Magnon BEC at finite momentum

From textbook knowledge towards BEC of magnons

Andreas Kreisel, Francesca Sauli, Johanes Hick, Lorenz Bartosch, Peter Kopietz
Institut für Theoretische Physik
Goethe Universität Frankfurt
Germany

Christian Sandweg, Matthias Jungfleisch, Vitaliy Vasyucka, Alexander Serga, Peter Clausen, Helmut Schultheiss, Burkard Hillebrands
Fachbereich Physik
Technische Universität Kaiserslautern
Germany

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AG MAGNETISMUS TU KAISERSLAUTERN

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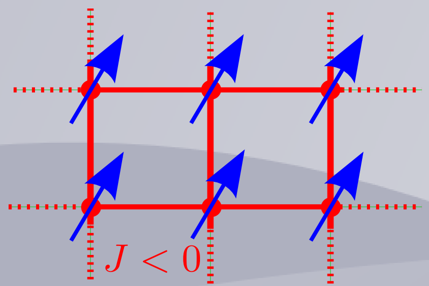
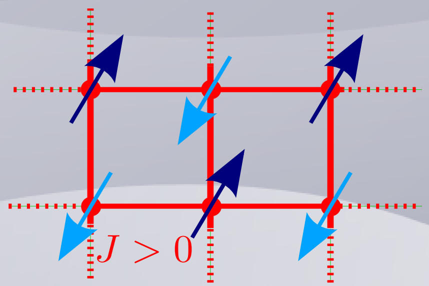
1. Introduction: Spin-wave theory

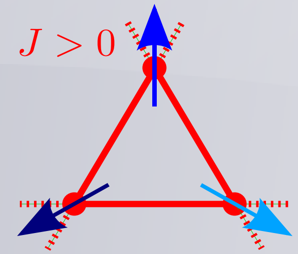
- Heisenberg model $H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$
- determine ordered classical groundstate
  - ferromagnet
    classical groundstate=quantum groundstate



  - anti-ferromagnet
    (2 sublattices, Néel groundstate)
    AK, Hasselmann, Kopietz, '07
    AK, Sauli, Hasselmann, Kopietz, '08



  - triangular anti-ferromagnet
    (3 sublattices, frustration)
    Chernychev, Zhitomirsky ('09)
    Veillette *et al.* ('05)
    AK, Kopietz (in preparation)



1. Spin wave theory

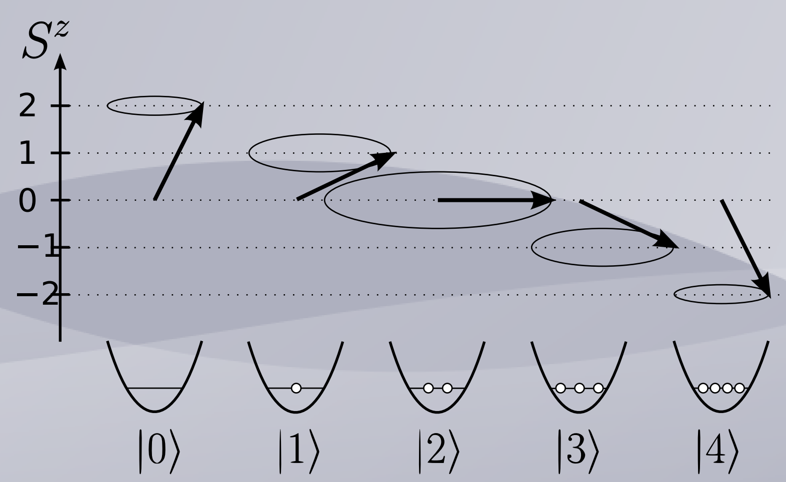
- expand in terms of bosons (1/S expansion), Holstein-Primakoff transformation

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\hat{S}^z = S - \hat{n} \qquad \hat{n} = \hat{b}^\dagger \hat{b} \qquad \left[\hat{b}, \hat{b}^\dagger\right] = 1$$

$$\hat{S}^+ = \sqrt{2S}\sqrt{1 - \frac{\hat{n}}{2S}}\hat{b}$$

$$\hat{S}^- = \sqrt{2S}\hat{b}^\dagger\sqrt{1 - \frac{\hat{n}}{2S}} \qquad \sqrt{1 - \frac{\hat{n}}{2S}} = 1 - \frac{\hat{n}}{4S} + \mathcal{O}(\frac{1}{S^2})$$



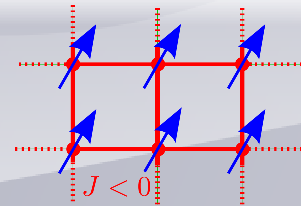
Holstein, Primakoff, Phys. Rev. **58**, 1098 (1940)

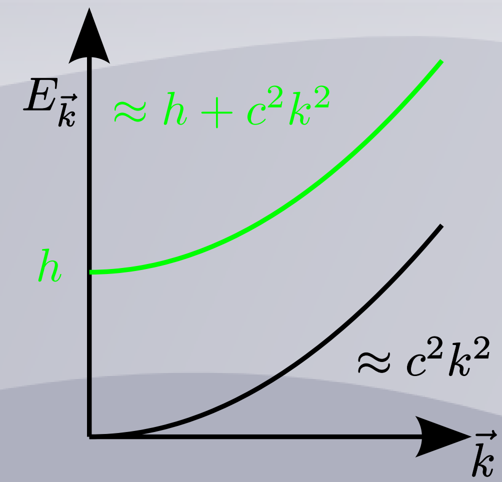
- determine properties of resulting interacting theory of bosons

$$H = \sum_{\vec{k}} E_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} \Gamma^3(\vec{k}_1, \vec{k}_2, \vec{k}_3) b^\dagger_{\vec{k}_1} b_{\vec{k}_2} b_{\vec{k}_3} + \sum_{1,2,3,4} \Gamma^4(1,2;3,4) b^\dagger_1 b^\dagger_2 b_3 b_4 + \ldots$$

1. Spin wave theory: General results

- ferromagnet

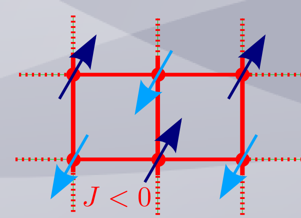




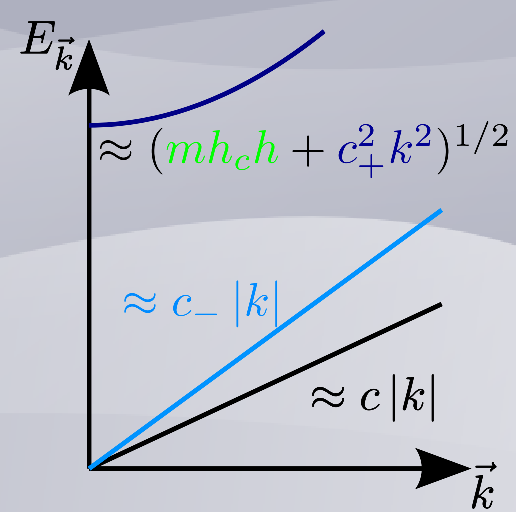
  - quadratic excitation spectrum
  - vanishing interaction vertices

$$\Gamma^4 \sim -(\vec{k}_1 \cdot \vec{k}_2 + \vec{k}_3 \cdot \vec{k}_4)$$

- antiferromagnet



  - linear spectrum (Goldstone mode)
  - two modes in magnetic field (2 sublattices)
  - divergent interaction vertices



$$\Gamma^4 \sim \sqrt{\frac{|\vec{k}_1||\vec{k}_2|}{|\vec{k}_3||\vec{k}_4|}} \left(1 \pm \frac{\vec{k}_1 \cdot \vec{k}_2}{|\vec{k}_1||\vec{k}_2|} \right)$$

Hasselmann, Kopietz ('06)

2.1 Spin-wave theory for thin film ferromagnets

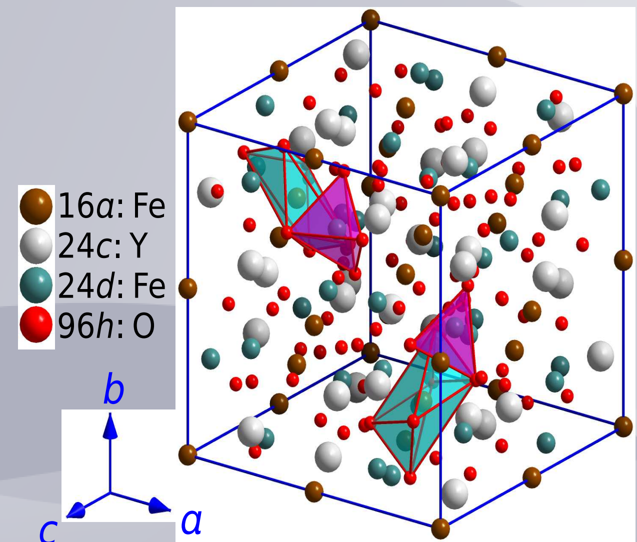
- Motivation: Experiments on YIG
  - Crystal structure:

space group: **Ia3d**
Y: 24(c) white
Fe: 24(d) green
Fe: 16(a) brown
O: 96(h) red

Gilleo *et al.* '58

Magnetic system:
40 magnetic ions in elementary cell
40 magnetic bands

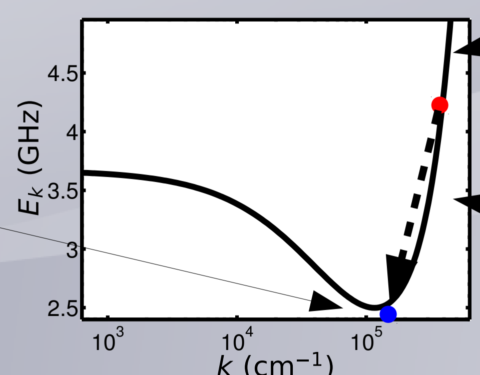
Elastic system:
160 atoms in elementary cell
3x160 phonon bands



  - low spin wave damping
  - good experimental control

Observation of the occupation number using microwave antennas or Brillouin Light Scattering (BLS)
Sandweg, *et al.*, Rev. Sci. Instrum. **81**, 073902 (2010)

BEC of magnons at room temperature!
Demokritov *et al.* Nature **443**, 430 (2006)

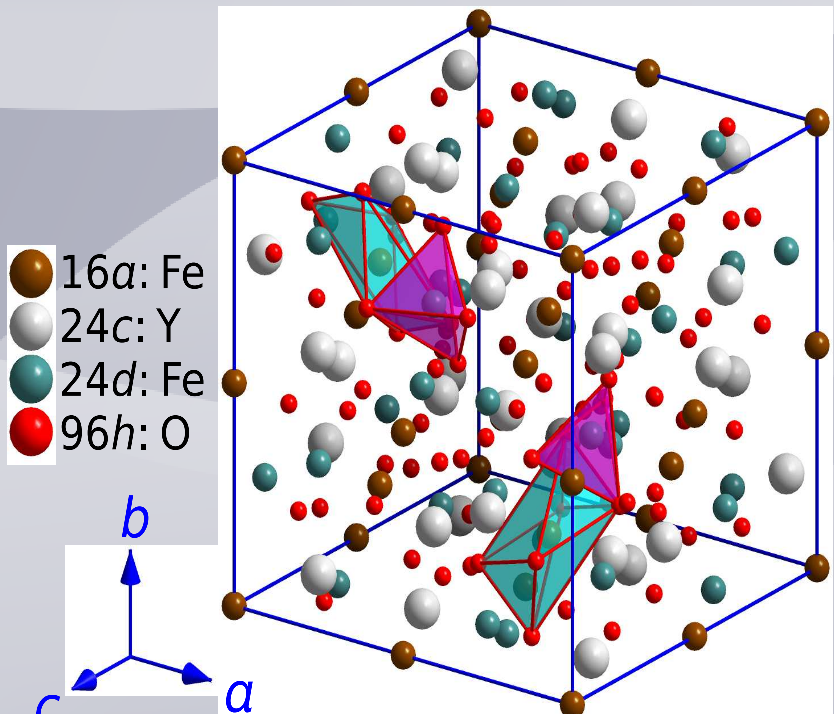


Parametric pumping of magnons at high k-vectors creates magnetic excitations

Question:
Time evolution of magnons:
Non-equilibrium physics of interacting quasiparticles

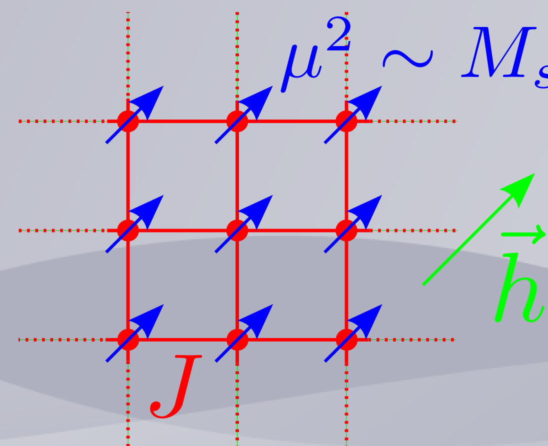
2.1 Simplifications to relevant physical properties

crystal structure of YIG



AK, Sauli, Bartosch, Kopietz ('09)

microscopic Hamiltonian



quantum spin S ferromagnet

Zeeman term

dipole-dipole interactions

$$\hat{H}_{\text{mag}} = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - h\sum_i S_i^z$$

$$-\frac{1}{2}\sum_i \sum_{j\neq i} \frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right]$$

2.1 Linear Spin Wave Theory

- classical groundstate for stripe geometry
- Holstein Primakoff transformation (bosons)

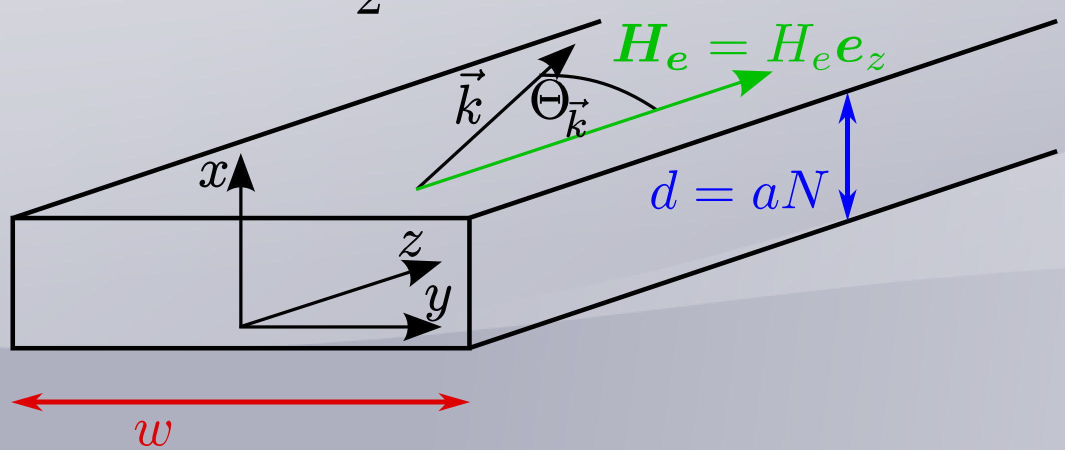
$$\hat{H}_2 = \sum_{ij} \left[A_{ij} b_i^\dagger b_j + \frac{B_{ij}}{2} \left(b_i b_j + b_i^\dagger b_j^\dagger \right) \right]$$

Filho Costa *et al.* Sol. State Comm. **108**, 439 (1998)

$$A_{ij} = \delta_{ij} h + S(\delta_{ij} \sum_n J_{in} - J_{ij}) + S \left[\delta_{ij} \sum_n D_{in}^{zz} - \frac{D_{ij}^{xx} + D_{ij}^{yy}}{2} \right],$$

$$B_{ij} = -\frac{S}{2} [D_{ij}^{xx} - 2i D_{ij}^{xy} - D_{ij}^{yy}]$$

dipolar tensor



- partial Fourier transformation (quasi 2D)

2.1 Linear Spin-wave theory: Numerical approach

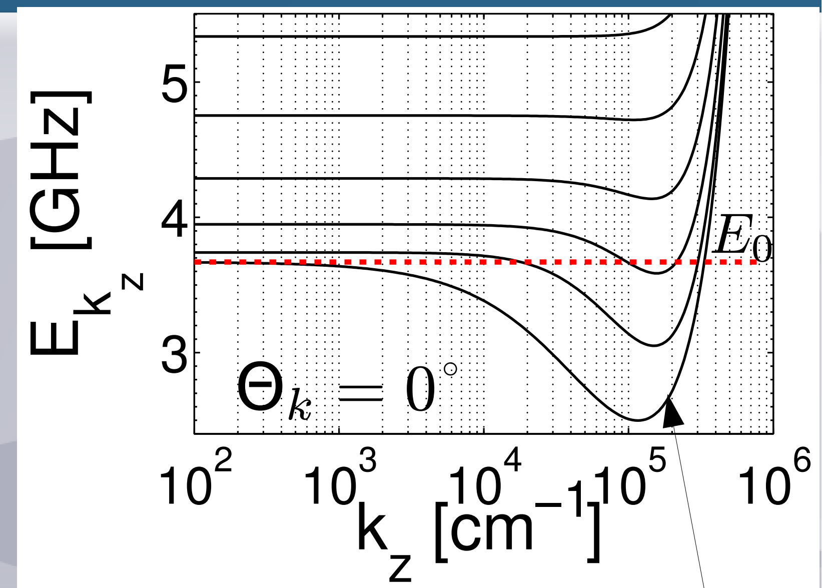
1) numerical diagonalization of 2Nx2N matrix

2) evaluation of dipol sums (Ewald summation technique)

Bartosch *et al.* '06

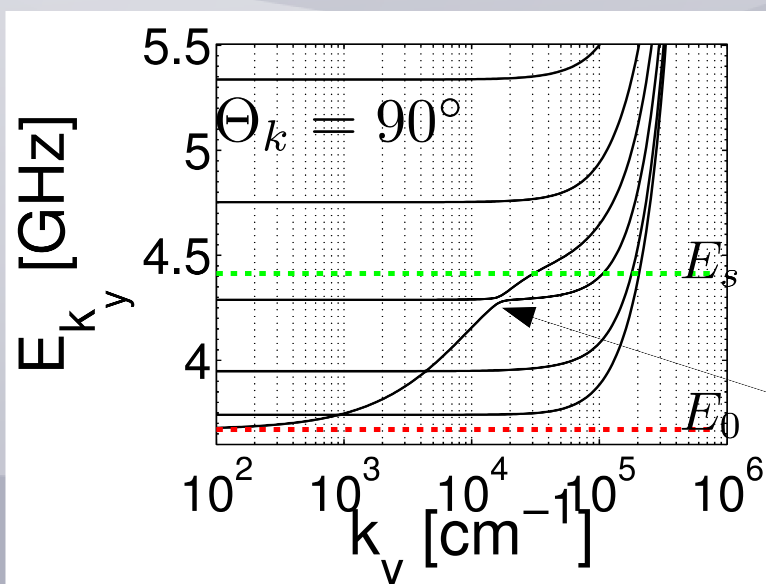
$$H_2 = \begin{pmatrix} A_{\vec{k}} & B_{\vec{k}} \\ -B^T_{\vec{k}} & -A_{\vec{k}} \end{pmatrix}$$

$d = 400a \approx 0.5\mu\mathrm{m} \qquad N = 400 \quad H_e = 700\ \mathrm{Oe}$



minimum for BEC

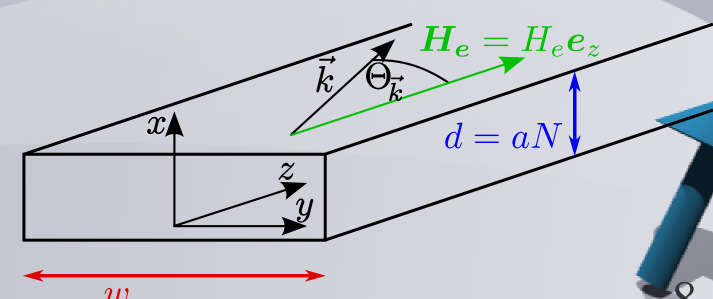
Demokritov *et al.* '06



$$E_0 = \sqrt{h(h + 4\pi\mu M_s)}$$

$$E_s = h + 2\pi\mu M_s$$

hybridization: surface mode



2.1 Linear spin-wave theory: Analytical approach

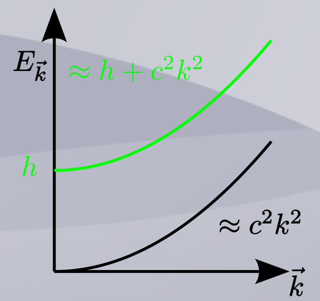
$$\hat{H} = \sum_{\vec{k}} \left[A_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \frac{B_{\vec{k}}}{2} b_{\vec{k}} b_{\vec{k}} + \frac{B^*_{\vec{k}}}{2} b^\dagger_{\vec{k}} b^\dagger_{\vec{k}} \right]$$

- dispersion via Bogoliubov transformation

$$E_{\vec{k}} = \sqrt{[h + \rho_{\text{ex}} \vec{k}^2 + \Delta(1 - f_{\vec{k}}) \sin^2 \Theta_{\vec{k}}][h + \rho_{\text{ex}} \vec{k}^2 + \Delta f_{\vec{k}}]} \qquad \Delta = 4\pi\mu M_S$$

- no dipolar interaction: $\Delta = 0$

$$E_{\vec{k}} = h + \rho_{\text{ex}} \vec{k}^2$$



- uniform mode approximation

$$f_{\vec{k}} = \frac{1 - e^{-|\vec{k}|d}}{|\vec{k}|d}$$

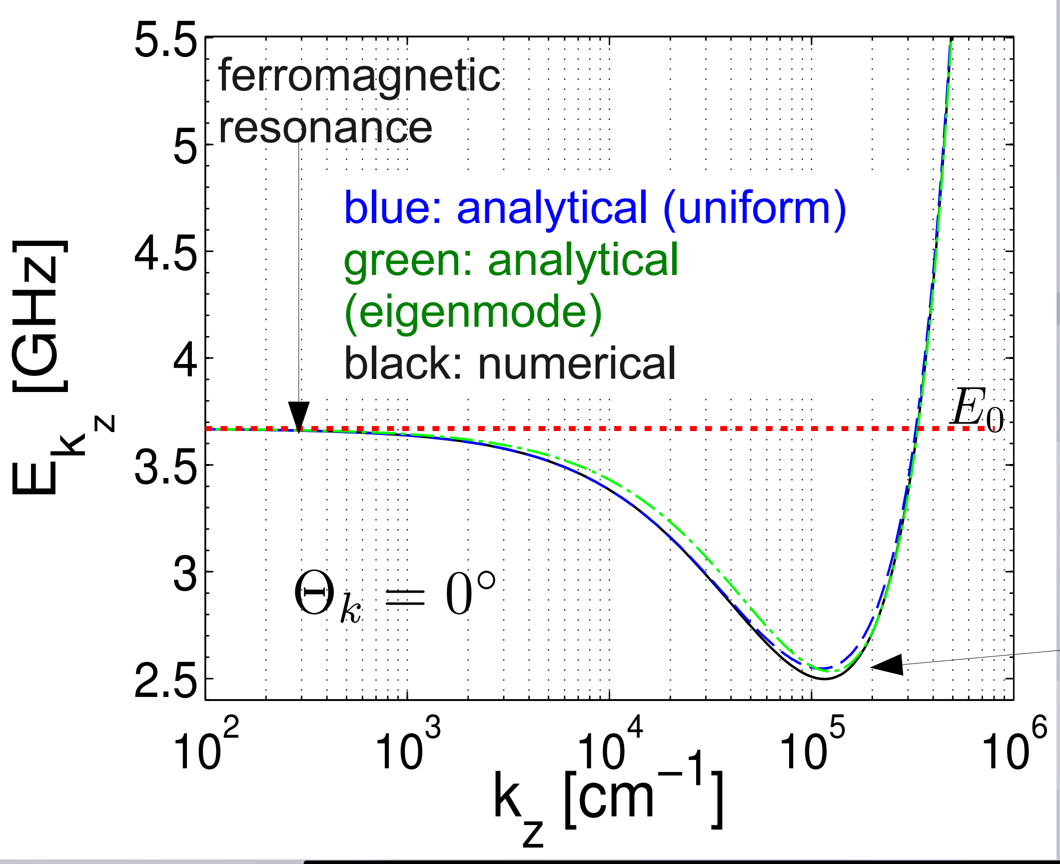
compare: Kalinikos *et al.* '86
Tupitsyn *et al.* '08

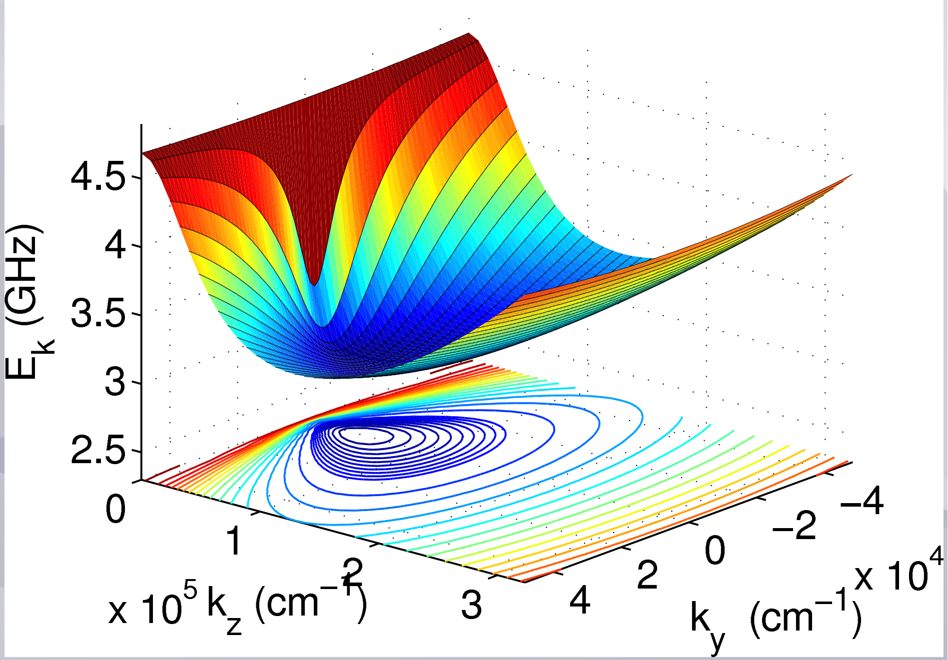
- lowest eigenmode approximation

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1 + e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$

AK, Sauli, Bartosch, Kopietz ('09)

2.1 Comparison





small deviations

$$d = 400a \approx 0.5\mu\text{m}$$

$$H_e = 700 \text{ Oe}$$

$$E_{\vec{k}} = \sqrt{[h + \rho_{\text{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\text{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

2.2 Condensate in YIG: BEC at finite momentum

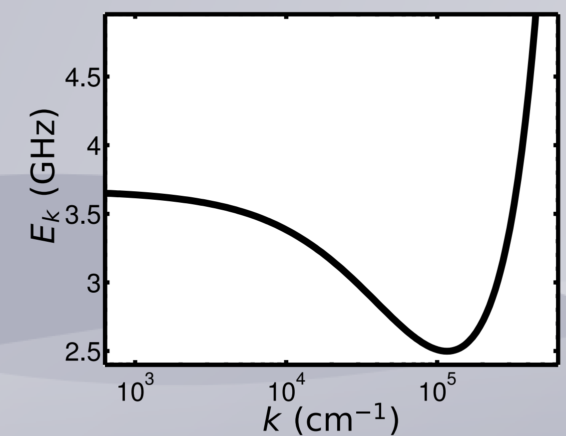
parallel pumping

- Hamiltonian

$$H_2 = \sum_{\vec{k}} \epsilon_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \frac{1}{2} \sum \left(\gamma b^\dagger b^\dagger + \gamma^* b b \right)$$

- new features for YIG system
  - condensate at finite wave-vectors

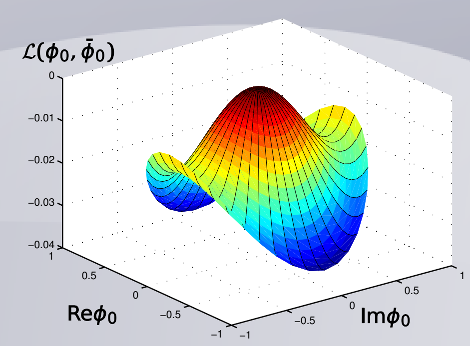
$$\phi_k = \delta_{k,k_{\min}} \phi_0$$



  - possible 2 condensates $\epsilon_{\vec{k}} = \epsilon_{-\vec{k}}$

$$\phi_k = \delta_{k,k_{\min}} \phi_0^+ + \delta_{k,-k_{\min}} \phi_0^-$$

  - explicitly symmetry breaking term



Napoleons hat potential

2.2 BEC at finite momentum

- Euclidean action $S[\Phi] = S_2[\Phi] + S_3[\Phi] + S_3[\Phi]$

$$S_2[\Phi] \;=\; \frac{1}{2}\int_0^\beta d\tau \sum_{\mathsf{k}} (\Phi^{\bar{a}}_{-\mathsf{k}}, \Phi^{a}_{-\mathsf{k}}) \begin{pmatrix} \partial_\tau + \epsilon_{\mathsf{k}} - \mu & \gamma_{\mathsf{k}} \\ \gamma_{\mathsf{k}} & -\partial_\tau + \epsilon_{\mathsf{k}} - \mu \end{pmatrix} \begin{pmatrix} \Phi^{a}_{\mathsf{k}} \\ \Phi^{\bar{a}}_{\mathsf{k}} \end{pmatrix}$$

- Bogoliubov shift $\Phi_{\vec{k}}(\tau) = \phi_{\vec{k}} + \delta\Phi_{\vec{k}}(\tau)$
- Gross-Pitaevskii equation from stationary point

$$\begin{aligned} 0 \;&=\; \left.\frac{\delta S[\Phi]}{\delta \Phi^{\sigma}_{\mathsf{k}}(\tau)}\right|_{\Phi=\phi} = (\epsilon_{\mathsf{k}} - \mu)\phi^{\bar{\sigma}}_{-\mathsf{k}} + \gamma_{\mathsf{k}}\phi^{\sigma}_{-\mathsf{k}} \\ &+\; \frac{1}{2\sqrt{N}} \sum_{\mathsf{k}+\mathsf{k}_1+\mathsf{k}_2=0} \sum_{\sigma_1\sigma_2} \Gamma_3(\mathsf{k}\sigma, \mathsf{k}_1\sigma_1, \mathsf{k}_2\sigma_2)\phi^{\sigma_1}_{\mathsf{k}_1}\phi^{\sigma_2}_{\mathsf{k}_2} \\ &+ \frac{1}{3!N} \sum_{\mathsf{k}+\mathsf{k}_1+\mathsf{k}_2+\mathsf{k}_3=0} \sum_{\sigma_1\sigma_2\sigma_3} \Gamma_4(\mathsf{k}\sigma, \mathsf{k}_1\sigma_1, \mathsf{k}_2\sigma_2, \mathsf{k}_3\sigma_3)\phi^{\sigma_1}_{\mathsf{k}_1}\phi^{\sigma_2}_{\mathsf{k}_2}\phi^{\sigma_3}_{\mathsf{k}_3} \end{aligned}$$

- two component BEC does not solve GPE

$$\phi^{\sigma}_{\mathsf{k}} = \delta_{\mathsf{k},\mathsf{q}}\psi^{\sigma}_{n} + \delta_{\mathsf{k},-\mathsf{q}}\psi^{\sigma}_{n}$$

2.2 BEC at finite momentum

- interactions provoke condensation at integer multiples of $\vec{k}_{\min}$

$$\phi^\sigma_{\mathsf{k}} = \sqrt{N} \sum_{n=-\infty}^{\infty} \delta_{\mathsf{k},n\mathsf{q}} \psi^\sigma_n$$

- discrete Gross-Pitaevskii equation for Fourier components

$$-(\epsilon_{n\mathsf{q}} - \mu)\psi^{\bar{\sigma}}_n - \gamma_n \psi^\sigma_n \quad = \quad \frac{1}{2} \sum_{n_1 n_2} \sum_{\sigma_1 \sigma_2} \delta_{n,n_1+n_2} V^{\sigma\sigma_1\sigma_2}_{n n_1 n_2} \; \psi^{\sigma_1}_{n_1} \psi^{\sigma_2}_{n_2}$$

$$+\frac{1}{3!} \sum_{n_1 n_2 n_3} \sum_{\sigma_1 \sigma_2 \sigma_3} \delta_{n,n_1+n_2+n_3} U^{\sigma\sigma_1\sigma_2\sigma_3}_{n n_1 n_2 n_3} \; \psi^{\sigma_1}_{n_1} \psi^{\sigma_2}_{n_2} \psi^{\sigma_3}_{n_3} \; .$$

- condensate density

$$\begin{aligned} \rho(\mathsf{r}) &= |\phi^a(\mathsf{r})|^2 \\ &= 4 \sum_n |\psi_n|^2 \cos^2(n\mathsf{q} \cdot \mathsf{r}) \end{aligned}$$

2.2 Interaction Vertices for YIG

- no $U(1)$ symmetry

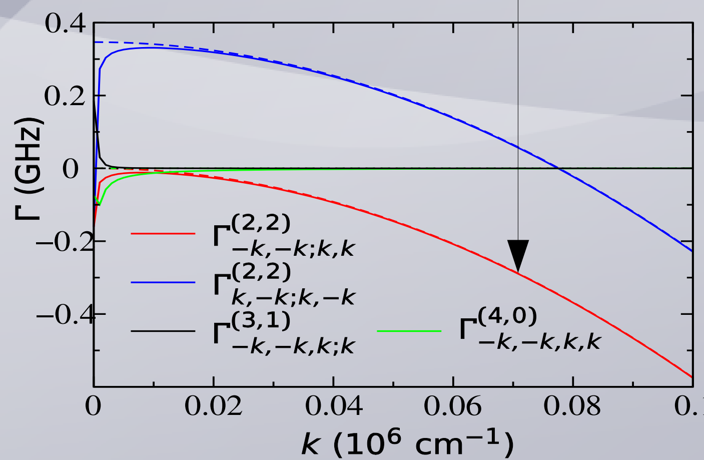
F. Sauli (in preparation)

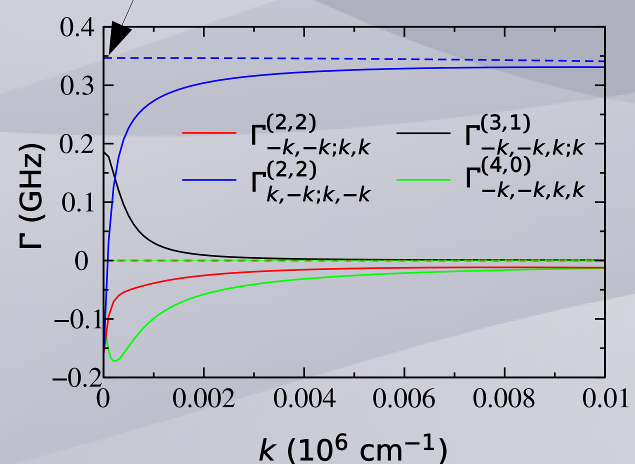
$$H_4 = \frac{1}{N} \sum_{\vec{k}_1 \cdots \vec{k}_4} \left(\frac{1}{(2!)^2} \Gamma^{(2,2)} b^\dagger b^\dagger bb + \frac{1}{3!} \left\{ \Gamma^{(3,1)} b^\dagger bbb + \text{h.c.} \right\} + \frac{1}{4!} \left\{ \Gamma^{(4,0)} bbbb + \text{h.c.} \right\} \right)$$

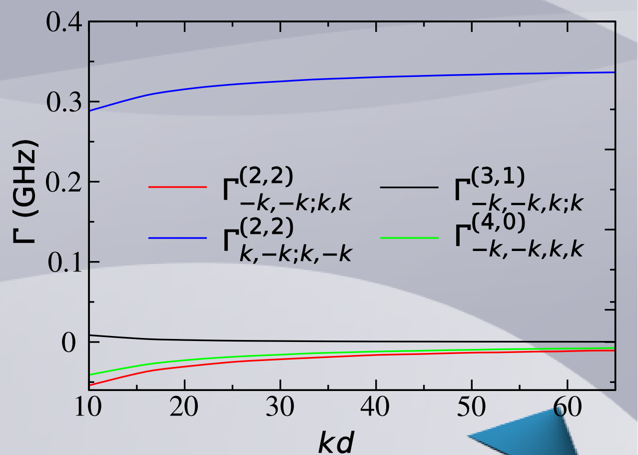
$\Gamma \sim -Jk^2$ ferromagnetic magnons

$\Gamma \sim M_s$ Rezende '09

$k = k_{\min}$ finite size effects





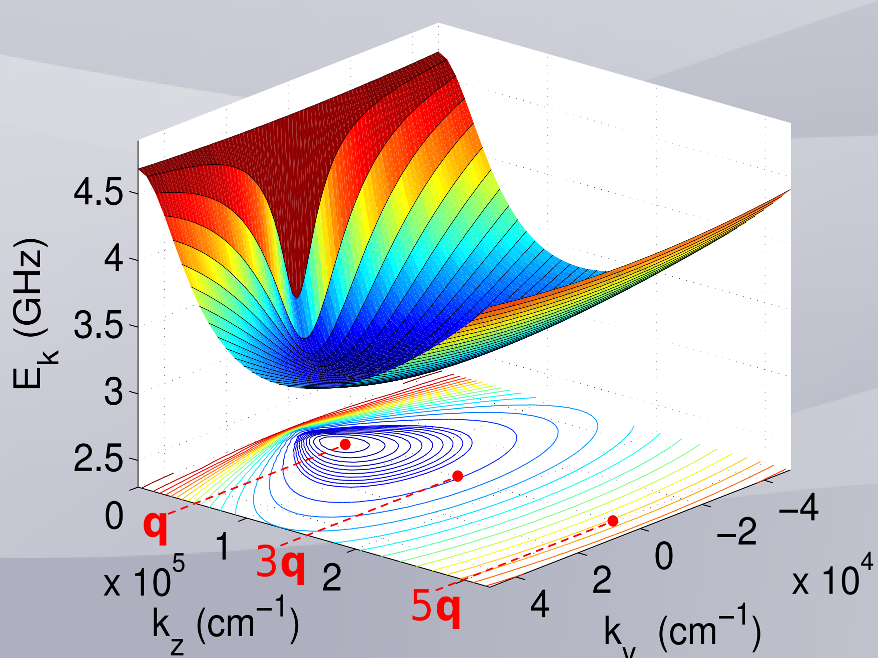


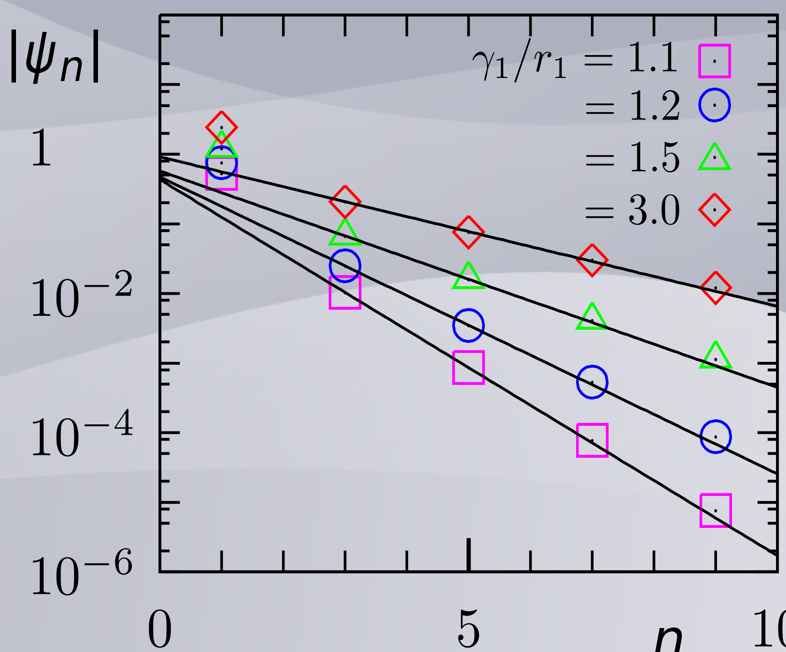
2.2 Solution of GPE

- solve discrete Gross-Pitaevskii equation

$$-(\epsilon_{n\mathrm{q}} - \mu)\psi_n^{\bar{\sigma}} - \gamma_n \psi_n^{\sigma} = \frac{1}{2}\sum_{n_1 n_2}\sum_{\sigma_1\sigma_2} \delta_{n,n_1+n_2} V_{nn_1n_2}^{\sigma\sigma_1\sigma_2}\, \psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}$$

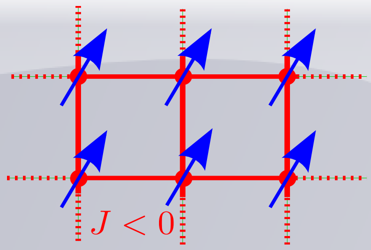
$$+\frac{1}{3!}\sum_{n_1 n_2 n_3}\sum_{\sigma_1\sigma_2\sigma_3} \delta_{n,n_1+n_2+n_3} U_{nn_1n_2n_3}^{\sigma\sigma_1\sigma_2\sigma_3}\, \psi_{n_1}^{\sigma_1}\psi_{n_2}^{\sigma_2}\psi_{n_3}^{\sigma_3}\,.$$

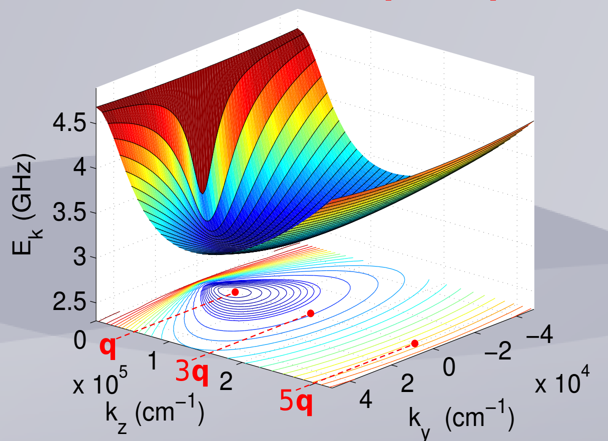




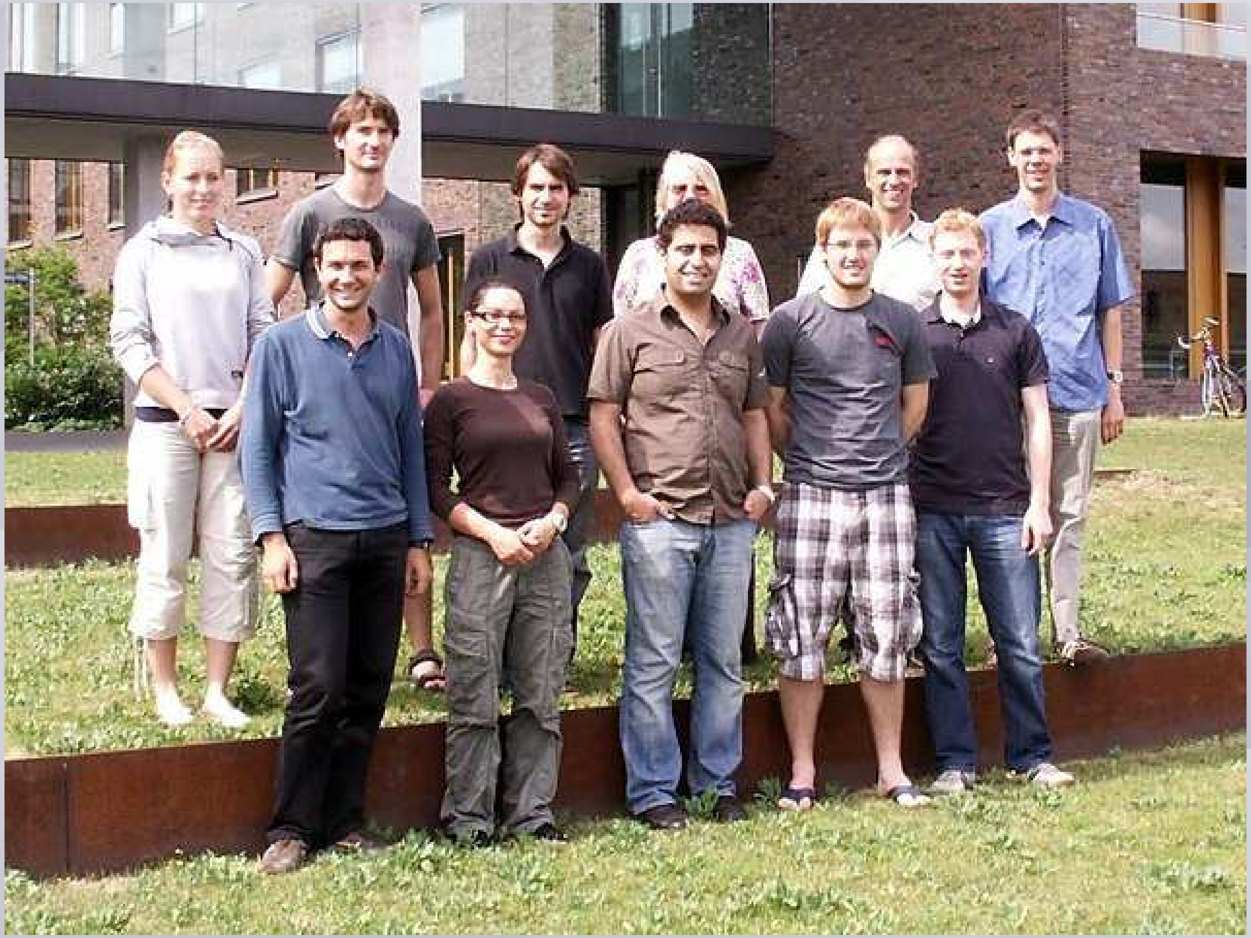
3 Summary

- description of magnetic insulators: Spin-wave theory
- development of **interacting spin-wave theory** with dipole dipole interactions
- interesting properties of the energy dispersion
- interactions: possible **condensation** of bosons at finite wave-vectors and integer multiples





4 Acknowledgement



Group of Peter Kopietz at ITP in Frankfurt (Germany)

www.itp.uni-frankfurt.de/~kreisel/en