Applications of Quantum Hamilton equations of Motion

Poster – Presentation

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Motivation

- Particles described as classical ones following conservative Brownian motion
- Derived Quantum Hamilton equations of motion, which can be solved unattached from the Schrödinger equation

The triangle of quantum mechanics

Quantum Hamilton principle

$$J[u, v] = \mathrm{E}\left[\int_0^T \left\{\frac{1}{2} m(v - \mathrm{i}u)^2 - V(x, t)\right\} \mathrm{d}t + \Phi_0(x_0)\right]$$

*M. Pavon, J. Math. Phys., 36: 6774-6800, **1995***

Schrödinger equation

$$\mathrm{i}\hbar\,\partial_t \Psi(x, t) = \left[-\frac{\hbar^2}{2m}\partial_{xx} + V(x, t)\right] \Psi(\mathbf{x}, t)$$

$$v(x, t) = \frac{1}{m}\partial_x \mathfrak{Im}\,[\ln \Psi],\; u(x, t) = \frac{\hbar}{m}\partial_x \mathfrak{Re}\,[\ln \Psi]$$

Quantum Hamilton equations of motion

$$\mathrm{d}x(t) = [v(x(t), t) + u(x(t), t)]\,\mathrm{d}t + \sqrt{\frac{\hbar}{m}}\,\mathrm{d}W_f(t)\ ,$$

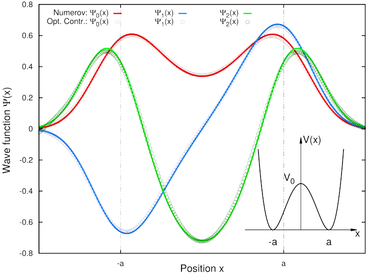
$$\mathrm{d}x(t) = [v(x(t), t) - u(x(t), t)]\,\mathrm{d}t + \sqrt{\frac{\hbar}{m}}\,\mathrm{d}W_b(t)\ ,$$

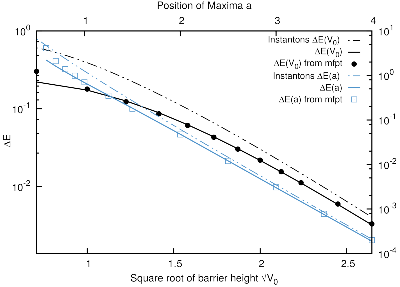
$$\mathrm{d}\,m\,[v(t) + u(t)] = \frac{\partial V(\mathbf{x}(t))}{\partial x}\mathrm{d}t + \sqrt{\frac{\hbar}{m}}\,\frac{\partial m\,[v(x(t), t) + u(x(t), t)]}{\partial x}\,\mathrm{d}W_b(t)$$

*Köppe et al., Ann. Phys, 529: 1600251, **2017***

Numerical results

One-dimensional double-well potential





Higher dimensions