Non-universality of aging during phase separation of the two-dimensional long-range Ising model

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Physical Aging

- Dynamical property
- Intuition about biological aging in humans:
  - Less flexible with age
  - More difficult to change things
  - E.g., learning new language when moving to different country may take more time (effort)
- Analogy physical aging:
  - The system's dynamical properties depend on age of the system
  - Autocorrelation function $C(t, t_w)$ becomes function of $y = t/t_w$ where t_w is the "age" of the system to which we compare the system at time t
  - Originates from glassy systems but is also observed in non-glassy systems out of equilibrium

Model

- Long-range Ising model (LRIM) with Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{i\neq j} J_{i,j} s_i s_j, \quad s_i = \pm 1 \tag{1}$$

- Distance-dependent interactions

$$J_{i,j} = |r(i,j)|^{-(d+\sigma)}. \tag{2}$$

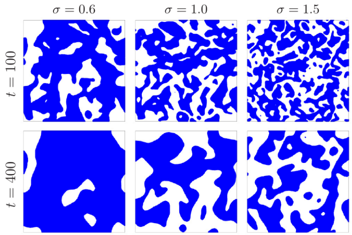
dimension d and tuneable parameter σ

$\Rightarrow$ Rich (dynamical) behavior in dependence of σ!

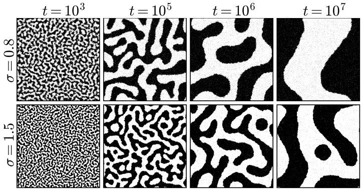
Phase Ordering in the LR Ising Model

Protocol:

- Start with random ($T = \infty$) configuration at $t = 0$
- Quench system to $T < T_c$
- NCOP: evolve system through single spin flips
- **COP**: evolve system through spin exchanges
- System undergoes symmetry breaking tries to relax to a state with $m = m_{\mathrm{eq}}$
- Coarsening: growing, structures
- Completely different ordering mechanisms



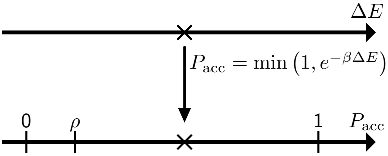
NCOP



COP

Traditional Metropolis:

1. Propose update
2. Calculate ΔE
3. $P_{\mathrm{acc}} = \min\left(1, e^{-\beta \Delta E}\right)$
4. Draw random number $\rho \in [0, 1)$
5. Accept if $\rho \leq P_{\mathrm{acc}}$



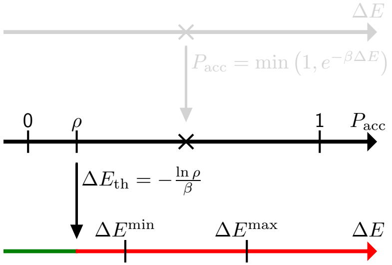
[1] FM, H. Christiansen, S. Schnabel, and W. Janke, PRX **13**, 031006 (2023).

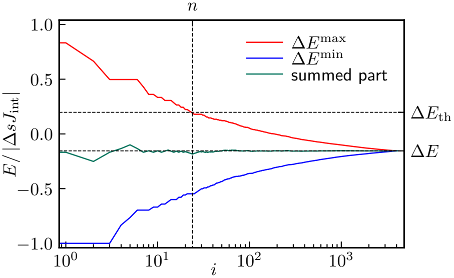
Simulation method[1]

Inverted Metropolis:

1. Propose update
2. calculate ΔE
3. $P_{\mathrm{acc}} = \min\left(1, e^{-\beta \Delta E}\right)$
2. Draw random number $\rho \in [0, 1)$
5. Accept if $\rho \leq P_{\mathrm{acc}}$
3. Calculate $\Delta E_{\mathrm{th}} = -\frac{\ln \rho}{\beta}$
4. Decide upon strict lower and upper bounds $\Delta E^{\mathrm{min/max}}$

   Accept if $\Delta E^{\mathrm{max}} < \Delta E_{\mathrm{th}}$ or reject if $\Delta E^{\mathrm{min}} > \Delta E_{\mathrm{th}}$





[1] FM, H. Christiansen, S. Schnabel, and W. Janke, PRX **13**, 031006 (2023).

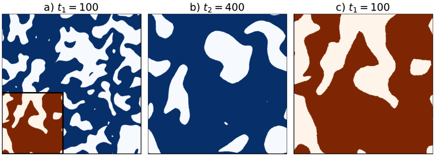
Example Evolution

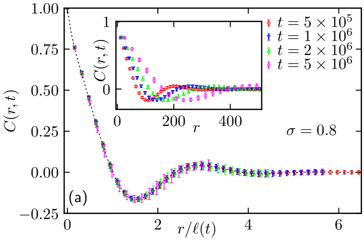
Characteristic length $\ell(t)$

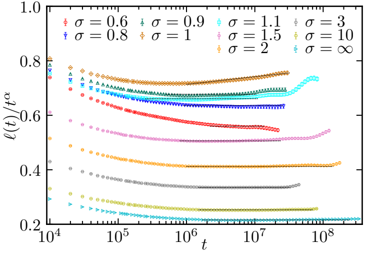
- Spatial correlation function

$$C(r,t) = \langle s_i(t) s_j(t) \rangle$$

- Scaling hypothesis: system is self-similar in time
- $C(r,t) = f\,[r/\ell(t)]$
- $\ell(t)$ determined from zero intersect of $C(r,t)$
- Typically, $\ell(t) \sim t^{\alpha} = t^{1/z}$







Aging

- Describes behavior of autocorrelation function

$$C(t, t_w) = \langle s_i(t) s_i(t_w) \rangle - \langle s_i(t) \rangle \langle s_i(t_w) \rangle$$

- Dynamical scaling:

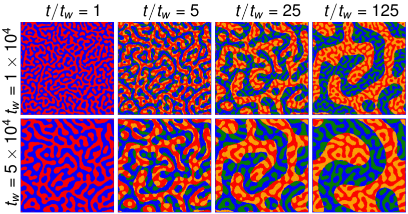
$$C(t, t_w) \sim (t/t_w)^{-\lambda/z}$$

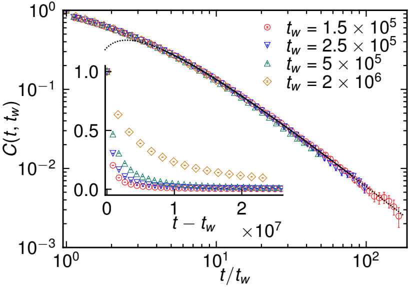
- If $\ell(t) \propto t^{1/z}$ equivalently:

$$C(t, t_w) \sim (\ell/\ell_w)^{-\lambda}$$

- Extract λ with fits to

$$f = Ay^{-\lambda\alpha}(1 - B/y)$$

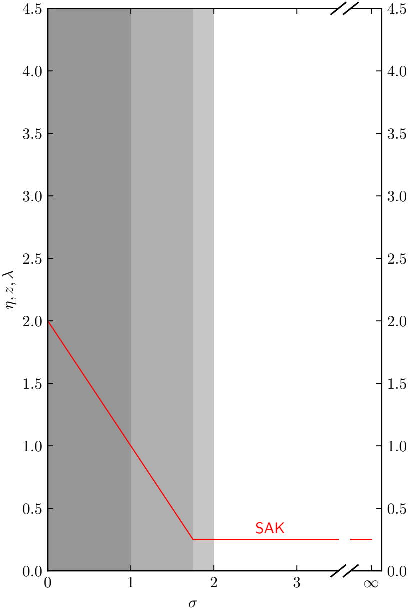




Equilibrium Properties

- For $\sigma < 1$ MF universality class
- For $d/2 \leq \sigma \leq \sigma_*$ nontrivial σ-dependent η
- For $\sigma > \sigma_*$ same universality class as NN model
- In 2D:

  Sak: $\sigma_* = 2 - \eta_{\mathrm{SR}} = 1.75$[2]
  Picco[3]: $\sigma_* = 2$



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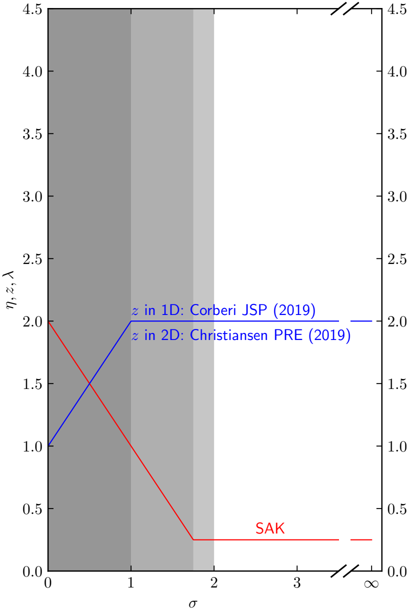
[2] J. Sak, PRB **8**, 281 (1973). E. Luijten and H. W. J. Blöte, PRL **89**, 025703 (2002). M. C. Angelini, G. Parisi, and F. Ricci-Tersenghi, PRE **89**, 062120 (2014).
[3] M. Picco, arXiv:1207.1018.

Lengthscale NCOP

- Theoretical prediction[4]

$$\ell(t) \sim t^{\alpha} = \begin{cases} t^{1/2} & \sigma > 1 \\ (t \ln t)^{1/2} & \sigma = 1 \\ t^{1/(1+\sigma)} & \sigma < 1 \end{cases}, \quad (3)$$

- Recovered in simulations both in 1D[5] and in 2D[6]
- Motion of domain walls well-described.



[4] A. J. Bray and A. D. Rutenberg, PRE **49**, R27 (1994).
[5] F. Corberi, E. Lippiello, and P. Politi, JSP **176**, 510 (2019).
[6] H. Christiansen, S. Majumder, and W. Janke, PRE **99**, 011301(R) (2019).

Lengthscale COP

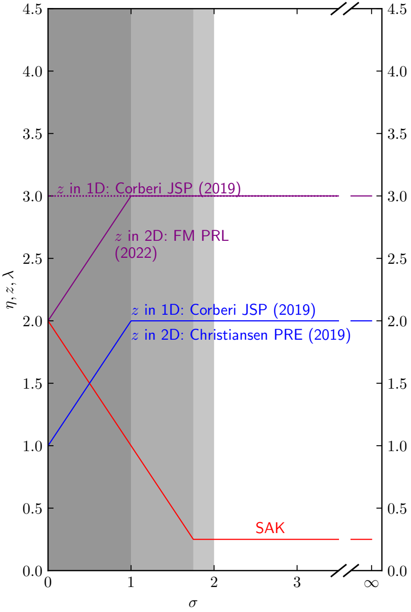
- Theoretical prediction[4]

$$\ell(t) \sim t^{\alpha} = \begin{cases} t^{1/3} & \sigma > 1 \\ (t \ln t)^{1/3} & \sigma = 1 \\ t^{1/(2+\sigma)} & \sigma < 1 \end{cases},$$

- Violated in 1D. Instead σ-independent growth[5]

$\Rightarrow$ Argument: Particle diffusion not enhanced

- Confirmed in 2D[7]



[5] A. J. Bray and A. D. Rutenberg, PRE **49**, R27 (1994).

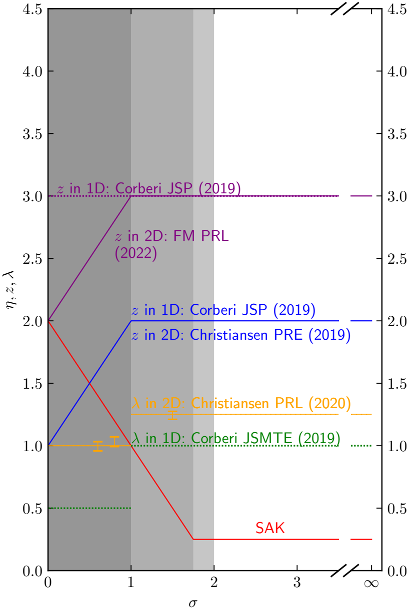
[6] F. Corberi, E. Lippiello, and P. Politi, JSP **176**, 510 (2019).

[7] FM, H. Christiansen, and W. Janke, PRL **129**, 240601 (2022).

- No theoretical predictions

$$C(t, t_w) \sim \left(\frac{t}{t_w}\right)^{-\lambda/z}$$

- Same picture in 1D[8] and 2D[9]:
  - $\lambda = d/2$ (FH bound[10]) for $\sigma < 1$
  - λ takes value of corr. NN-model for $\sigma > 1$



[8] F. Corberi, E. Lippiello, and P. Politi, J. Stat. Mech. (2019) 074002.
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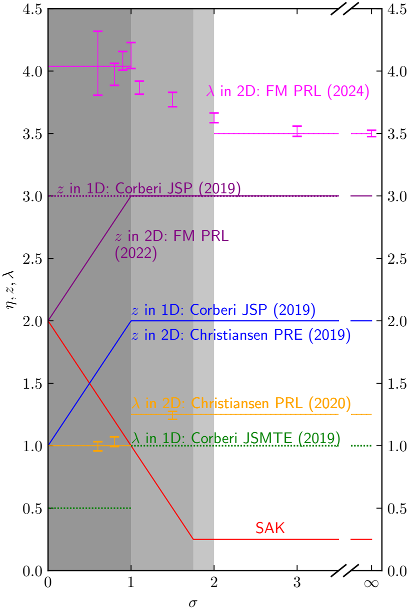
[10] D. S. Fisher and D. A. Huse, PRB **38**, 373 (1988).

New results – Aging COP[11]

- No theoretical predictions

$$C(t, t_w) \sim \left(\frac{t}{t_w}\right)^{-\lambda/z}$$

- No results for 1D
- In 2D:
  - For $\sigma > 2$: λ compatible with 3.500(26) NN value
  - For $\sigma \leq 1$: $\lambda \approx 4$
  - Non-trivial σ-dependence of λ in between



[11] FM, H. Christiansen, and W. Janke, https://arxiv.org/pdf/2409.08050