Spin Phonon Interactions in Triangular Antiferromagnets

A spin-wave approach for the ordered phase

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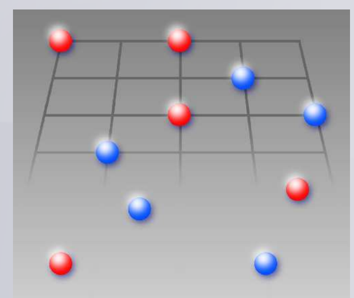
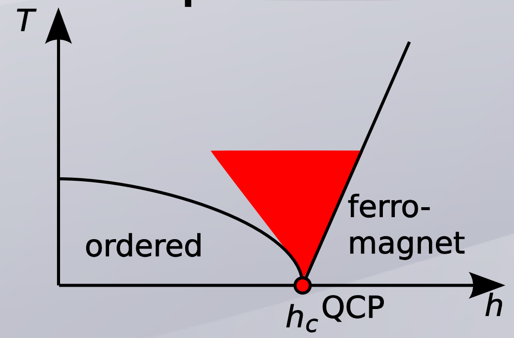


Motivation

- Triangular antiferromagnet:
  - possible realization of spin liquid, Anderson ‘73
    but also ordered phase with magnons as elementary excitations Veillette *et al.* ‘05 Dalidovich *et al.* ‘06 Chernyshev *et al.* ‘09
  - Quantum critical: influences of quantum critical point due to quantum fluctuations at T>0



Radu *et al.* ‘05

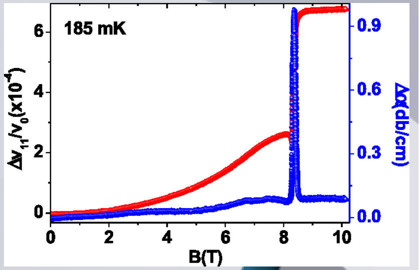
  - First: understand properties of the phases away from the quantum critical point!

Probing Magnetic Properties

- direct measurements:
  Neutron scattering experiments
  - poor resolution at low energies; detection of gap in excitation spectrum difficult
  - Not all informations about properties of phase:
    Broadening in dynamical structure factor
    Spinons or damped spin waves?

Konho *et al.* ‘07
Starykh *et al.* ‘07

- indirect measurements
  Specific heat, thermal expansion, ...
  Ultrasound technique
  - High resolution at small energies, momenta
  - But: not much information about properties of magnetic excitations



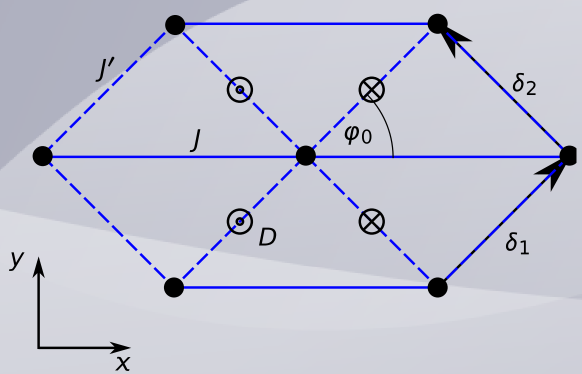
Talk: P. T. Cong
TT 15.3 Tue 14:30

Model

- antiferromagnet on a triangular lattice $\mathrm{Cs_2CuCl_4}$

$$H = \frac{1}{2}\sum_{ij}\left[J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \mathbf{D}_{ij}\cdot(\mathbf{S}_i\times\mathbf{S}_j)\right] - \sum_i \mathbf{h}\cdot\mathbf{S}_i$$

exchange — DM anisotropy



$$+\sum_{\mathbf{k}\lambda}\left[\frac{P_{-\mathbf{k}\lambda}P_{\mathbf{k}\lambda}}{2M} + \frac{M}{2}\omega^2_{\mathbf{k}\lambda}X_{-\mathbf{k}\lambda}X_{\mathbf{k}\lambda}\right]$$

phonon dispersion (acoustic)

Coldea *et al.* '02
Veillette *et al.* '05

- spin phonon coupling via expansion of exchange integrals

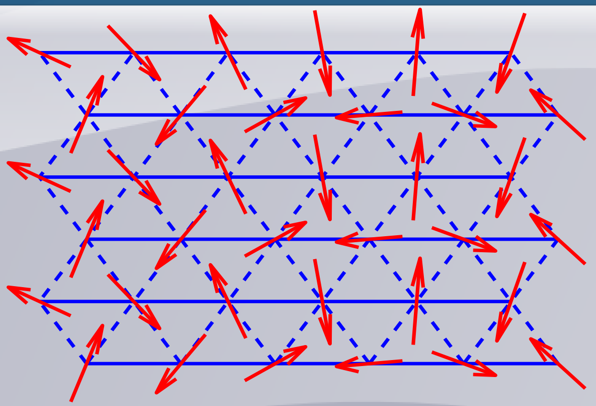
$$J_{ij} = J(\mathbf{r}_{ij}) + (\mathbf{X}_{ij}\cdot\nabla_{\mathbf{r}})\,J(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_{ij}} + \ldots = J(\mathbf{r}_{ij}) + X_{ij} + \ldots$$

bare exchange — magnon phonon coupling

Chakraborty *et al.* '87

Spin Wave Approach for the Ordered Phase

- classical groundstate spiral with ordering vector Q

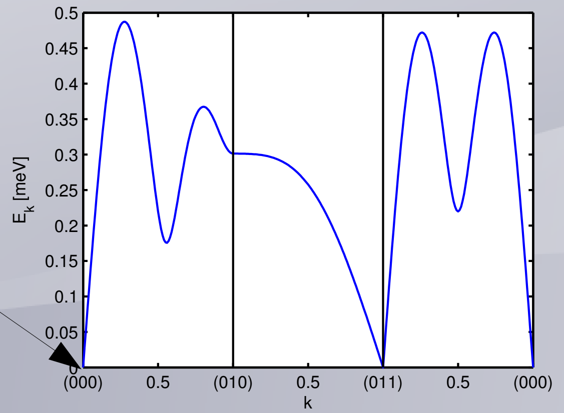


- linear spin wave theory (magnons)

$$H^s \approx \sum_{\mathsf{k}} E_{\mathsf{k}} \beta^\dagger_{\mathsf{k}} \beta_{\mathsf{k}} \qquad E_{\mathsf{k}} = \sqrt{A_{\mathsf{k}}^2 - B_{\mathsf{k}}^2} + C_{\mathsf{k}} \neq E_{-\mathsf{k}}$$

$$h = 0$$

Goldstone mode (gapless)
small corrections in 1/S



Veillette *et al.* '05

Magnon Phonon Interactions

- 1/S expansion of coupling term

$$H^{\mathrm{sp}} = H^{s} + H^{1p}_{1m} + H^{1p}_{2m} + \dots$$

- Hybridization (coupled magnoelastic waves)

$$H^{1p}_{1m} = \sum_{\mathsf{k}} \left(\tilde{\Gamma}^{(1)}_{\mathsf{k}} \cdot \mathsf{x}_{\mathsf{k}} \beta_{-\mathsf{k}} + \tilde{\Gamma}^{(2)}_{\mathsf{k}} \cdot \mathsf{x}_{\mathsf{k}} \beta^{\dagger}_{\mathsf{k}} \right) + h.c.$$

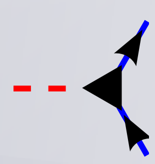
  - diagonalization: finite renormalization of sound velocity c

$$\delta c = \frac{v-c}{2} + \sqrt{\left(\frac{v-c}{2}\right)^2 + \partial_k \left(\tilde{\Gamma}^{(1)}_{k} + \tilde{\Gamma}^{(2)}_{-k} \right)^2 \Big|_{k=0}}\,, \quad c > v$$

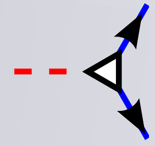
spin wave velocity

- three particle process

$$H^{1p}_{2m} = \frac{1}{\sqrt{N}} \sum_{\mathsf{k}\mathsf{k}'} \Big[\Gamma^{\beta^{\dagger}\beta}_{\mathsf{k},\mathsf{k}'} \cdot \mathsf{x}_{\mathsf{k}-\mathsf{k}'} \beta^{\dagger}_{\mathsf{k}} \beta_{\mathsf{k}'}$$



$$+ \frac{1}{2!} \left(\Gamma^{\beta^{\dagger}\beta^{\dagger}}_{\mathsf{k},\mathsf{k}'} \cdot \mathsf{x}_{\mathsf{k}+\mathsf{k}'} \beta^{\dagger}_{\mathsf{k}} \beta^{\dagger}_{\mathsf{k}'} + h.c.\Big] \right).$$



Effective Theory

- integrate out canonical momentum fields

Gaussian action

$$S[X_\lambda, \bar{\psi}, \psi] = \frac{\beta}{2} \sum_{K\lambda} M(\omega^2 + \omega_{\mathbf{k}\lambda}^2) X_{-K\lambda} X_{K\lambda} - \beta \sum_K (i\omega - E_{\mathbf{k}}) \bar{\psi}_K \psi_K$$

interactions

$$+ \frac{\beta}{2\sqrt{N}} \sum_{K,K',\lambda} \Big[(\Gamma^{\beta^\dagger \beta}_{\mathbf{k},\mathbf{k}'} \cdot \mathbf{e}_{\mathbf{k}-\mathbf{k}'\lambda}) X_{K-K'\lambda} \bar{\psi}_K \psi_{K'}$$

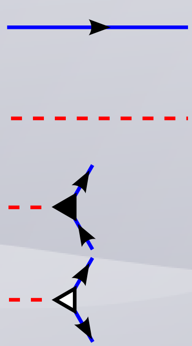
$$+ (\Gamma^{\beta^\dagger \beta^\dagger}_{\mathbf{k},\mathbf{k}'} \cdot \mathbf{e}_{\mathbf{k}+\mathbf{k}'\lambda}) X_{K+K'\lambda} \bar{\psi}_K \bar{\psi}_{K'} + c.c. \Big]$$

$G^0_\psi(K) = \frac{1}{i\omega - E_{\mathbf{k}}}$

$G^0_X(K\lambda) = \frac{1}{\omega^2 + \omega^2_{\mathbf{k}\lambda}}$

$\boldsymbol{\Gamma}^{\beta^\dagger \beta}_{\mathbf{k}',\mathbf{k}'-\mathbf{k}} \cdot \mathbf{e}_{\mathbf{k}\lambda}$

$\boldsymbol{\Gamma}^{\beta^\dagger \beta^\dagger}_{\mathbf{k}',\mathbf{k}-\mathbf{k}'} \cdot \mathbf{e}_{\mathbf{k}\lambda}$



Kreisel *et al.* ‘08

- full phonon propagator

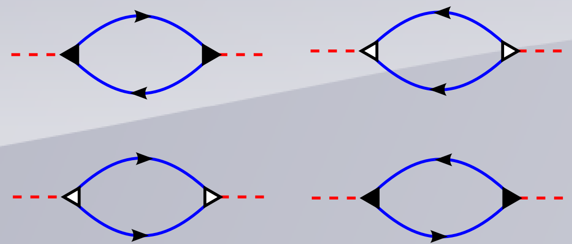
$$G_X(K\lambda) = \frac{1}{\omega^2 + \omega^2_{\mathbf{k}\lambda} + \Sigma_X(K\lambda)}$$

self-energy

Ultrasound Attenuation Rate

- self-energy

$\Sigma_X(\mathbf{k},\omega) =$



$G^0_\psi(K) = \frac{1}{i\omega - E_{\mathbf{k}}}$

$G^0_X(K\lambda) = \frac{1}{\omega^2 + \omega^2_{\mathbf{k}\lambda}}$

$\mathbf{\Gamma}^{\beta^\dagger\beta}_{\mathbf{k}',\mathbf{k}'-\mathbf{k}} \cdot \mathbf{e}_{\mathbf{k}\lambda}$

$\mathbf{\Gamma}^{\beta^\dagger\beta^\dagger}_{\mathbf{k}',\mathbf{k}-\mathbf{k}'} \cdot \mathbf{e}_{\mathbf{k}\lambda}$

$$\Sigma_X(K\lambda) = -\frac{1}{N}\sum_{\mathrm{k}'}\Big[\frac{|\Gamma^{\beta^\dagger\beta}_{\mathrm{k}',\mathrm{k}'-\mathrm{k}} \cdot \mathrm{e}_{\mathrm{k}\lambda}|^2}{2M}\frac{n(E_{\mathrm{k}'}) - n(E_{\mathrm{k}'-\mathrm{k}})}{i\omega - E_{\mathrm{k}'} + E_{\mathrm{k}'-\mathrm{k}}}$$

$$+\frac{|\Gamma^{\beta^\dagger\beta^\dagger}_{\mathrm{k}',\mathrm{k}-\mathrm{k}'} \cdot \mathrm{e}_{\mathrm{k}\lambda}|^2}{2M}\frac{n(E_{\mathrm{k}'}) + n(E_{\mathrm{k}-\mathrm{k}'}) + 1}{-i\omega + E_{\mathrm{k}'} + E_{\mathrm{k}-\mathrm{k}'}}\Big] + (K \to -K)$$

- attenuation rate

$$\gamma_{\mathrm{k}\lambda} = -\frac{\mathrm{Im}\Sigma_X(\omega_{\mathrm{k}\lambda} + i0, \mathrm{k}, \lambda)}{2\omega_{\mathrm{k}\lambda}}$$

- carry out integration:

contribution quadratic in momentum $\gamma_{\mathrm{k}\lambda} = I\,\mathrm{k}^2$
(magnons in the ordered phase)

Outlook

- finite magnetic field $h \neq 0$
  - linear spin wave theory obtains gapless spectrum although a gap is not forbidden by symmetry
  - overcome divergencies in spin wave approach which have to cancel in physical quantities

    Zhitomirsky *et al.* '99, Maleyev '00, Hasselmann *et al.* '06
  - more sophisticated approach

    (A. Kreisel, P. Kopietz, in preparation)

Summary

- extension of spin wave theory with Spin phonon interactions
- perturbative calculation of sound velocity and phonon damping in 1/S expansion
- possible indirect measurement of magnetic properties with help of phonons:
  detect contribution to frequency shift and attenuation rate in experiments