ✗ Homogeneous system, thermodynamic limit

➔ Idea: *Local* Bethe Ansatz

Density in the trap

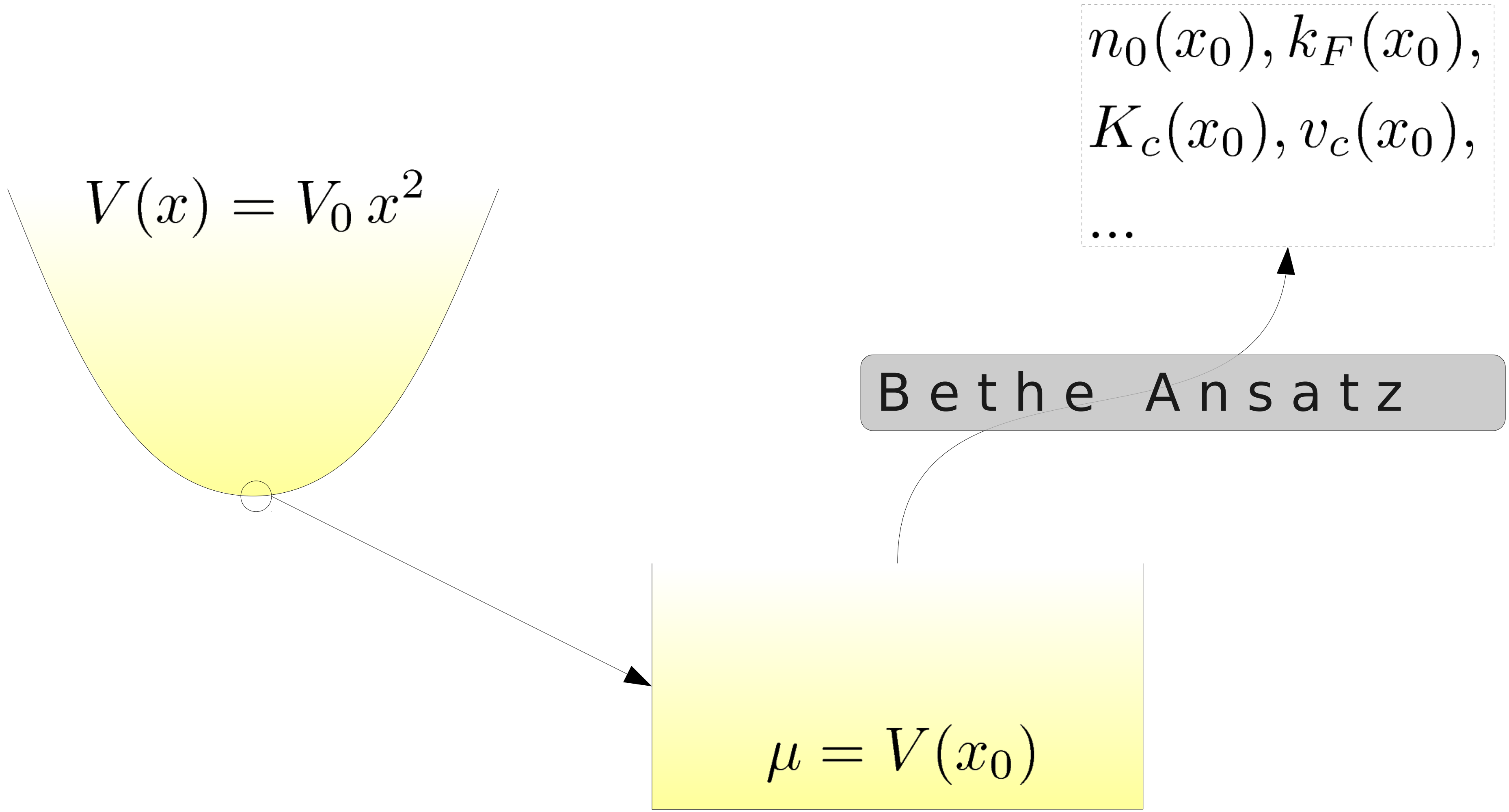
Local Bethe Ansatz



Density in the trap

Local Bethe Ansatz

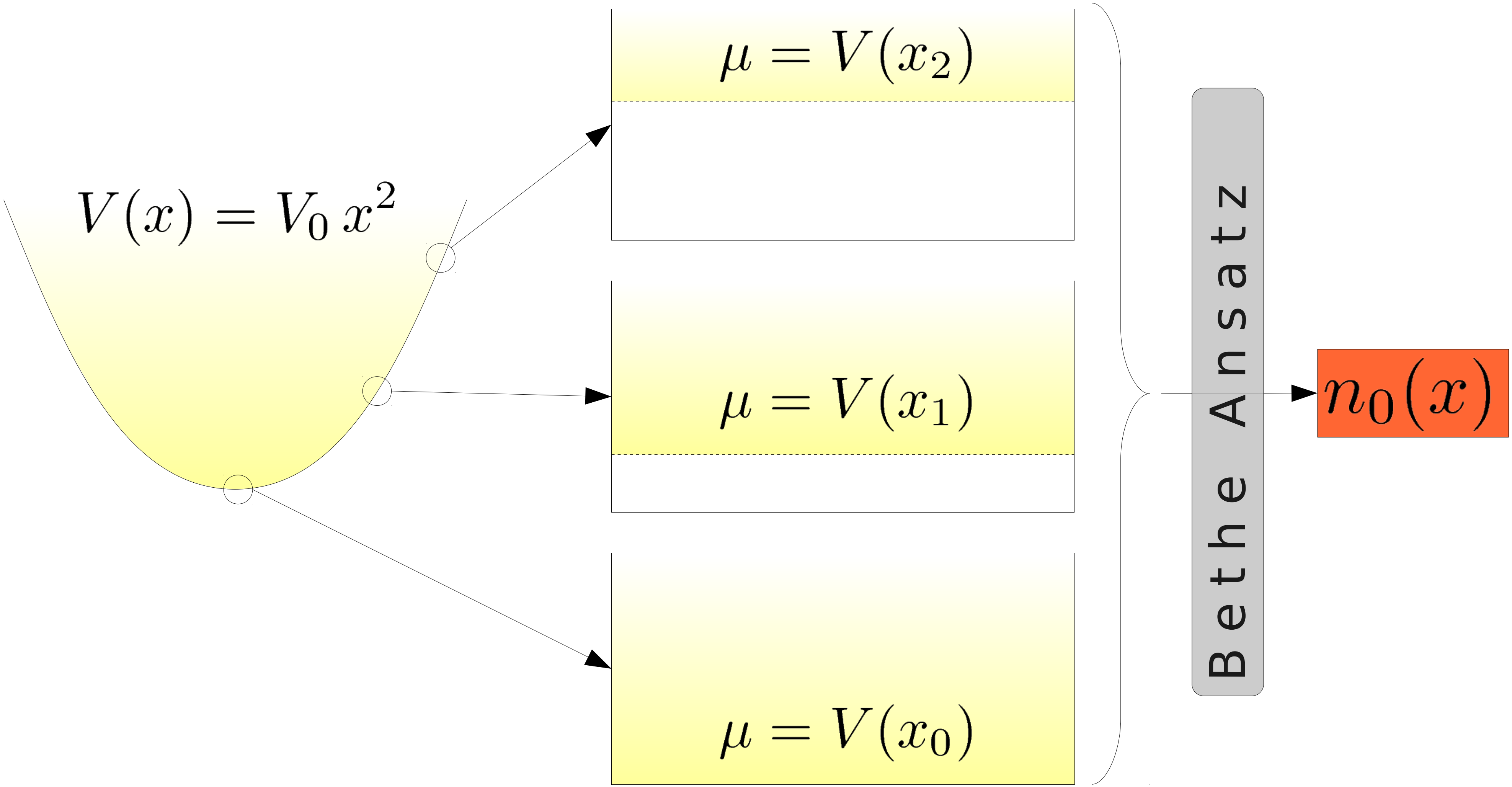
- Solve auxiliary system for each lattice point



Density in the trap

Local Bethe Ansatz

- Solve auxiliary system for each lattice point



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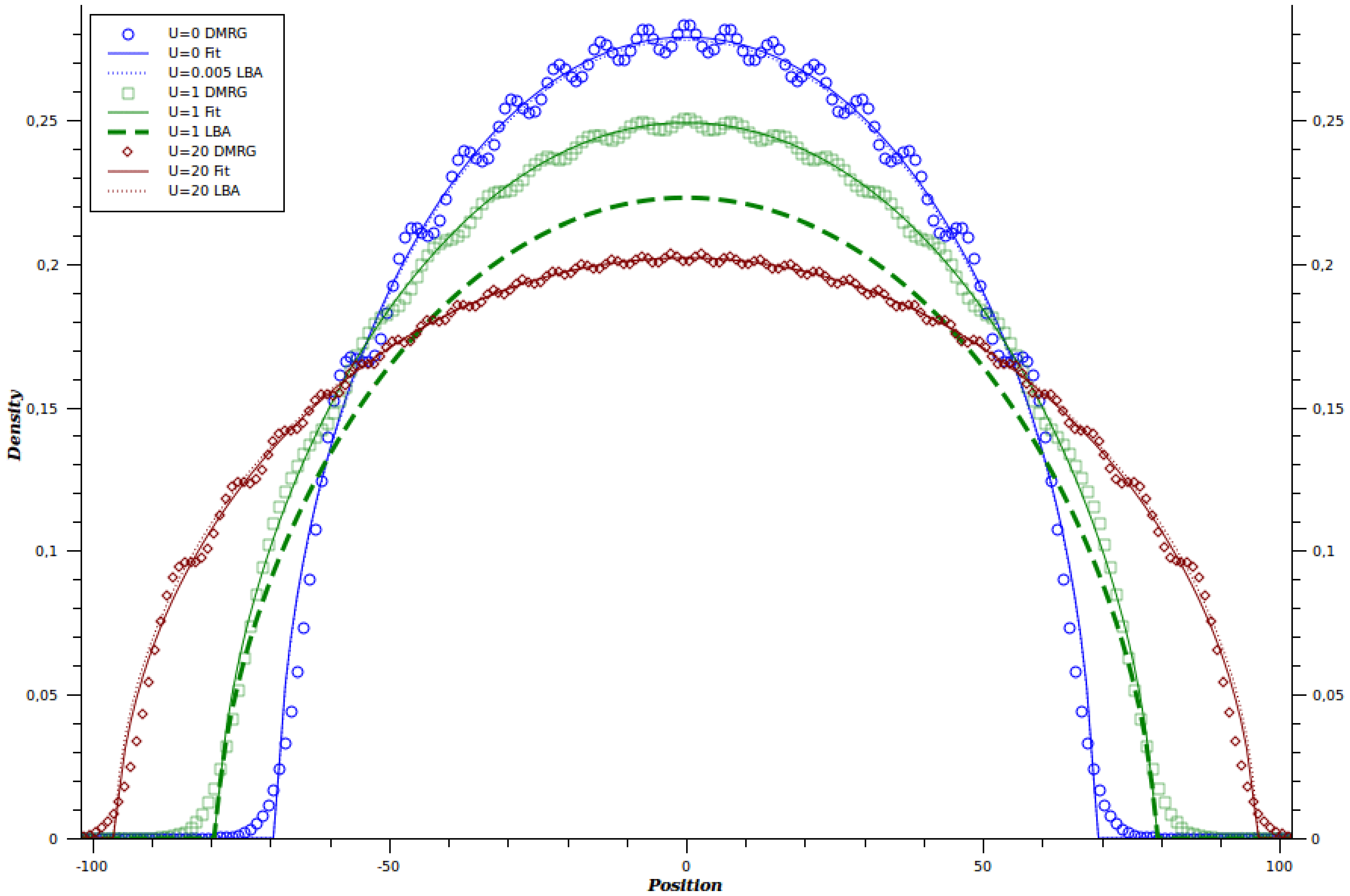
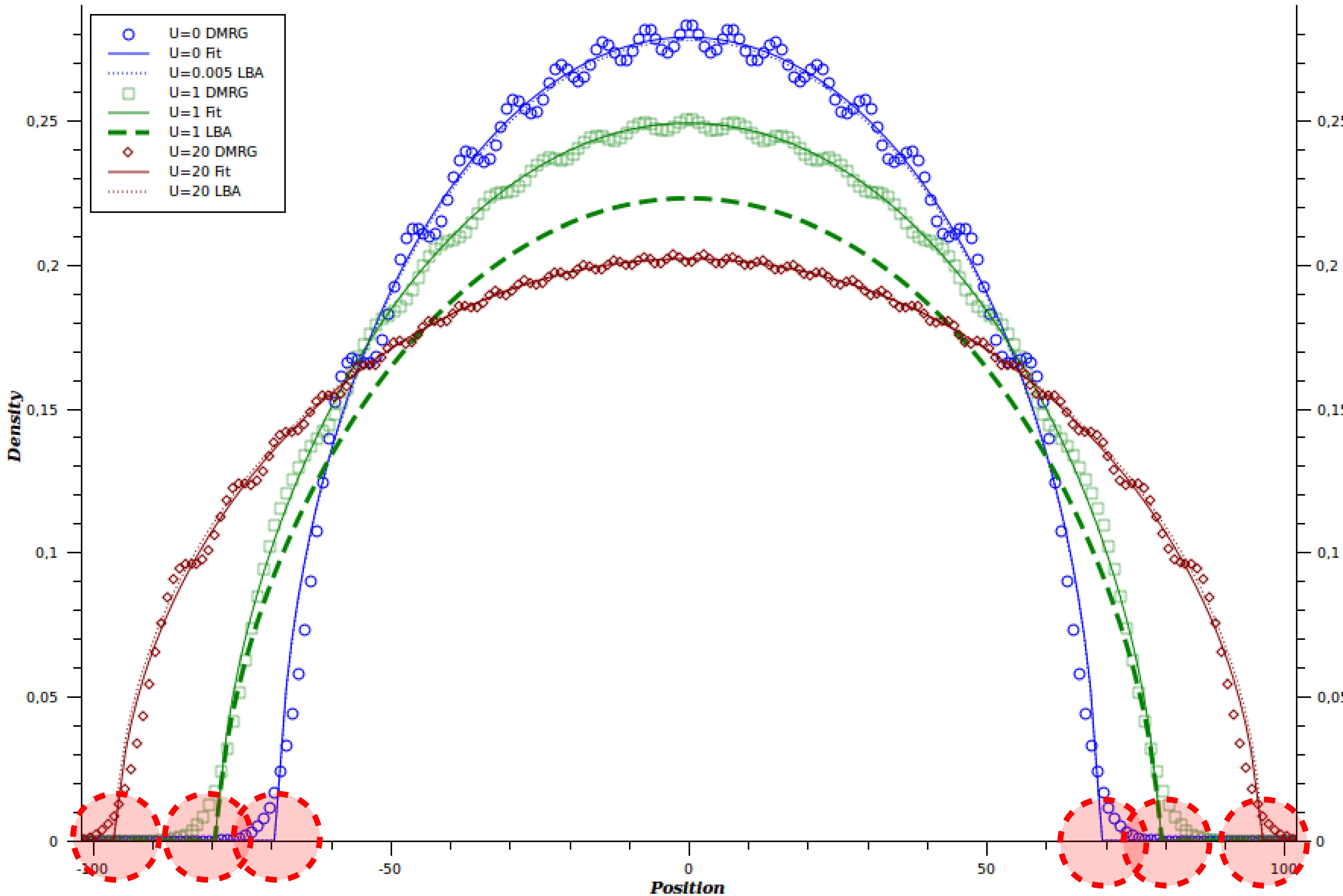
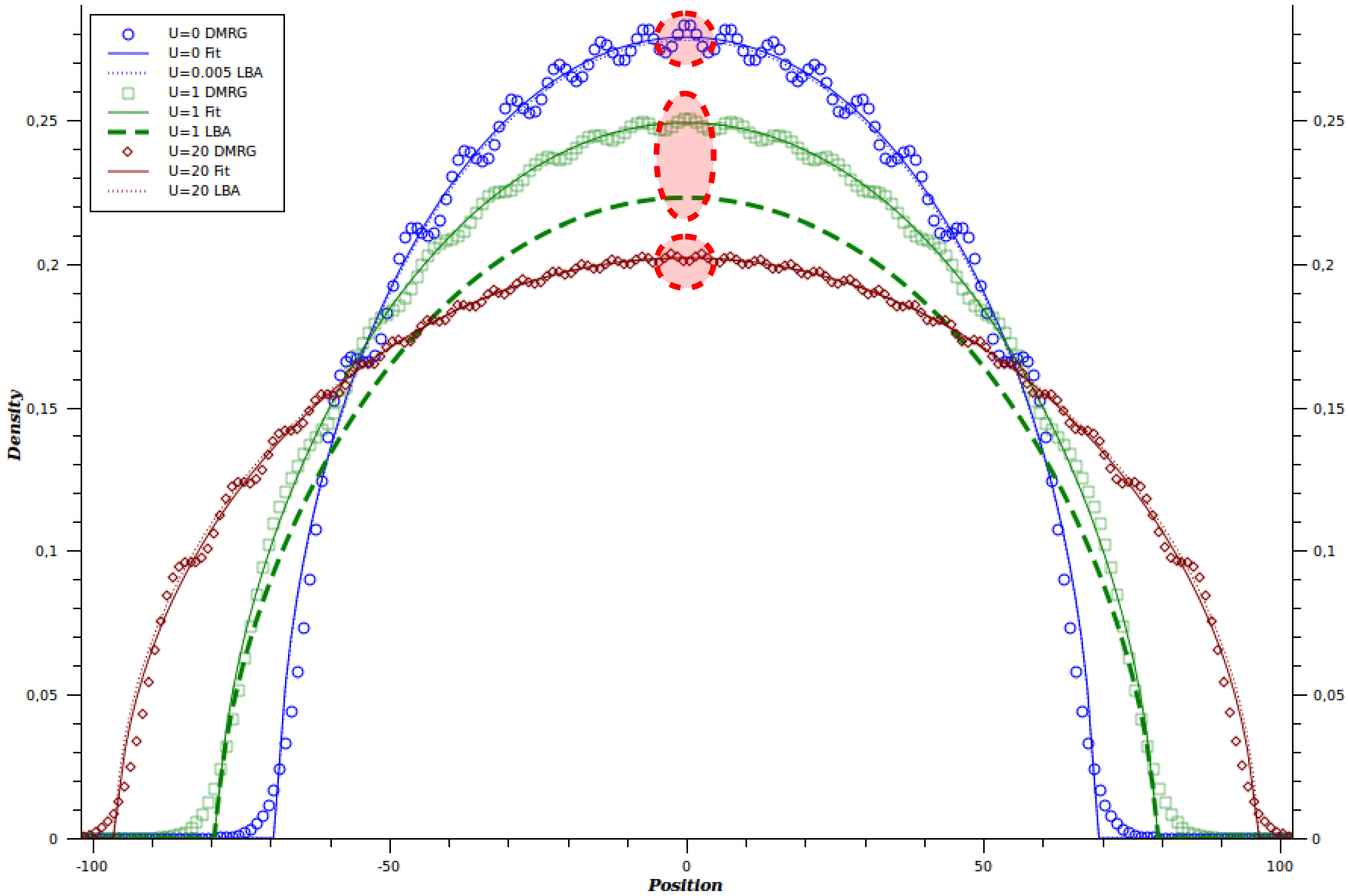
Density in the trap



Density in the trap

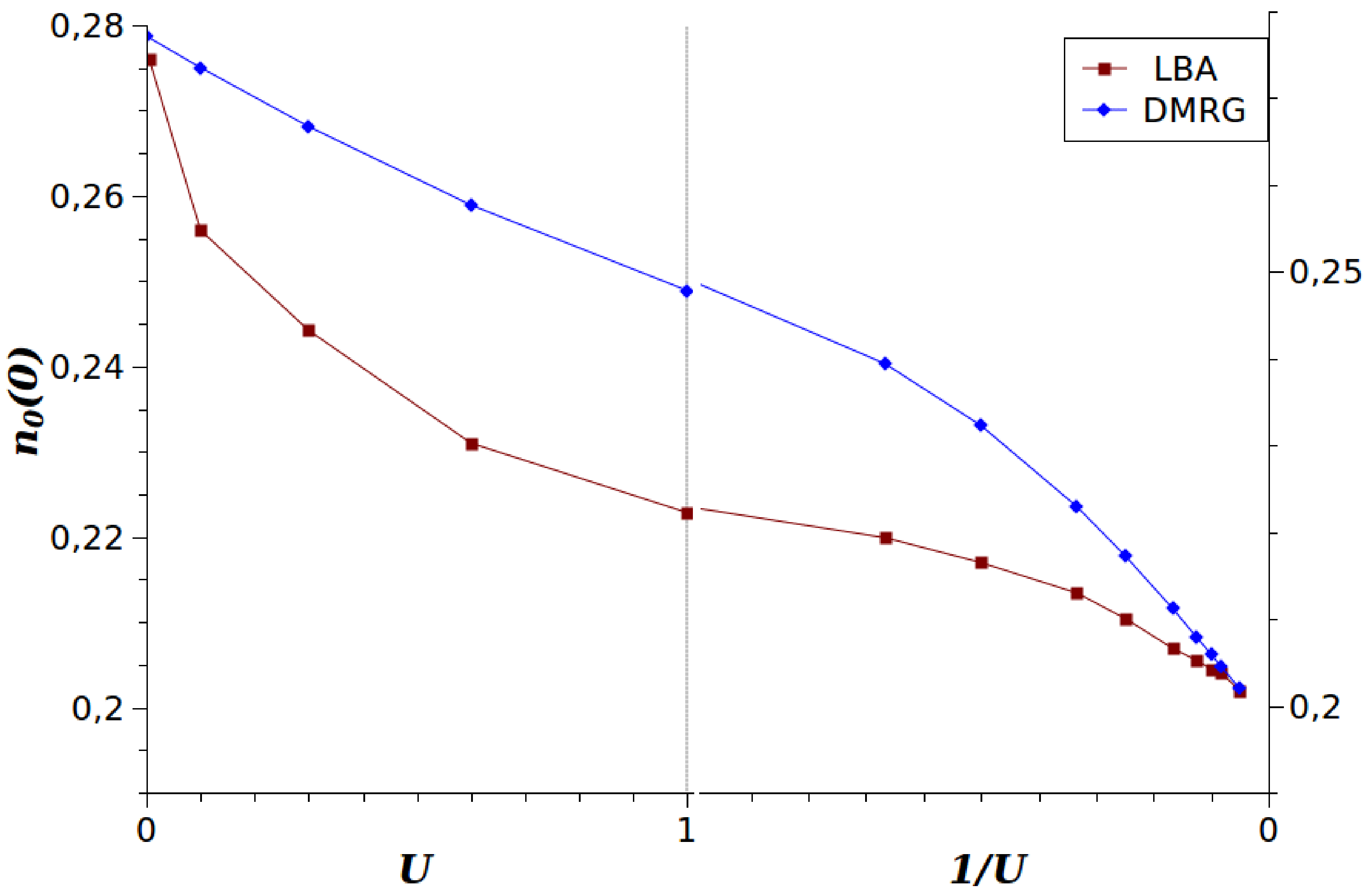


Density in the trap



Density in the trap

- Density in trap center:



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Density in the trap

Oscillations:

- Known from homogeneous case:
  - $U = 0$: Friedel oscillations
  - $U \to \infty$: Wigner crystal oscillations
- On top of the slowly varying part

Density in the trap

Oscillations:

- $$\delta n(x) = A_1 \frac{\cos\left(2\tilde{k}_F(x)\, x\right)}{\left[1-(x/L_F)^2\right]^{K_1}}$$

Friedel

Density in the trap

Oscillations:

- $$\delta n(x) = A_1 \frac{\cos\left(2\tilde{k}_F(x)\,x\right)}{\left[1-(x/L_F)^2\right]^{K_1}} + A_2 \frac{\cos\left(4\tilde{k}_F(x)\,x\right)}{\left[1-(x/L_F)^2\right]^{K_2}}$$

Wigner

Density in the trap

Oscillations:

- $$\delta n(x) = A_1 \frac{\cos\left(2\tilde{k}_F(x)\,x\right)}{[1-(x/L_F)^2]^{K_1}} + A_2 \frac{\cos\left(4\tilde{k}_F(x)\,x\right)}{[1-(x/L_F)^2]^{K_2}}$$

- Position dependent Fermi wave vector

$$2\tilde{k}_F(x) = \frac{2\pi}{x}\int_0^x n_0(y)\,dy = k_F^{(0)}\left[Z(x) + L_F/x\,\sin\left(x/L_F\right)\right]$$

$$k_F^{(0)} = \alpha\sqrt{2N-1}$$

Density in the trap

Oscillations:

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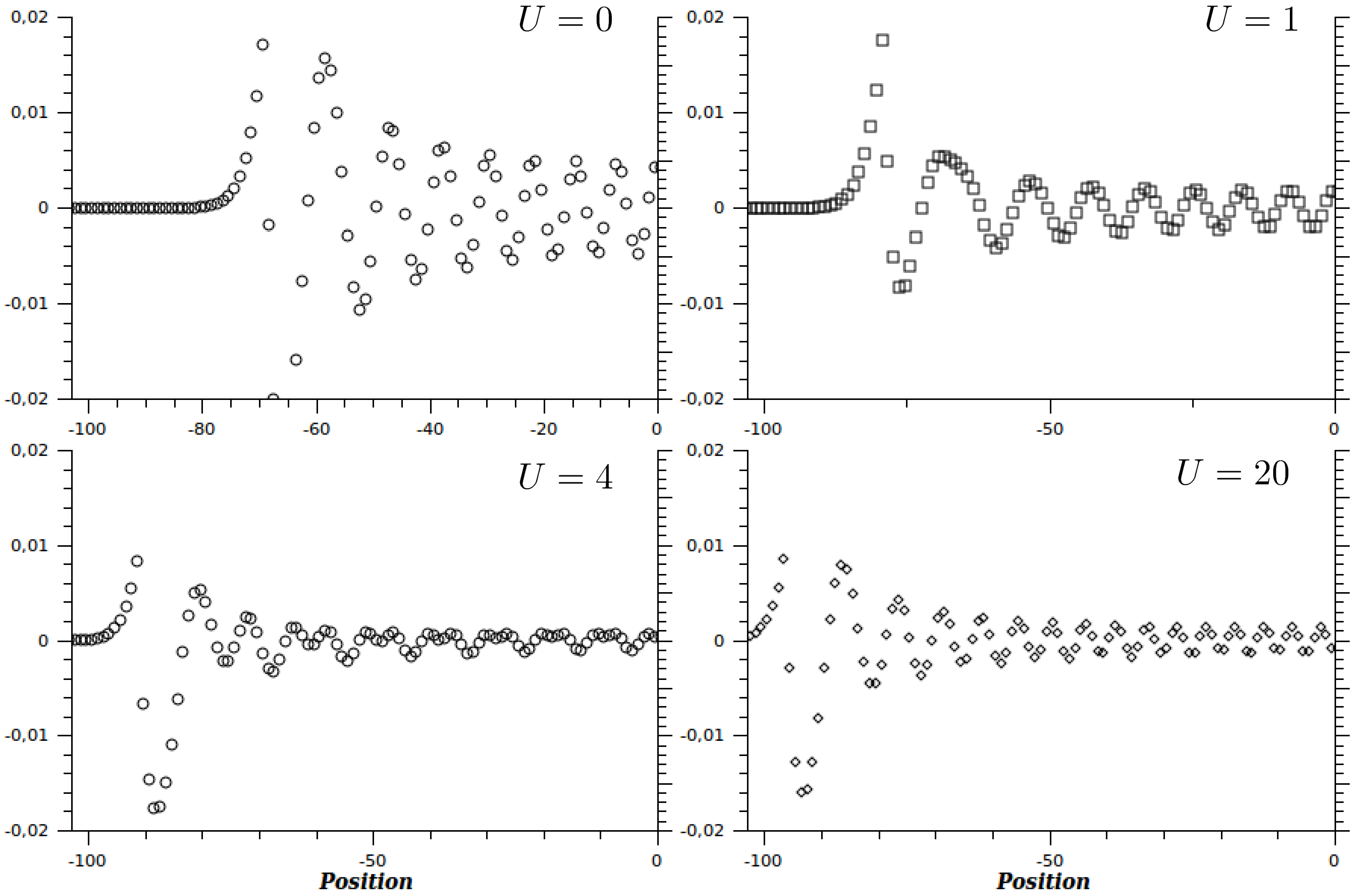
$$k_F^{(0)} = \alpha\sqrt{2N-1}$$

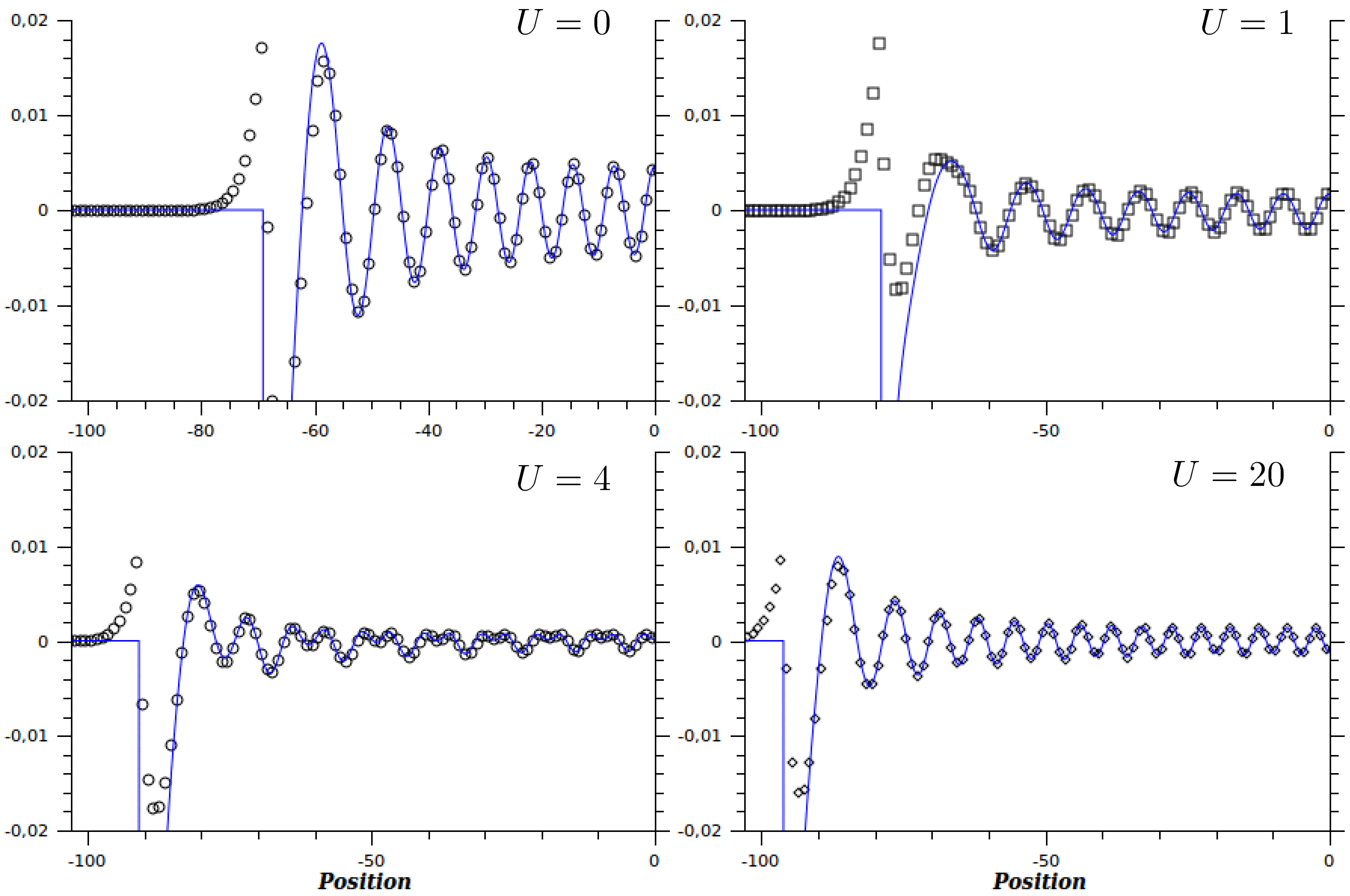
- Unknown amplitude and exponent

➔ Fit parameters $A_{1,2},\ K_{1,2},\ L_F,\ k_F$

Density in the trap

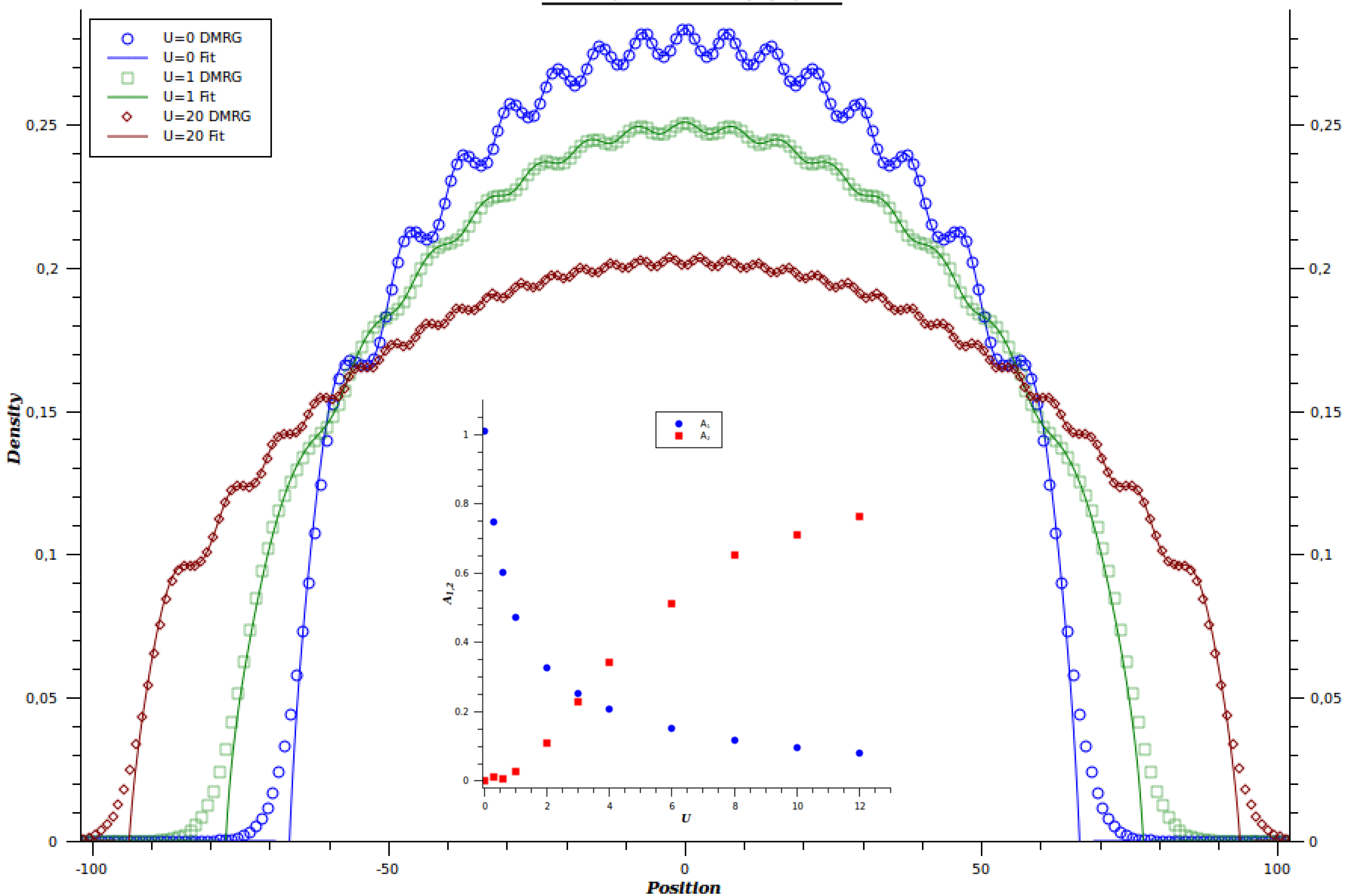


Density in the trap



Density in the trap

Final fit result



Outlook

- More electronic properties:

  (Local) Density of States

  - Tunneling experiments
  - Photoemission
  - Probe single particle wavefuntions

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Conclusions

- Slowly varying part of the density:
  - Fit (Thomas-Fermi approach)
    vs.
  - Local Bethe Ansatz (fails at intermediate U)
- Oscillations described by analytical expression (adapted from homogeneous case)
- Crossover from Friedel into Wigner crystal regime well described within a trap