BEC at finite momenta

Johannes Hick

Goethe Universität Frankfurt
Institut für Theoretische Physik
Prof. Dr. P. Kopietz

Student workshop
21.07.2010

Outline

Motivation

- Experiment: BEC of pumped magnons at room temperatur in thin films of YIG[1]
- Dispersion of thin YIG films has two generated minima at finite momenta $\pm \boldsymbol{q}$
- BEC observed at $\pm \boldsymbol{q}$

[1] O. Dzyapko *et al.* '07

Hamiltonian

Dilute Bose gas with parametric pumping

$$
\begin{aligned}
H_2 &= \sum_k (\epsilon_k - \mu)\, a_k^\dagger a_k + \sum_k \left(\frac{\gamma_k^*}{2} a_{-k} a_k + \frac{\gamma_k}{2} a_k^\dagger a_{-k}^\dagger \right) \\
H_3 &= \frac{1}{\sqrt{N}} \sum_{1,2,3} \delta_{1+2+3,0} \left\{ \frac{1}{2} \Gamma_3^{\bar{a}aa} a_{-1}^\dagger a_2 a_3 + \frac{1}{2} \Gamma_3^{\bar{a}\bar{a}a} a_{-1}^\dagger a_{-2}^\dagger a_3 \right. \\
&\quad \left. + \frac{1}{3!} \Gamma_3^{aaa} a_1 a_2 a_3 + \frac{1}{3!} \Gamma_3^{\bar{a}\bar{a}\bar{a}} a_{-1}^\dagger a_{-2}^\dagger a_{-3}^\dagger \right\} \\
H_4 &= \frac{1}{N} \sum_{1,2,3,4} \delta_{1+2+3+4} \left\{ \frac{1}{4} \Gamma_{12;34}^{\bar{a}\bar{a}aa} a_{-1}^\dagger a_{-2}^\dagger a_3 a_4 \right. \\
&\quad + \frac{1}{3!} \Gamma_{1;234}^{\bar{a}aaa} a_{-1}^\dagger a_2 a_3 a_4 + \frac{1}{3!} \Gamma_{123;4}^{\bar{a}\bar{a}\bar{a}a} a_{-1}^\dagger a_{-2}^\dagger a_{-3}^\dagger a_4 \\
&\quad \left. + \frac{1}{4!} \Gamma_{1234}^{aaaa} a_1 a_2 a_3 a_4 + \frac{1}{4!} \Gamma_{1234}^{\bar{a}\bar{a}\bar{a}\bar{a}} a_{-1}^\dagger a_{-2}^\dagger a_{-3}^\dagger a_{-4}^\dagger \right\}
\end{aligned}
$$

Which modes can condense?

Euclidean action

$$S_2[\Phi] = \frac{1}{2}\int_0^\beta \mathrm{d}\tau \sum_k \begin{pmatrix} \Phi^{\bar{a}}_{-k} & \Phi_k \end{pmatrix} \begin{pmatrix} \partial_\tau + \epsilon_k - \mu & \gamma_k \\ \gamma_k^* & -\partial_\tau + \epsilon_k - \mu \end{pmatrix} \begin{pmatrix} \Phi^a_k \\ \Phi^{\bar{a}}_{-k} \end{pmatrix}$$

$$S_3[\Phi] = \frac{1}{\sqrt{N}}\int_0^\beta \mathrm{d}\tau \sum_{1,2,3} \sum_{\sigma_1,\sigma_2,\sigma_3} \delta_{1+2+3,0} \frac{1}{3!} \Gamma_3(1\sigma_1, 2\sigma_2, 3\sigma_3) \Phi_1^{\sigma_1}\Phi_2^{\sigma_2}\Phi_3^{\sigma_3}$$

$$S_4[\Phi] = \frac{1}{N}\int_0^\beta \mathrm{d}\tau \sum_{1,2,3,4} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} \delta_{1+2+3+4,0} \frac{1}{4!} \Gamma_4(1\sigma_1, 2\sigma_2, 3\sigma_3, 4\sigma_4) \Phi_1^{\sigma_1}\Phi_2^{\sigma_2}\Phi_3^{\sigma_3}\Phi_4^{\sigma_4}$$

Notation

$$\begin{aligned} a_k &\to \Phi^a_k(\tau) \\ a_k^\dagger &\to \Phi^{\bar{a}}_{-k}(\tau) \end{aligned}$$

Symmetrized vertices, for example

$$\Gamma_3(1a, 2\bar{a}, 3a) = \Gamma^{\bar{a}aa}_{1;23}$$

BEC

Fields of condensed modes have finite expectation values

$$\langle \Phi_k^\sigma \rangle = \phi_k^\sigma$$

Shifting fields by expectation values

$$\Phi_k^\sigma = \phi_k^\sigma + \delta\Phi_k^\sigma$$

$$S[\phi + \delta\Phi] = S[\phi] + \int_0^\beta \mathrm{d}\tau \sum_{k\sigma} \left.\frac{\delta S[\Phi]}{\delta \Phi_k^\sigma}\right|_{\Phi=\phi} \delta\Phi_k^\sigma + \ldots$$

Extremum condition

$$0 = \left.\frac{\delta S[\Phi]}{\delta \Phi_k^\sigma}\right|_{\Phi=\psi} = (\epsilon_k - \mu)\,\phi_{-k}^{\bar{\sigma}} + \gamma_k^\sigma \phi_{-k}^\sigma + \frac{1}{\sqrt{N}} \sum_{1,2} \sum_{\sigma_1,\sigma_2} \delta_{1+2+k,0} \Gamma_3\left(\boldsymbol{k}\sigma, \boldsymbol{1}\sigma_1, \boldsymbol{2}\sigma_2\right) \phi_1^{\sigma_1} \phi_2^{\sigma_2} + \frac{1}{N} \sum_{1,2,3} \sum_{\sigma_1,\sigma_2,\sigma_3} \delta_{1+2+3+k,0} \Gamma_4\left(\boldsymbol{k}\sigma, \boldsymbol{1}\sigma_1, \boldsymbol{2}\sigma_2, \boldsymbol{3}\sigma_3\right) \phi_1^{\sigma_1} \phi_2^{\sigma_2} \phi_3^{\sigma_3}$$

BEC at $\boldsymbol{k} = 0$

Consider $\phi_{\boldsymbol{k}}^{\sigma} = \sqrt{N}\delta_{\boldsymbol{k},0}\psi_0^{\sigma}$ with $\Gamma_{00;00}^{\bar{a}\bar{a}aa}$ only

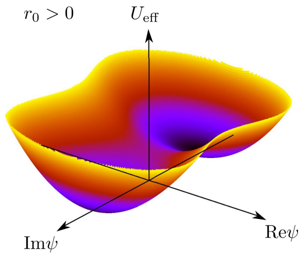
$$\frac{1}{N}S[\psi] = (\epsilon_0 - \mu)\psi_0^*\psi_0 + \frac{|\gamma_0|}{2}(\psi_0\psi_0 + \psi_0^*\psi_0^*) + \frac{\Gamma}{4}\psi_0^*\psi_0^*\psi_0\psi_0$$

Extremum condition

$$\begin{aligned} 0 &= (\epsilon_0 - \mu)\psi^* + |\gamma_0|\psi_0 \\ &\quad + \frac{\Gamma}{2}\psi_0^*\psi_0^*\psi_0 \end{aligned}$$

Extrema

$$\psi_0 = \pm i\sqrt{\frac{2(\gamma_0 - \epsilon_0 + \mu)}{\Gamma}}$$



BEC at $\boldsymbol{q}$

Momenta conservation couples momenta multiple to $\boldsymbol{q}$

$$
\begin{aligned}
-\left(\epsilon_n-\mu\right)\psi_n^{\bar\sigma}-\gamma_n\psi_n^{\sigma} \;=\;& \frac{1}{2}\sum_{n_1,n_2}\sum_{\sigma_1,\sigma_2}\delta_{n,n_1+n_2}V_{nn_1n_2}^{\sigma\sigma_1\sigma_2}\psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}\\
&+\frac{1}{3!}\sum_{n_1,n_2,n_3}\sum_{\sigma_1,\sigma_2,\sigma_3}\delta_{n,n_1+n_2+n_3}U_{nn_1n_2n_3}^{\sigma\sigma_1\sigma_2\sigma_3}\psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}\psi_{n_3}^{\sigma_3}
\end{aligned}
$$

Consider $\psi_n^{\sigma}=\delta_{n,1}\psi_1^{\sigma}+\delta_{n,-1}\psi_{-1}^{\sigma}$, for $n=0,\pm2,\pm3$ left-hand side

$$
-\left(\epsilon_n-\mu\right)\psi_n^{\bar\sigma}-\gamma_n\psi_n^{\sigma}=0
$$

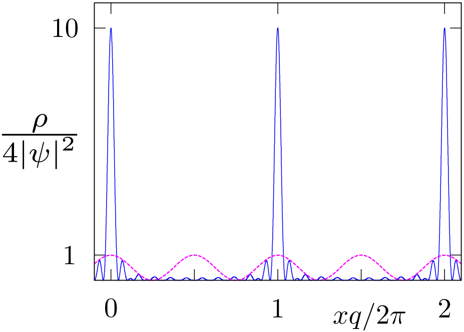
but right-hand side

$$
\begin{aligned}
&\frac{1}{2}\sum_{n_1,n_2}\sum_{\sigma_1,\sigma_2}\delta_{n,n_1+n_2}V_{nn_1n_2}^{\sigma\sigma_1\sigma_2}\psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}\\
+&\frac{1}{3!}\sum_{n_1,n_2,n_3}\sum_{\sigma_1,\sigma_2,\sigma_3}\delta_{n,n_1+n_2+n_3}U_{nn_1n_2n_3}^{\sigma\sigma_1\sigma_2\sigma_3}\psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}\psi_{n_3}^{\sigma_3}\neq 0
\end{aligned}
$$

All integer multiples of $\boldsymbol{q}$ have to condense!

Real space BEC

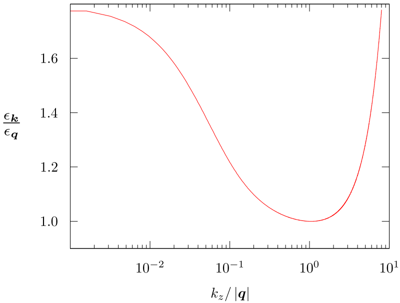
If many $\psi_{\boldsymbol{n}}^{\sigma}$ contribute with same order of magnitude, BEC localized at sides of 1-dim lattice with spacing $2\pi/\left|\boldsymbol{q}\right|$



BEC resembles liquid-solid transition

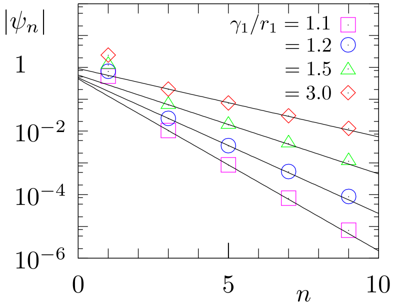
Magnons in YIG

- Effective Hamiltonian for magnons in YIG has the before discussed form
- Quadratic $U(1)$ symmetry breaking terms due to parallel pumping
- Vertices studied by A. Kreisel *et al.* '09
- Dispersion has two degenerated minima at $\pm \boldsymbol{q}$



Magnons in YIG

- Three-point vertices $V^{\sigma\sigma_1\sigma_2}_{nn_1n_2} = 0$ at $\boldsymbol{k} = n \cdot \boldsymbol{q}$ $\Rightarrow$ only components with odd multiples of $\boldsymbol{q}$ have to be finite
- system of equations truncated at $n = 9$ for nummerical calculations
- first Fourier component dominant



Conclusion

- Due to the time-independent Gross-Pitaeskii equation, all Fouier components of the BEC with momentum multiple to the lowest momentum has to be finite in generic interactions
- In the case of YIG, only the odd multiples condense and the Fourier components decay rapidly