Microscopic theory of spin waves in ferromagnetic films

Quantum theory for magnons in yttrium-iron garnet (YIG)

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Kreisel *et al.* arXiv: 0903.2847

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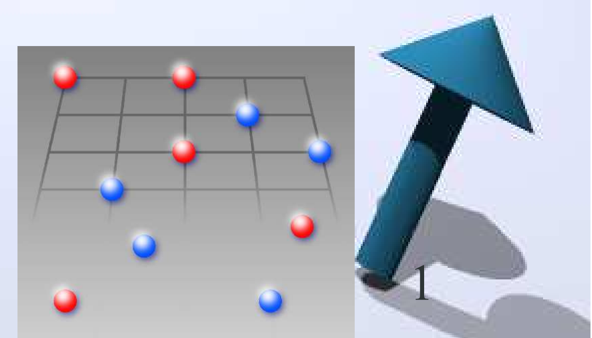


Heisenberg model

quantum spin S ferromagnet

Zeeman term

dipole-dipole interactions

$$
\begin{aligned}
\hat{H} &= -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - h\sum_i S_i^z \\
&\quad - \frac{1}{2}\sum_i\sum_{j\neq i}\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right] \\
&= -\frac{1}{2}\sum_{ij}\sum_{\alpha\beta}\left[J_{ij}\delta^{\alpha\beta} + D_{ij}^{\alpha\beta}\right] S_i^\alpha S_j^\beta - h\sum_i S_i^z
\end{aligned}
$$

- Dipolar tensor

$$D_{ij}^{\alpha\beta} = (1-\delta_{ij})\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3\hat{r}_{ij}^\alpha\hat{r}_{ij}^\beta - \delta^{\alpha\beta}\right]$$

material parameters

$a = 12.376$ Å
Gilleo *et al.* '58

$4\pi M_s = 1750$ G
Tittmann '73

$\frac{\rho_{\rm ex}}{\mu} = 5.17\cdot 10^{-13}\,\mathrm{Oe\ m}^2$
Cherepanov *et al.* '93

alternatively

$J = 1.29$ K

$S = 14.2 \qquad \mu = 2\mu_B$

Tupitsyn *et al.* '08

Spin wave approximation

- classical groundstate for stripe geometry
- Holstein Primakoff transformation (bosons)

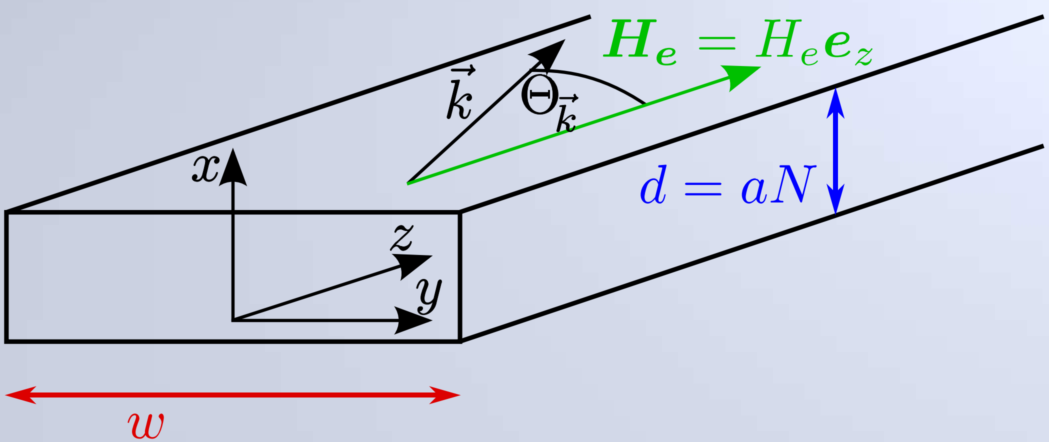
$$\hat{H}_2 = \sum_{ij} \left[A_{ij} b_i^\dagger b_j + \frac{B_{ij}}{2} \left(b_i b_j + b_i^\dagger b_j^\dagger \right) \right]$$

Costa *et al.* '98

$$A_{ij} = \delta_{ij} h + S(\delta_{ij} \sum_n J_{in} - J_{ij}) + S \left[\delta_{ij} \sum_n D_{in}^{zz} - \frac{D_{ij}^{xx} + D_{ij}^{yy}}{2} \right],$$

$$B_{ij} = -\frac{S}{2} [D_{ij}^{xx} - 2iD_{ij}^{xy} - D_{ij}^{yy}]$$

dipolar tensor



Stripe geometry

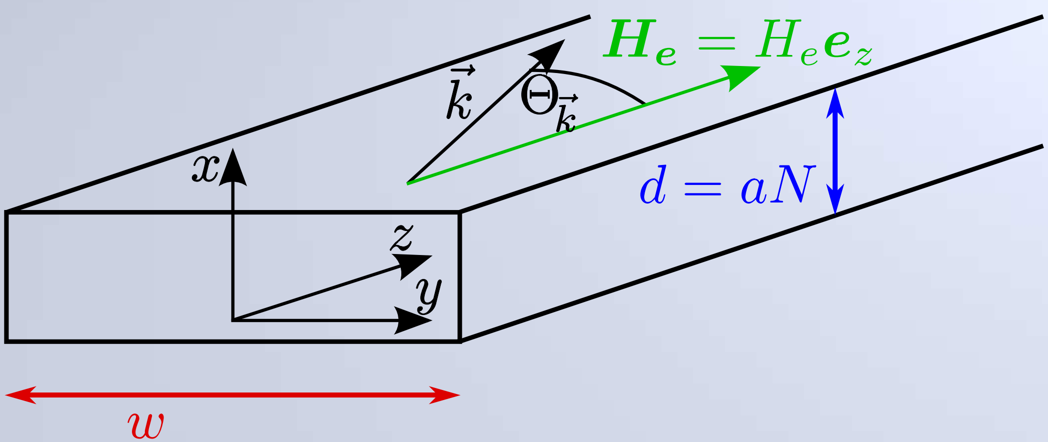
- partial Fourier transformation

$w \to \infty$

$$b_i = \frac{1}{\sqrt{N_y N_z}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_i} b_{\vec{k}}(x_i)$$

- find all branches

$$\det \begin{pmatrix} E_{\vec{k}}\mathsf{I} - \mathsf{A}_{\vec{k}} & -\mathsf{B}_{\vec{k}} \\ -\mathsf{B}^*_{\vec{k}} & -E_{\vec{k}}\mathsf{I} - \mathsf{A}_{\vec{k}} \end{pmatrix} = 0$$



Problems:
1) dipolar sums
2) large matrices

Numerical approach

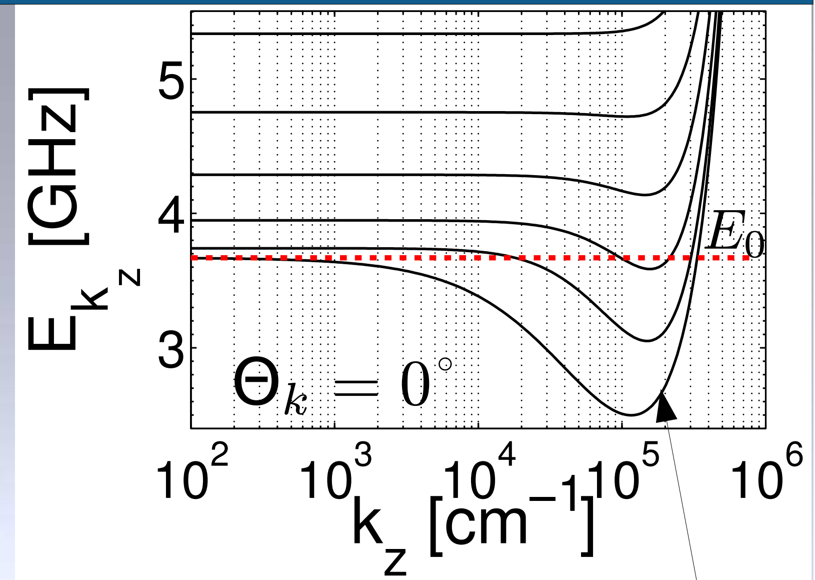
1) numerical diagonalization of 2Nx2N matrix

$$H_2 = \begin{pmatrix} A_{\vec{k}} & B_{\vec{k}} \\ -B^T_{\vec{k}} & -A_{\vec{k}} \end{pmatrix}$$

2) evaluation of dipol sums (Ewald summation technique)

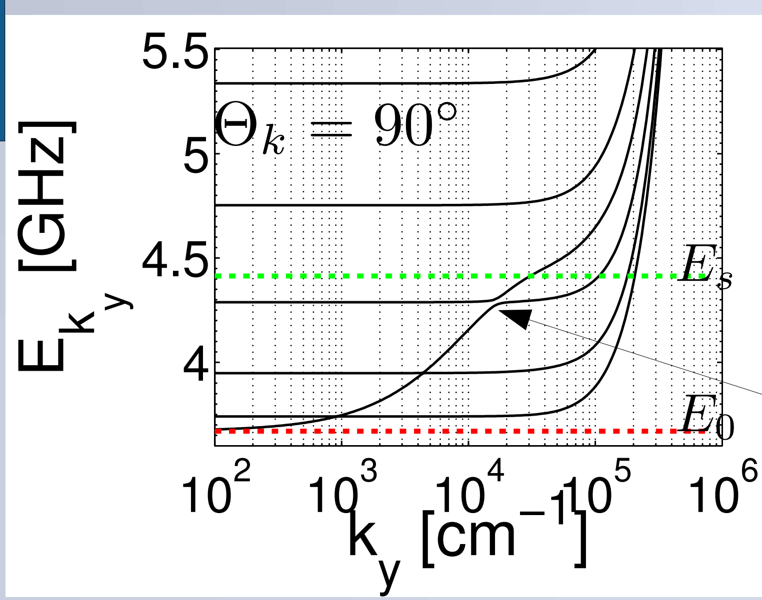
Bartosch *et al.* '06

$d = 400a \approx 0.5\mu\mathrm{m}$ $\quad N = 400 \quad H_e = 700$ Oe



minimum for BEC

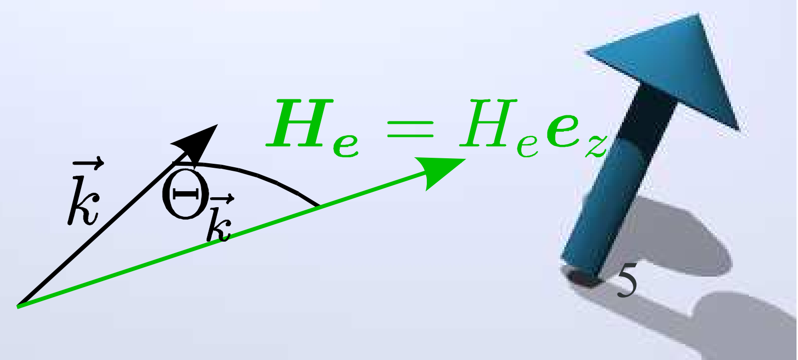
Demokritov *et al.* '06



$$E_0 = \sqrt{h(h + 4\pi\mu M_s)}$$

$$E_s = h + 2\pi\mu M_s$$

hybridization: surface mode



Analytical results with approximation

dispersion via Bogoliubov transformation

$$E_{\vec{k}} = \sqrt{[h + \rho_{\text{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\text{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

$$\Delta = 4\pi\mu M_S$$

- uniform approximation
  $\Rightarrow$ form factor

$$b_{\vec{k}}(x_i) \approx \frac{1}{\sqrt{N}} b_{\vec{k}}$$

$$f_{\vec{k}} = \frac{1 - e^{-|\vec{k}|d}}{|\vec{k}|d}$$

compare: Kalinikos *et al.* ‘86
Tupitsyn *et al.* ‘08

- eigenmode approximation
  $\Rightarrow$ different form factor:

$$b_{\vec{k}}(x_i) \approx \sqrt{\frac{2}{N}} \cos\left(\frac{\pi}{2d} x_i\right) b_{\vec{k}}$$

eigenfunction of
exchange matrix

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1 + e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$

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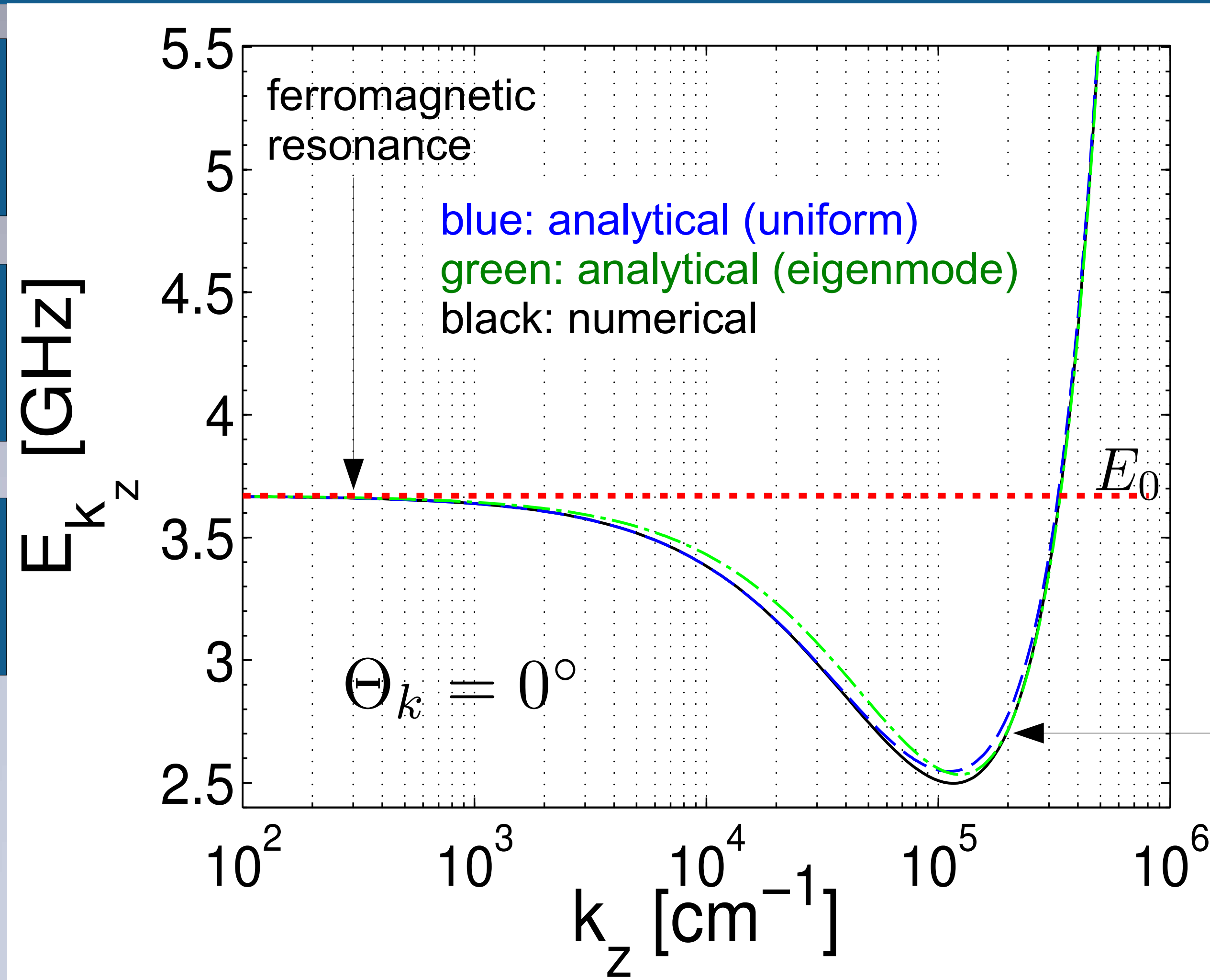
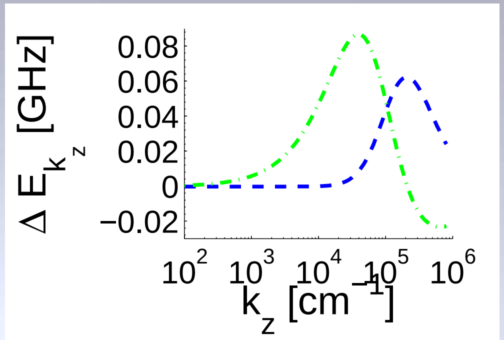
Comparison: lowest mode





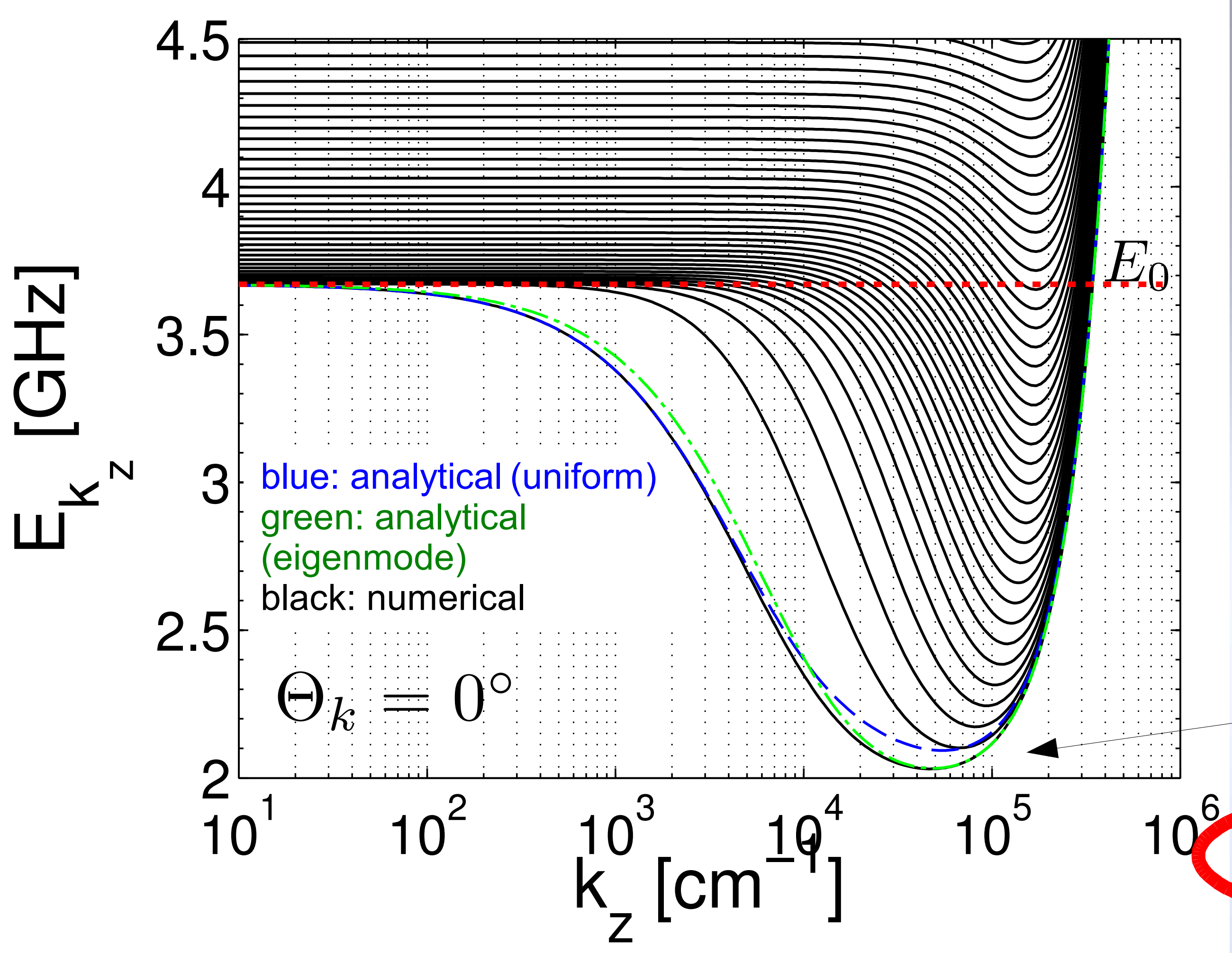
small deviations

$d = 400a \approx 0.5 \mu\mathrm{m}$

$H_e = 700$ Oe

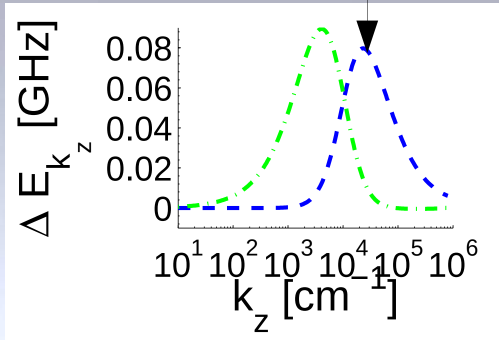
$$E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta(1 - f_{\vec{k}}) \sin^2 \Theta_{\vec{k}}][h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta f_{\vec{k}}]}$$

Real system: all modes



eigenmode approximation better

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1+e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$



minimum for BEC

Demokritov *et al.* '06

$d = 4040a \approx 5\mu\text{m}$

$H_e = 700$ Oe

$$E_{\vec{k}} = \sqrt{[h + \rho_{\text{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\text{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

Summary and Outlook

- Microscopic theory for YIG:
  numerical (all modes): summation of dipolar sums + diagonalization
  analytical (first mode): uniform / eigenmode approximation (good agreement for relevant parameters)

- Magnon-magnon interactions:
  momentum dependence can be taken into account

- Acknowledgements

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