

Gravity & the Big Bang

in the Context of Epidemics:

How Mobility Impacts Epidemics & Vice Versa

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<https://sites.google.com/view/fakhtehgh/>

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Gravity

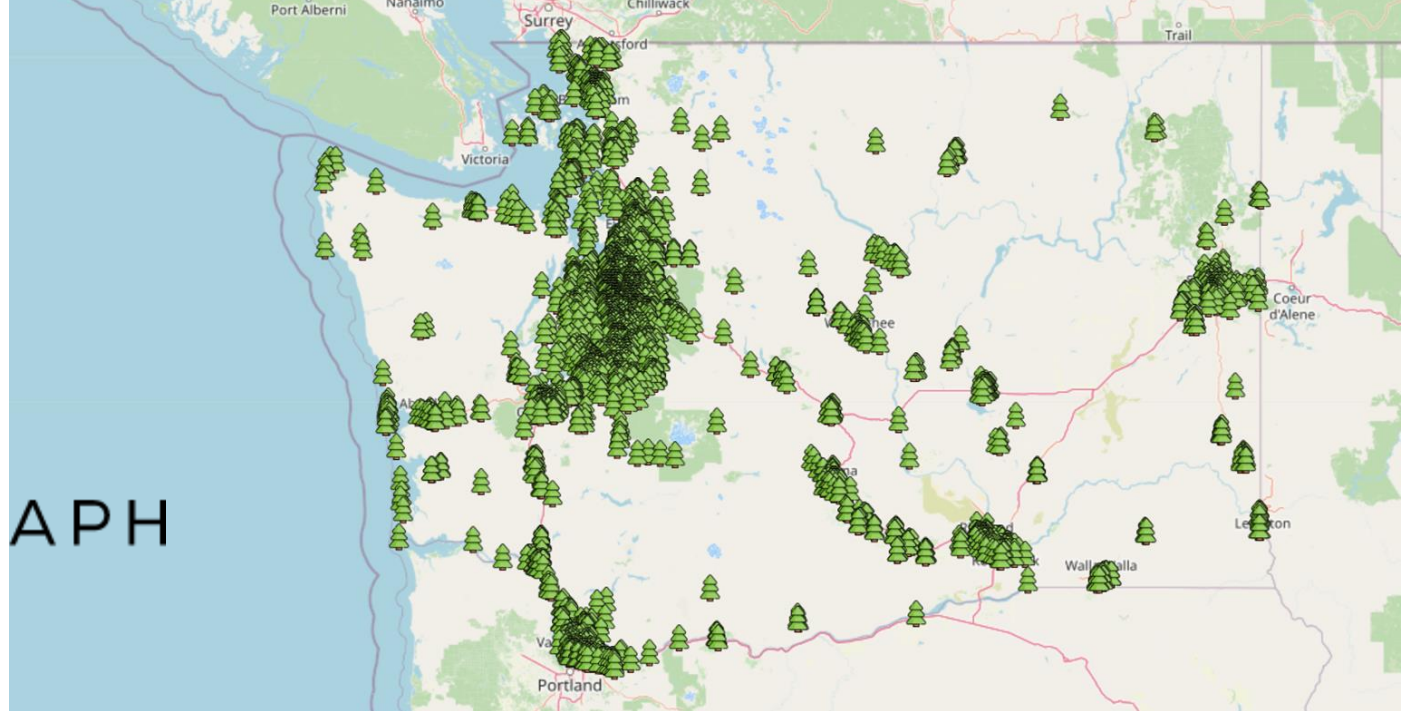
how epidemic impacts mobility

(Nature) Communications Physics volume 7, Article number: 55 (2024)

Data



SAFE GRAPH



Weekly visitation patterns from census block groups to the Washington state urban parks



SAFE GRAPH: <https://docs.safegraph.com/docs/weekly-patterns>

The Gravity Model

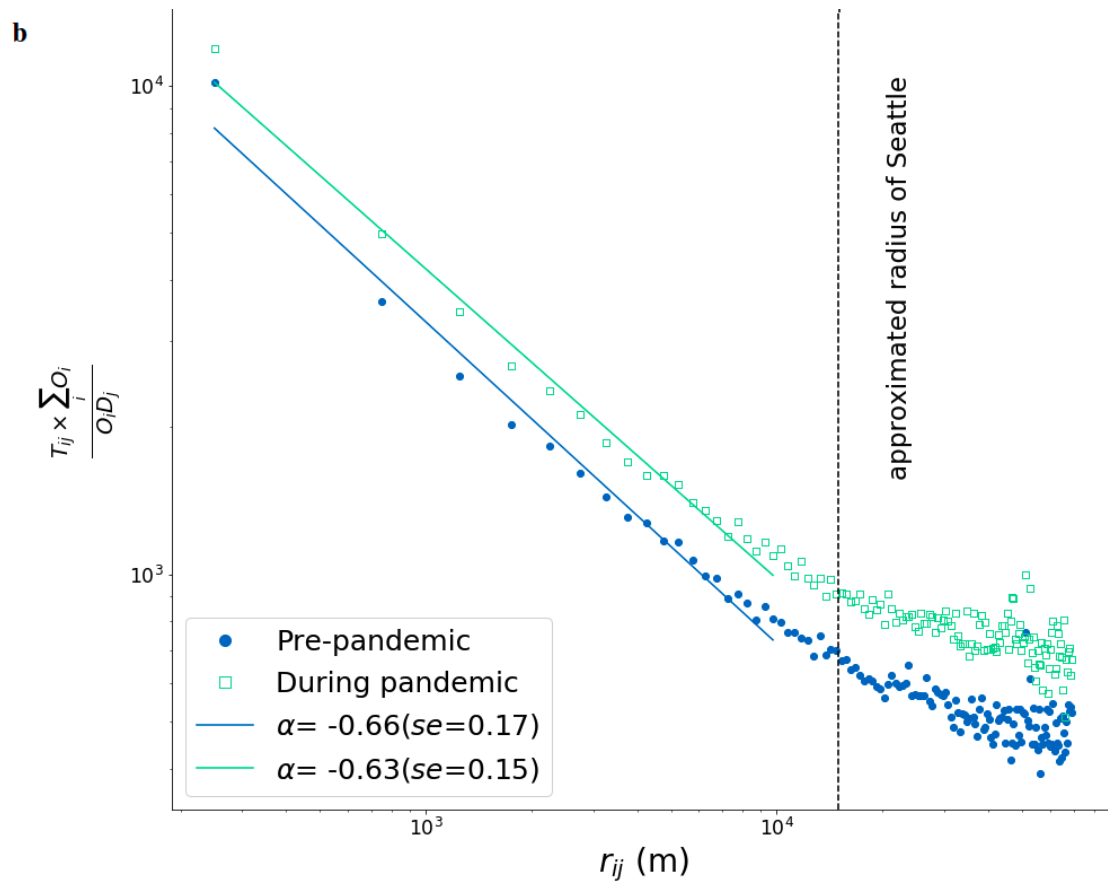
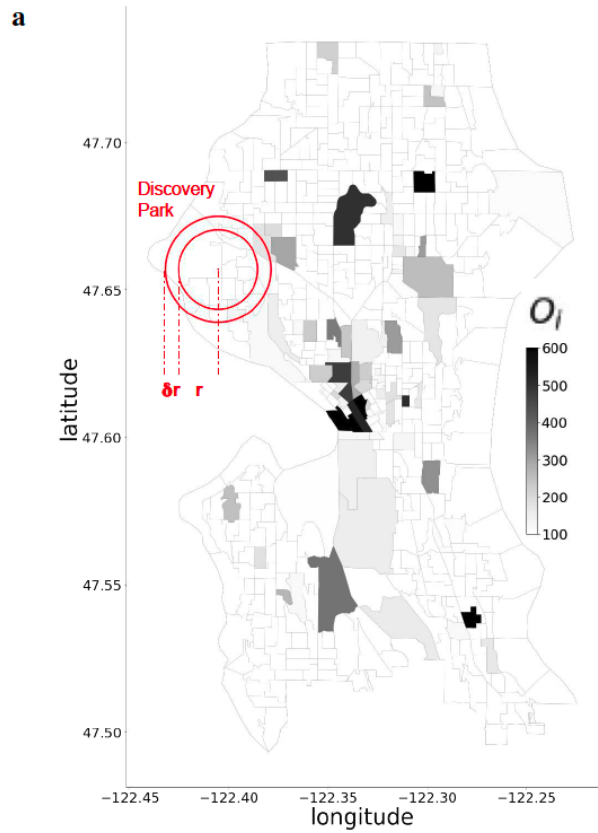
The diagram illustrates the Gravity Model equation, which predicts the number of visitors from a census block group (CBG) to a park. The equation is:

$$T_{ij} = \frac{O_i D_j}{\sum_i O_i K r_{ij}^\alpha}$$

The variables in the equation are defined as follows:

- T_{ij} : number of visitors from census block group i (CBG _{i}) to park j
- O_i : number of visitors residing in CBG _{i}
- D_j : total number of visitors for park j
- $\sum_i O_i$: total number of visitors (summed over all CBGs i)
- K : a constant
- r_{ij} : distance between CBG _{i} and park j
- α : a constant

Arrows in the diagram point from the labels to the corresponding variables in the equation.

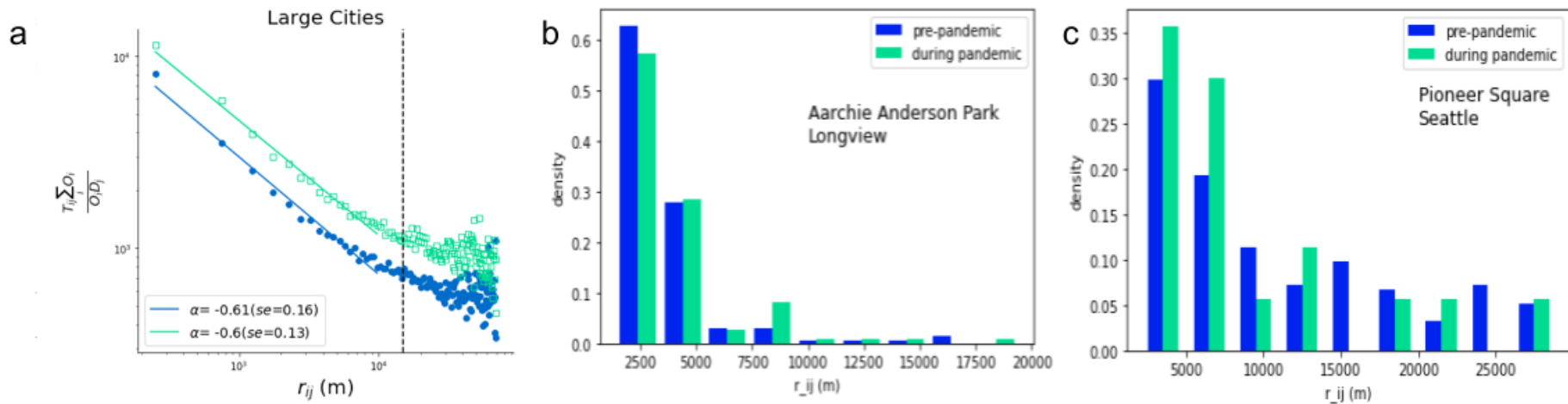


Question: Changes in Park Visitation Pattern

- What is the **pattern** that the park visitation obeys from?
- How the park visitation pattern has **changed** due to the **COVID-19 pandemic**?
- How the change in park visitation patterns varies between different
 - **seasonal patterns,**
 - **socio-economic groups,**
 - and **size of cities?**

Different Cities' Reaction (Equality)

Large vs Small Cities



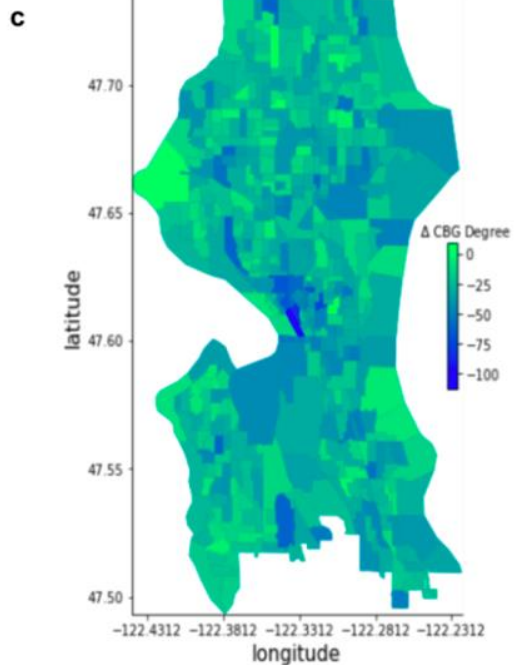
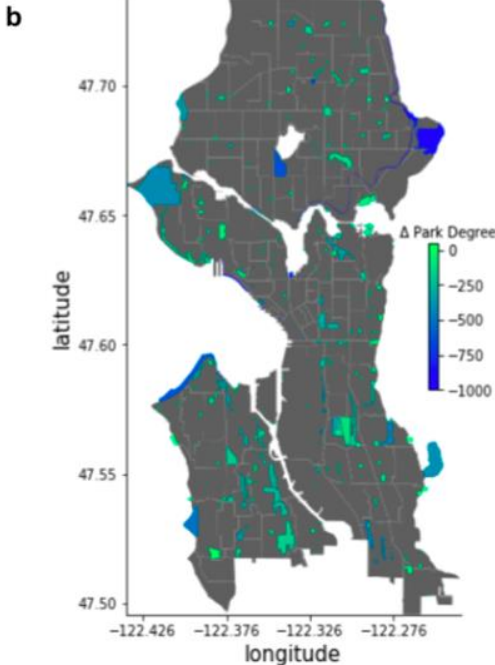
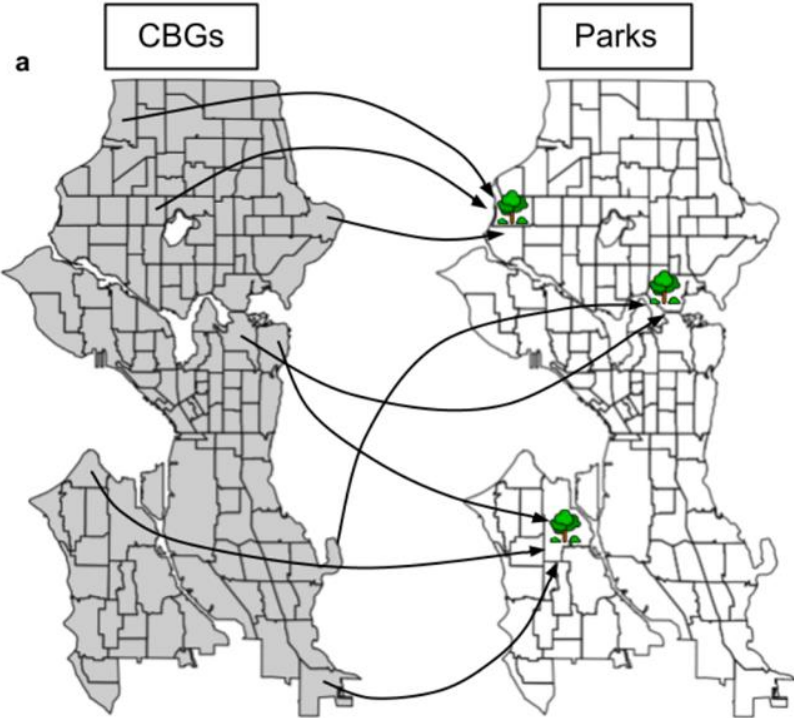
Different Groups' Reaction (Equality)

During (2020) vs Pre (2019) Pandemic:

- Gravity model is a robust pattern.
- The slope changes.
- People visit closer parks during pandemic, in both warm and cold seasons.
- Different socio-economic groups:
 - Closer parks become more popular among the least vulnerable visitors
 - The most vulnerable visitors have more interest in visiting distant parks
- Large vs small cities
 - Visitors from smaller cities prefer visiting further distance
 - Visitors from larger cities prefer closer parks

Propensity & Diversity of the visits

Park Visitation Network



Correlations:

Degree CBG	pre-pandemic	During the pandemic	Change in degree
SVI	-0.93	-0.9	0.87

Degree Park	pre-pandemic	During the pandemic	Change in degree
visitor/area	0.67	0.94	-0.72

- The higher the social vulnerability of an area, the residents of that area exhibit a lower propensity in park visitation.
- The parks in richer areas of Washington State did not exhibit a significant difference in their visitors' diversity.

The big bang of an outbreak

how mobility impacts epidemics

[arXiv:2405.03703v1](https://arxiv.org/abs/2405.03703v1)

COVID-19 in Iran?

Where? When? How?

What Was the Challenge?

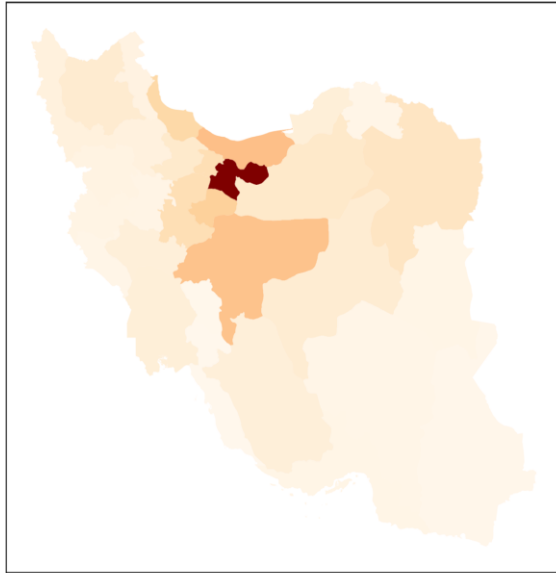


- Iran's population is almost **83 million** with almost **3 million** daily travels between its provinces.
- **Tehran** is the capital city of Iran whose population is around **15 million**.
- **Qom** is a relatively small province near Tehran, with almost **3 million** population.
- Imam Khomeini Airport (IKA) is a major international airport in Iran, located between **Qom** (89 km away) and **Tehran** (55 km away).

Isolate Qom or Tehran?

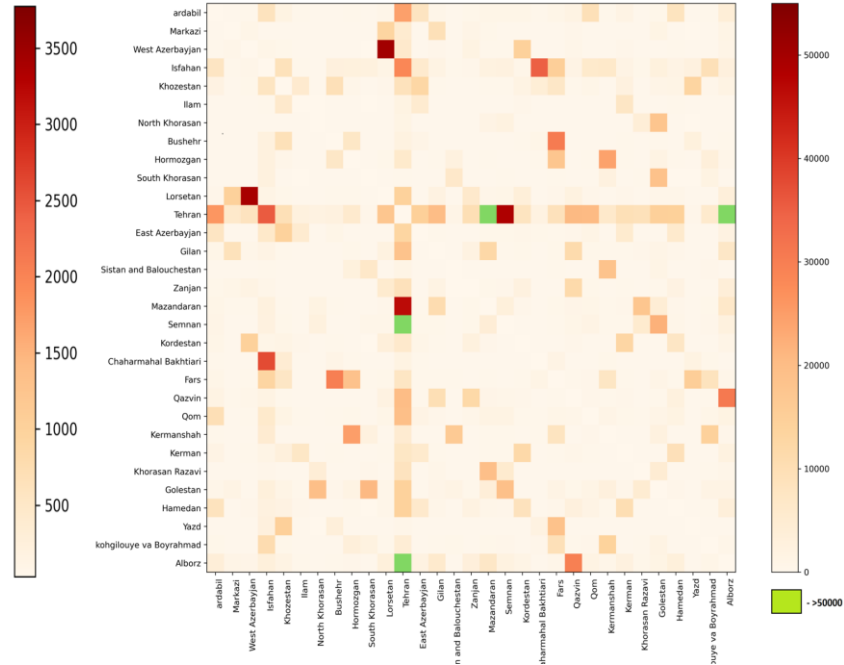
Our Data (23th February 2020):

A



of Coronavirus active cases

B



Daily passenger flow

A Well-Visualized Approach

Effective Distance



The Hidden Geometry of Complex, Network-Driven Contagion Phenomena

Dirk Brockmann and Dirk Helbing

Science **342**, 1337 (2013);

DOI: 10.1126/science.1245200

Empirical Data I: The Mobility Network

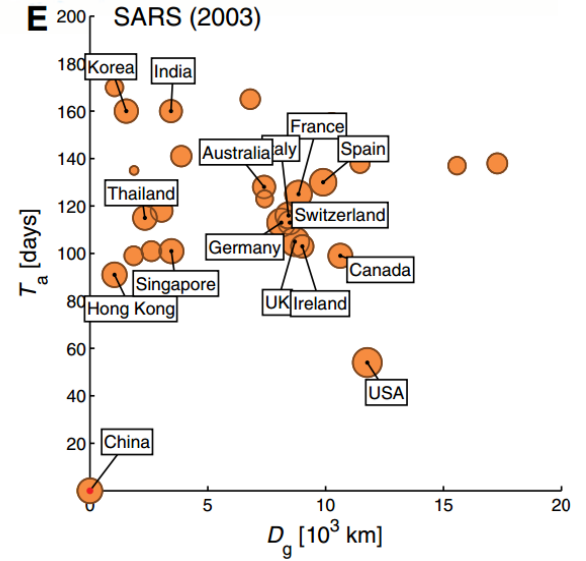
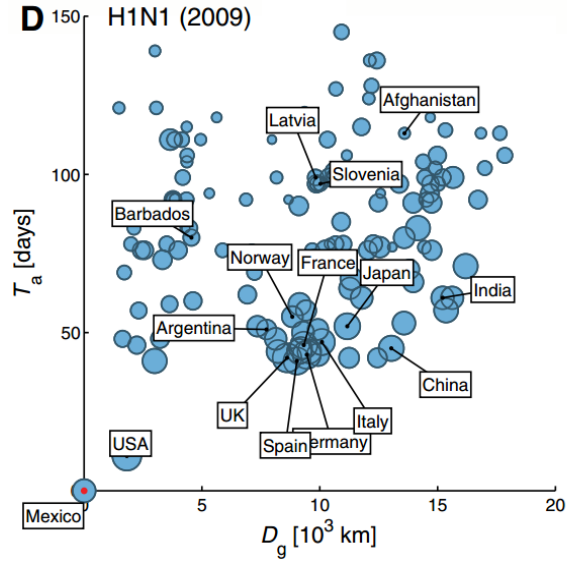
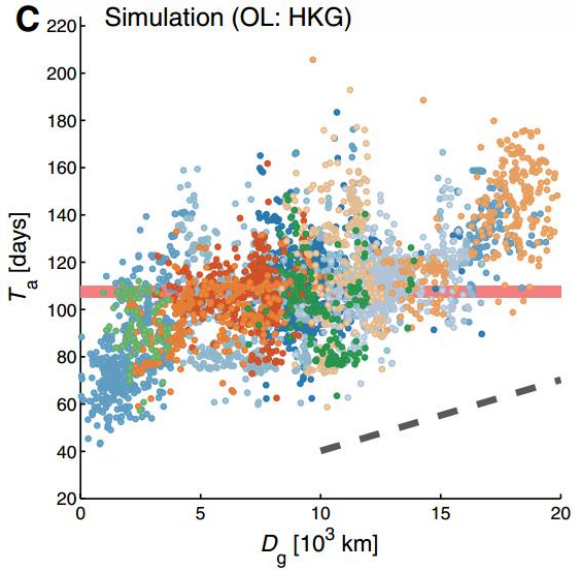


Empirical Data II: The Active Cases



105 days < T_arival < 110 days

Geographic Distance



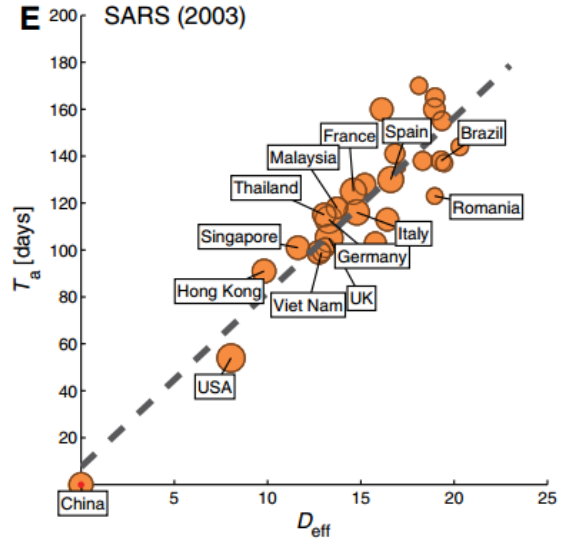
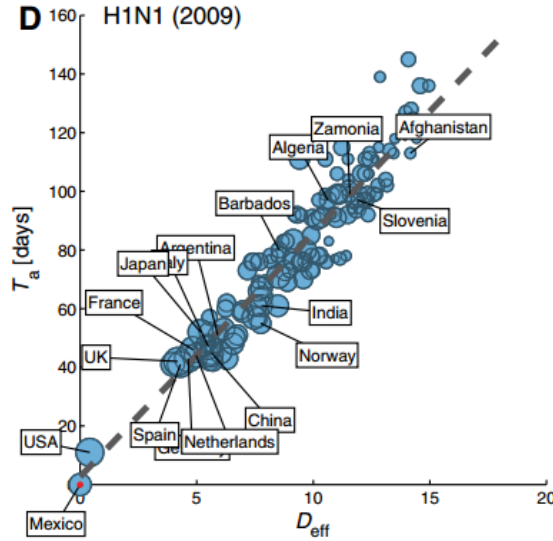
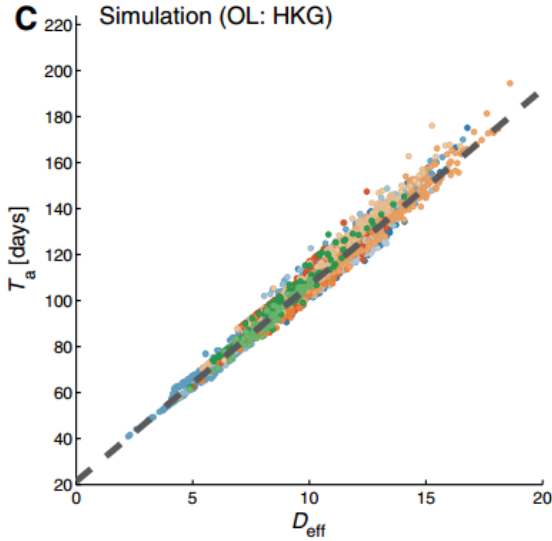
Effective Distance

$$d_{mn} = (1 - \log P_{mn}) \geq 1$$

$$P_{mn} = F_{mn}/F_n$$

indirect flights

$$D_{mn} = \min_{\Gamma} \lambda(\Gamma)$$



$$T_a = \underbrace{D_{\text{eff}}(\mathbf{P})}_{\text{eff. distance}} / \underbrace{v_{\text{eff}}(\alpha, R_0, \gamma, \varepsilon)}_{\text{eff. speed}}$$

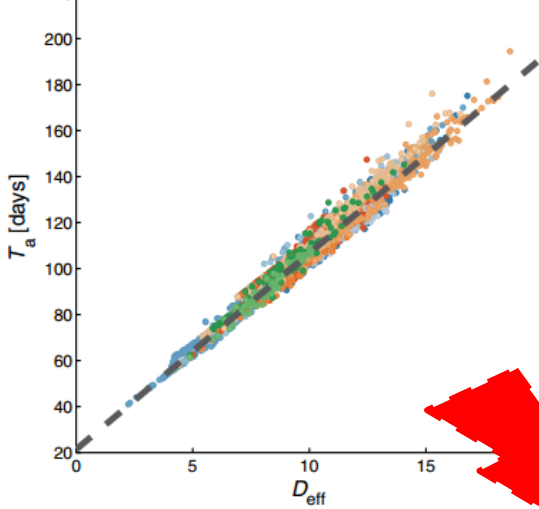
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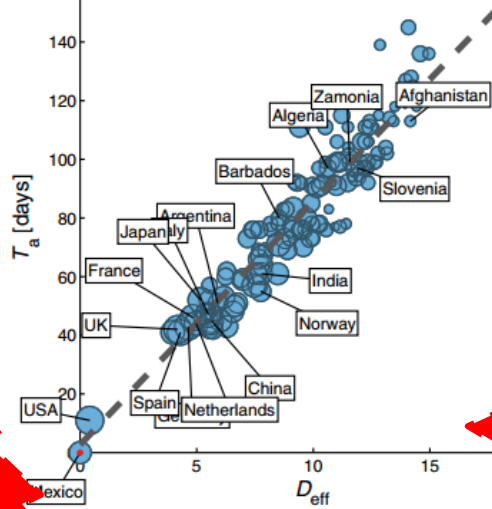
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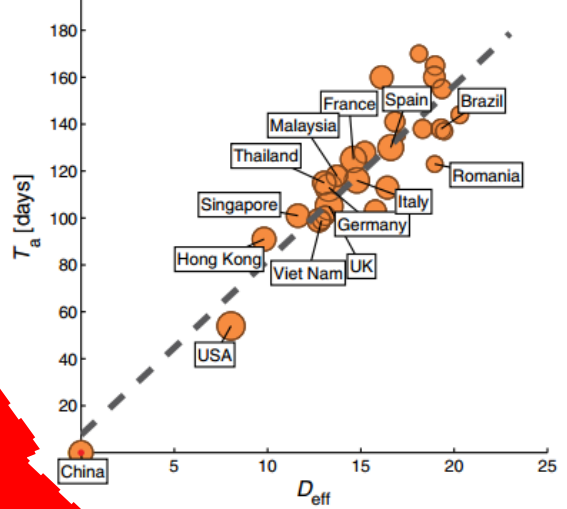
C Simulation (OL: HKG)



D H1N1 (2009)



E SARS (2003)



$$T_a = \underbrace{D_{\text{eff}}(\mathbf{P})}_{\text{eff. distance}} / \underbrace{v_{\text{eff}}(\alpha, R_0, \gamma, \varepsilon)}_{\text{eff. speed}}$$

Arrival Time?

The Hidden Geometry of Complex, Network-Driven Contagion Phenomena

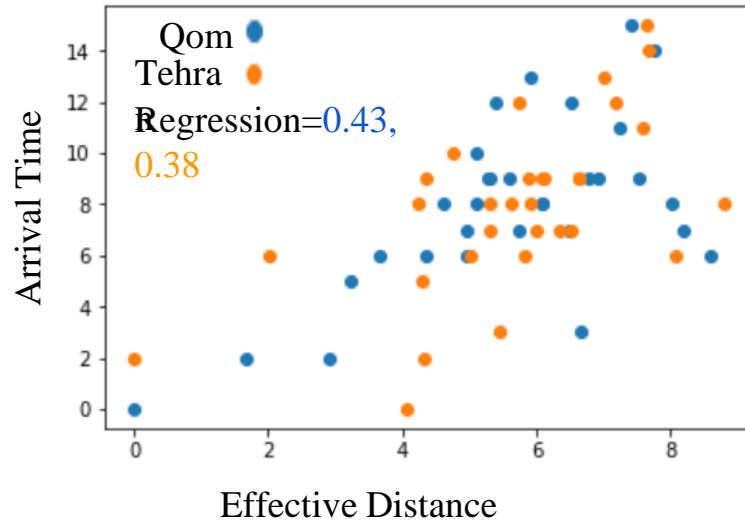
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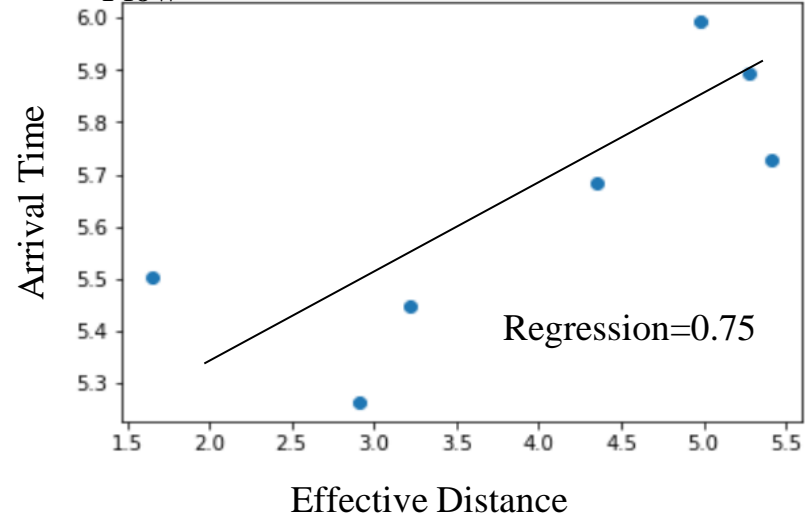
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$$T_a = \underbrace{D_{\text{eff}}(\mathbf{P})}_{\text{eff. distance}} / \underbrace{v_{\text{eff}}(\alpha, R_0, \gamma, \epsilon)}_{\text{eff. speed}}$$

DB & DH's Algorithm + data: I & Flow



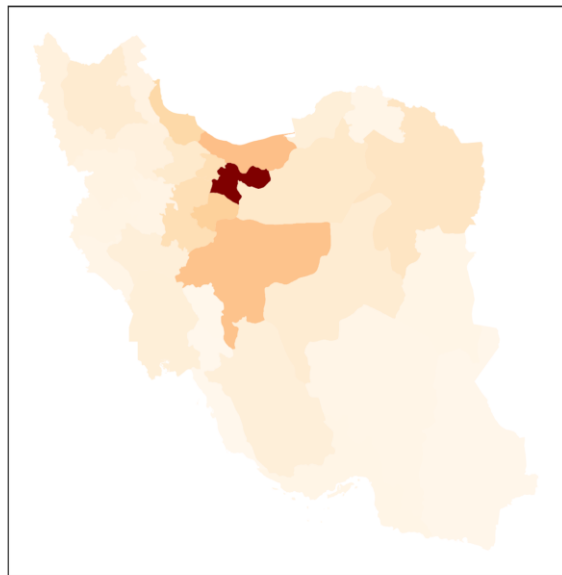
DB & DH's Algorithm + Cleaned data: I & Flow



Our Approach?

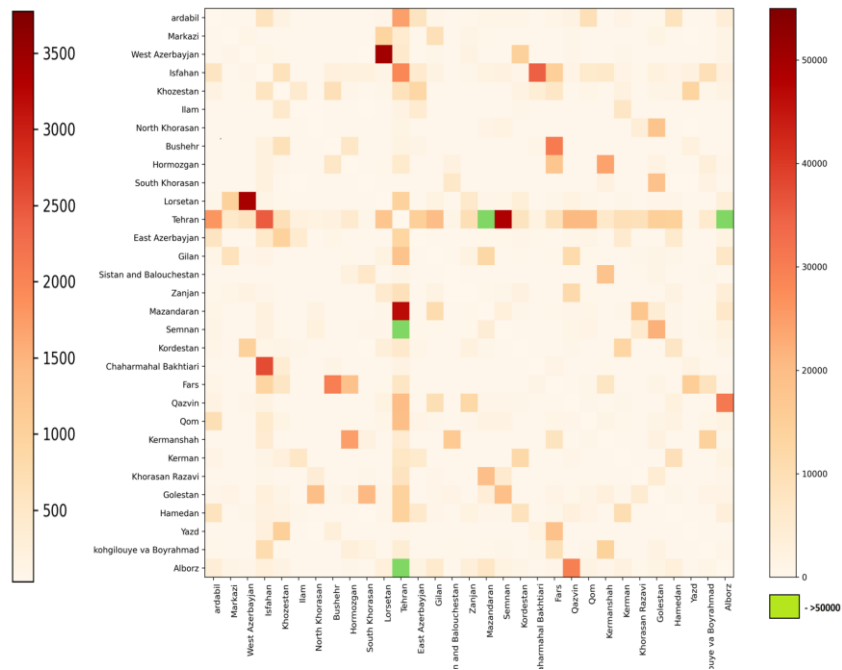
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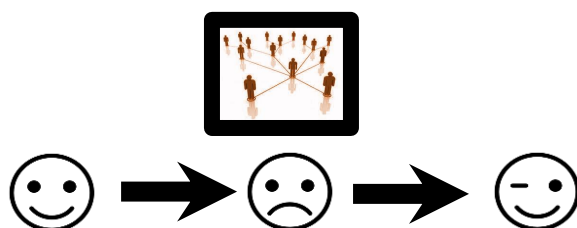


Daily passenger flow

Our Mathematical Formalism

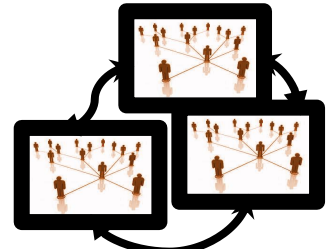
$$I^e(\vec{t}_e) = (I_1^e(t_e), I_2^e(t_e), \dots, I_n^e(t_e)),$$

$$\frac{dI_i}{dt} =$$



[Intra population (HMFA, SIR)]

+



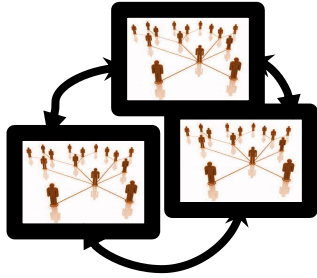
[InterPopulations (MetaPopulation)]

Intra population: SIR (Homogenous Mean Field Approximation)



$$\frac{dI_i}{dt} = [\beta_i S_i I_i - \gamma_i I_i]$$

Inter population: Mobility (passengers flow)



$$\frac{dI_i}{dt} = \left[\sum_j pP_{ji} I_j - pI_i \right]$$

Our Mathematical Formalism

$$I^e(\vec{t}_e) = (I_1^e(t_e), I_2^e(t_e), \dots, I_n^e(t_e)),$$

$$\frac{dI_i}{dt} = [\beta_i S_i I_i - \gamma_i I_i] + \left[\sum_j p_{ji} P_{ji} I_j - p I_i \right]$$

[Inter population (HMFA, SIR)]+ [Intra Populations (MetaPopulation)]

Our Mathematical Formalism

$S_i \sim N_i$

$$\frac{dI_i}{dt} = [\beta_i S_i I_i - \gamma_i I_i] + \left[\sum_j p P_{ji} I_j - p I_i \right]$$

$$I(\vec{t}) = e^{\hat{B}pt} I(\vec{0})$$

$$I(\vec{t}) = \left(\hat{I} + \hat{B}pt + \frac{(\hat{B}pt)^2}{2} + \dots \right) I(\vec{0}),$$

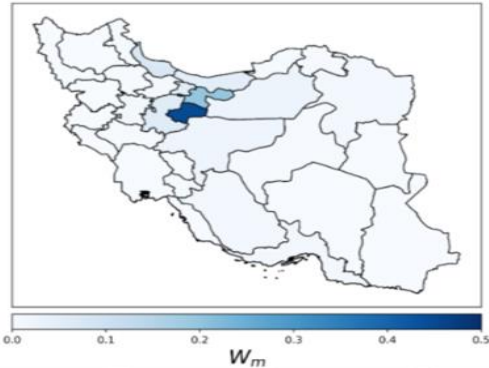
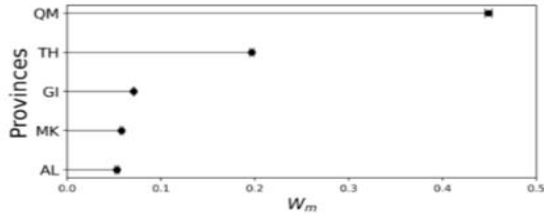
Where?

Basis of the space

$$\hat{m} = (0, 0, \dots, 1, \dots, 0)$$
$$\hat{m}' = e^{\hat{B}pt} \hat{m}$$

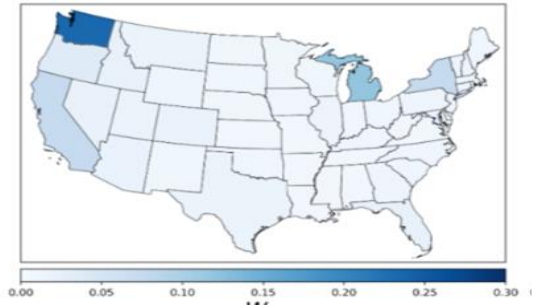
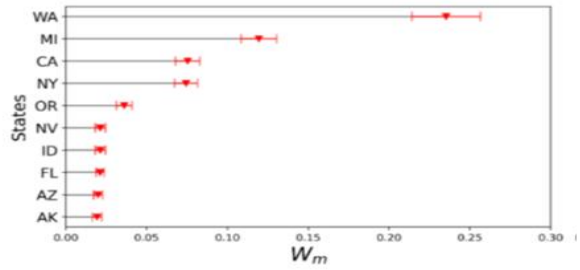
$$W_m = \frac{\hat{m}' \cdot I(\vec{t})}{\sum'_n \hat{n}' \cdot I(\vec{t})}$$

Data: Day = 5



Where?

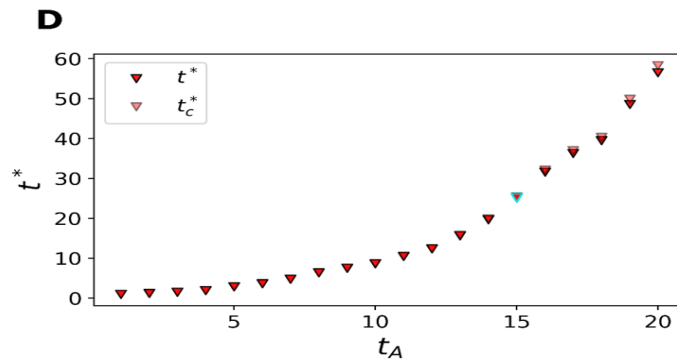
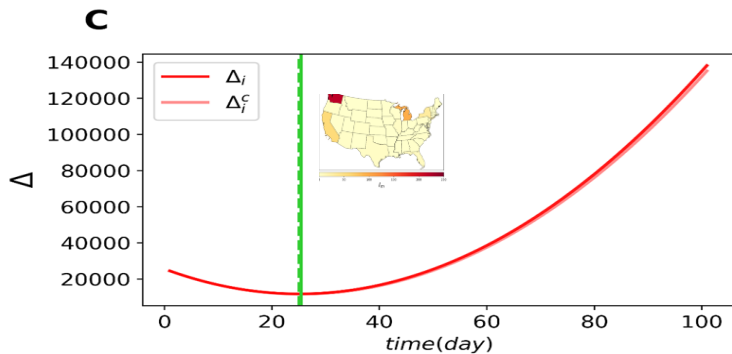
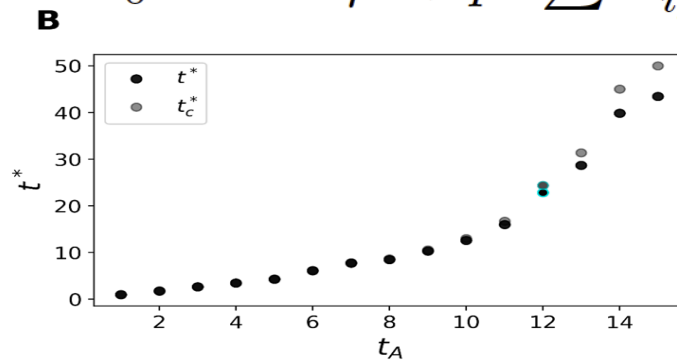
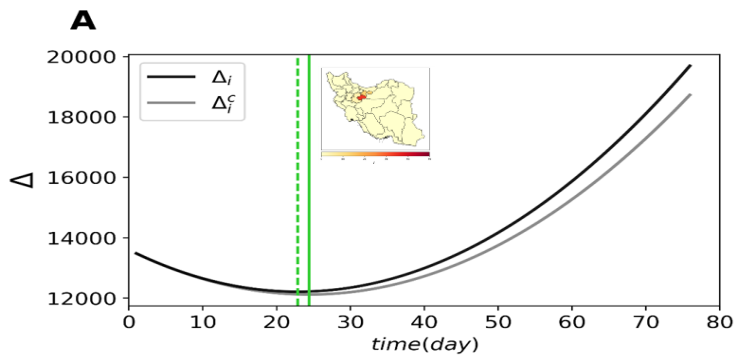
Data: Day = 5



When?

$$\Delta_i(t) = \frac{\sum_{j=1}^n (I_j(t) - I_j^e(t_e))^2}{n}$$

$$t_i^* = \frac{1}{i_0} \frac{-\eta(i_0 - I_i^e) + p \sum (P_{ij} I_j^e)}{\eta^2 + p^2 \sum P_{ij}^2}$$



How? Overtaking Time

$$(N_j \beta_j - \gamma_j) I_j = p \left(\sum P_{kj} I_k - I_j \right)$$

$$I(\vec{t}) = (\hat{I} + \hat{B}pt + \frac{(\hat{B}pt)^2}{2} + \dots) I(\vec{0}),$$

$$t^j = \frac{1}{p} \frac{1}{(2 - q_i - q_j) - \frac{P_{ij}^1}{P_{ij}}}$$

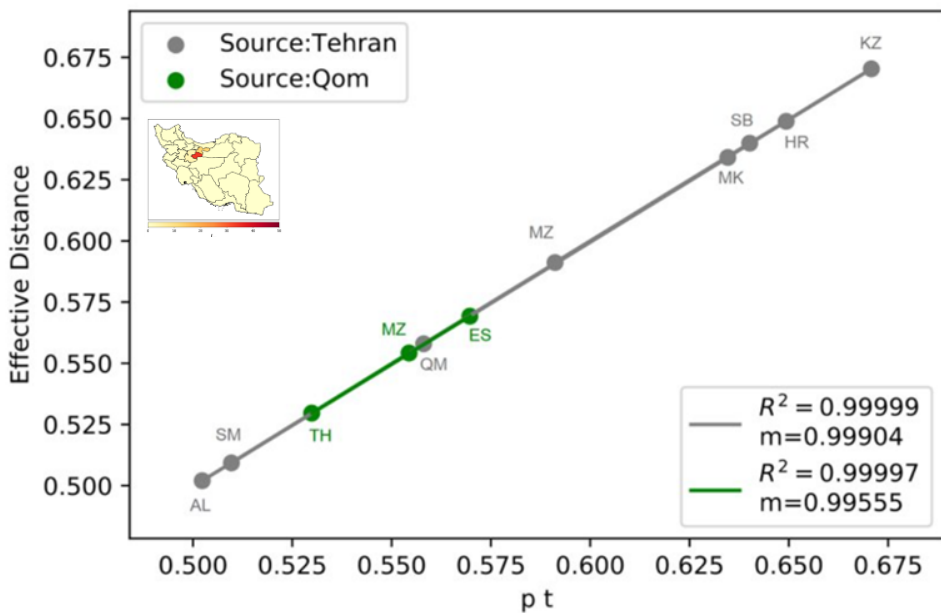
$$D_{ij} = \frac{1}{(2 - q_j - q_i) - \frac{P_{ij}^1}{P_{ij}}}$$

Effective Distance vs Overtaking Time

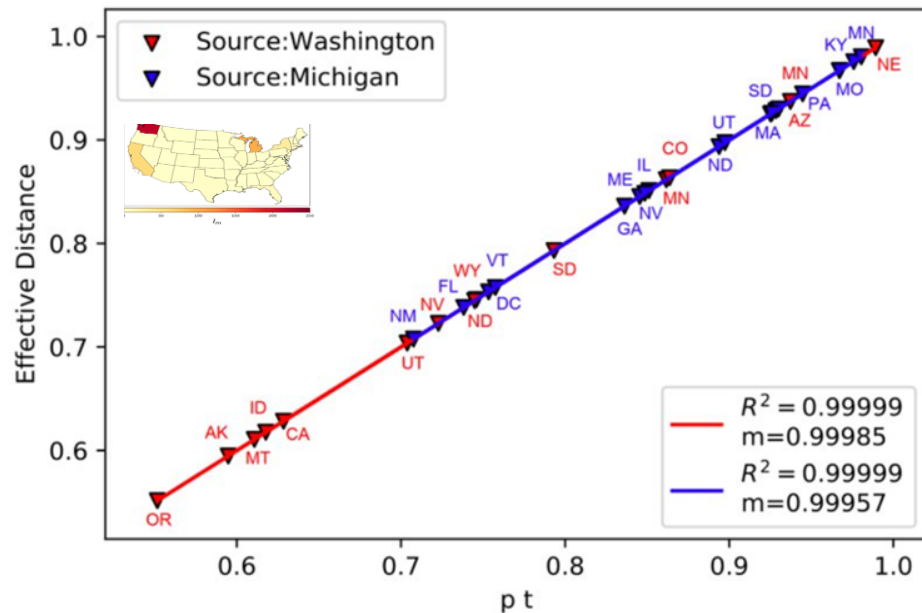
$$D = pt$$

Effective Distance vs Overtaking Time, Simulated I + Empirical Mobility Data

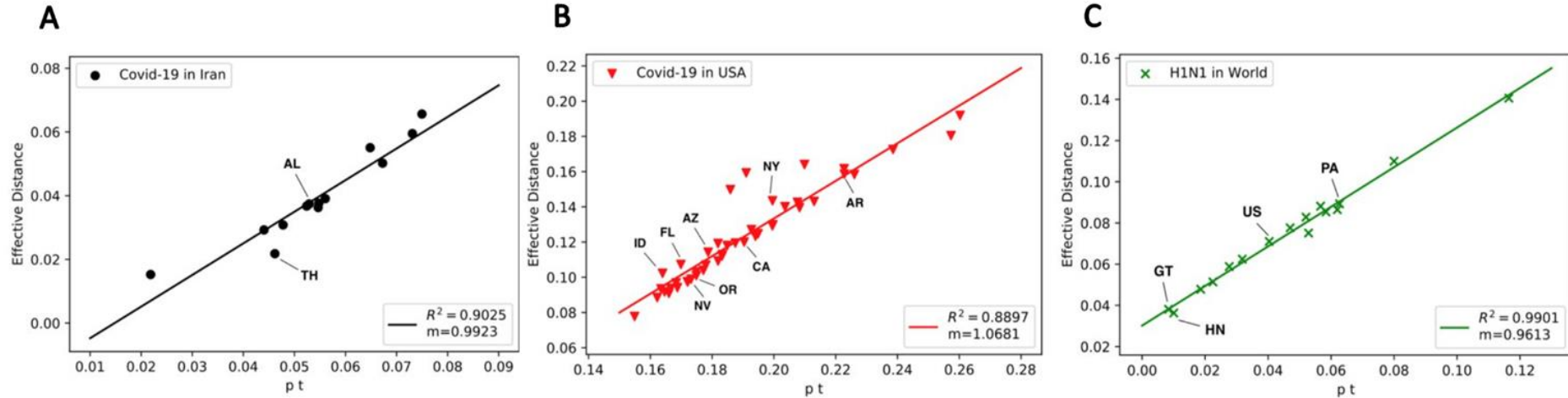
A



B



The Universal Geometry & Slope: Empirical Data



DPG deadline 01.12.2024! Join SOE session!
Looking for Lecturer/TA at SRH, Leipzig Campus

Thank you!



Gravity Team:



Zahra Ghadiri, Afra Mashhadi, Marc Timme & Fakhteh Ghanbarnejad

Big Bang Team:



Yazdan Babazadeh, Amin Safaesirat, Fakhteh Ghanbarnejad

Reaction to the pandemic

Gravity model
Socio-economic groups
Small vs. large cities

(Nature) Communications Physics volume 7, Article number: 55 (2024)

The big bang of an outbreak

Where was the origin?

When did it begin?

How does it spread?

data: 2 snapshots: active cases & mobility

cut-off errors

Multi sources

[arXiv:2405.03703v1](https://arxiv.org/abs/2405.03703v1)

Socio-economic Vulnerability

The Social Vulnerability Index uses U.S. Census data to determine the relative social vulnerability of every census tract.

1. **Poverty** - population living below Federal poverty level
2. **Unemployment** - age 16 and over seeking work
3. **Per capita income** - (2013 inflation-adjusted \$)
4. **Education** - age 25+ without a high school diploma
5. **Health insurance** - age less than 65 without insurance

SVI

The most vulnerable
SVI > 0.75

The least vulnerable
SVI < 0.25