Contact process in inhomogeneous environment

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Phys. Rev. E **95**, 012105 (2017)
arXiv:1711.03495

CompPhys17 , Leipzig, 1st December (2017)

Contact process

Prototype of stochastic lattice models

Lattice sites: empty $(\emptyset)$

occupied by one particle (A)

Continuous time Markov-process:

two processes:

- $(\mathbf{A} \to \emptyset)$ particle at site *l* disappears

with a rate $\mu(l)$

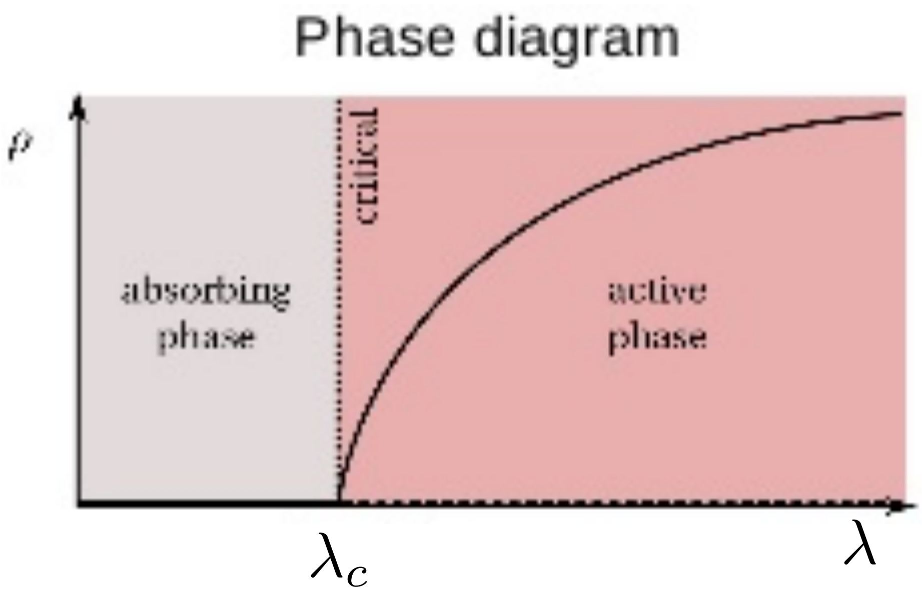
- $\emptyset\mathbf{A} \to \mathbf{AA}$ new particle is produced on empty sites

with a rate $p\Lambda(l)/n$

n: coordination number, *p*: number of occupied neighbours

local control parameter: $\lambda(l) = \Lambda(l)/\mu(l)$

Homogeneous contact process



Parameters in 1d

λ_c	$3.29785(2)$
β	$0.276486(8)$
β_1	$0.73371(2)$
$\nu_{\parallel} = \nu_{\parallel,1}$	$1.733847(6)$
$\nu_{\perp}$	$1.096854(4)$

$$\Delta = \lambda - \lambda_c, \quad \text{control} - \text{parameter}$$

$$\rho \sim \Delta^{\beta}, \quad \text{order} - \text{parameter}$$

$$\rho_1 \sim \Delta^{\beta_1}, \quad \text{surf.order} - \text{parameter}$$

$$\xi_{\parallel} \sim |\Delta|^{-\nu_{\parallel}}, \quad \text{correlation} - \text{length}$$

$$\xi_{\perp} \sim |\Delta|^{-\nu_{\perp}}, \quad \text{correlation} - \text{length}$$

Inhomogeneous contact process

Forms of inhomogeneity

- Free surface: $\rho \to \rho_1$

  $\beta \to \beta_1$

  $\nu_{\|}, \nu_{\perp} \to \quad \text{unchanged}$

- Internal defect:

  Irrelevant perturbation, exponents are unchanged

- Spatial disorder:

  Infinite-disorder criticality: $\xi_{\perp} \sim (\ln \xi_{\|})^{\psi}, \quad \psi = 1/2$

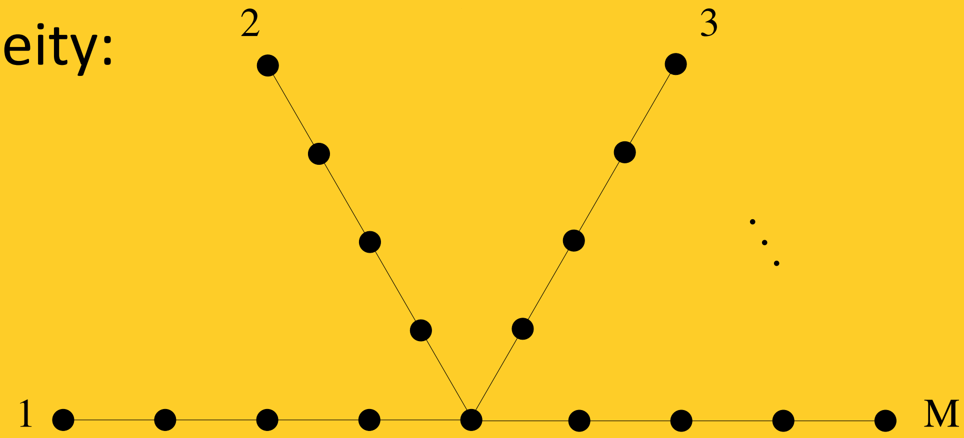
- Temporal disorder:

  Conventional disorder criticality: $\xi_{\|} \sim \xi_{\perp}^{z}$

- Fractals, complex networks

Inhomogeneous contact process

New types of inhomogeneity:



- Multiple junction:
  M=1 free surface
  M=2 internal defect
  M>2

$$M(\beta_1/\nu_\parallel) > 1$$ relevant perturbation.

- Smoothly varying (surface) inhomogeneity:

$$\lambda(l) - \lambda(\infty) = Al^{-s}$$

l distance from the surface

Scaling transformation:

$$\Delta\lambda'(l') = b^{1/\nu_\perp}\Delta\lambda(l) \longrightarrow A' = Ab^{1/\nu_\perp - s}$$

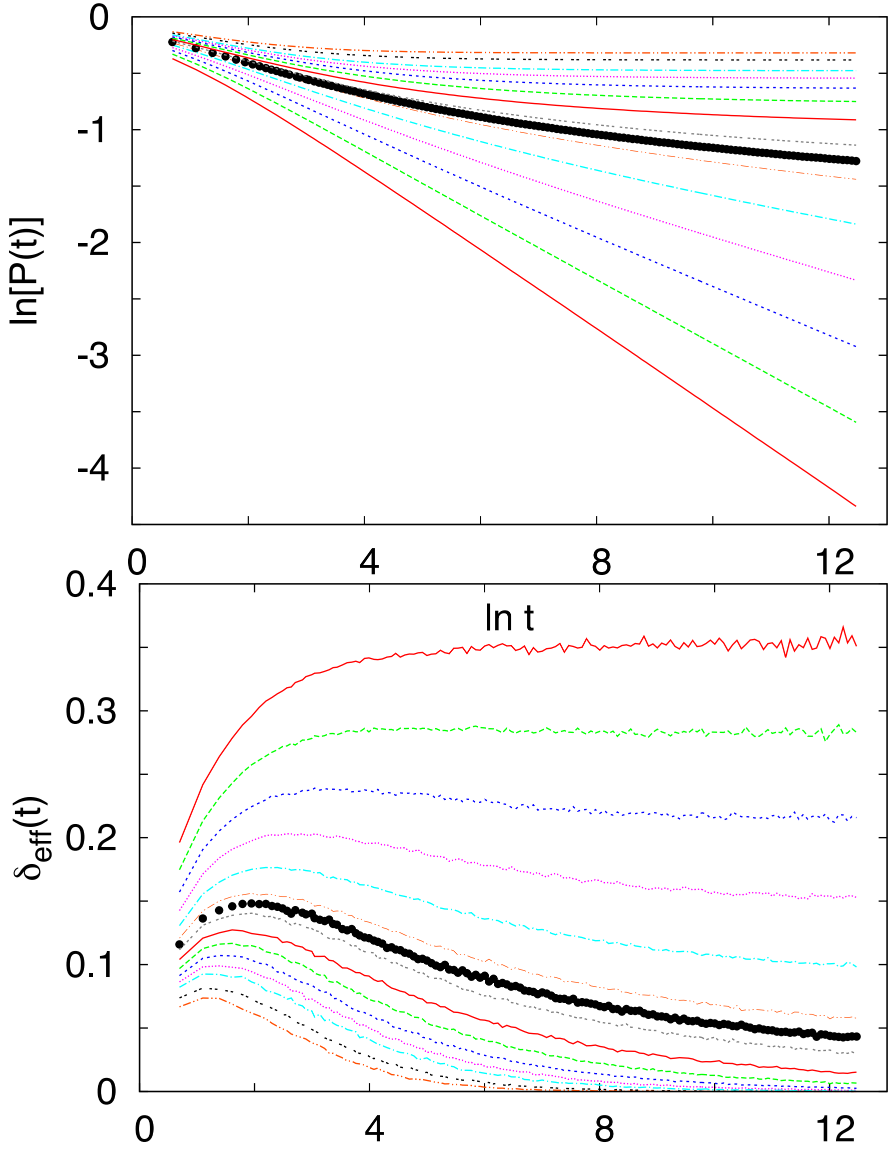
marginal perturbation with: $s = 1/\nu_\perp$

Numerical simulation

- Time-steps: t=1,2,…,10^6
- Nº active sites: N(t)
- Active sites picked randomly:
  - made inactive with pr. $1/[\lambda(l)+1]$
  - activate one neighboring site with pr. $\lambda(l)/[\lambda(l)+1]$
  - for end site activation with pr. $1/2\lambda(l)/[\lambda(l)+1]$
- Seed simulation:
  initially all but the surface site are inactive
- Measuring the survival probability: P(t) (fraction of active sites)
  in the long-time limit: $P(t) \to \rho_1$
- Effective decay exponent:

$$\delta_{\text{eff}}(t) = -\frac{\ln P(t) - \ln P(t')}{\ln t - \ln t'}$$

Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$



$P(t) \simeq b(A)t^{-\delta'(A)} + p(A)$

$A = 3.5, 4, 4.5, 5, 5.5, 6, 7, 8$

$A_c = 3.25$

$A = 0.5, 1, 1.5, 2, 2.5, 3$

$P(t) \sim t^{-\delta(A)}$

$A = 0.5, 1, 1.5, 2, 2.5, 3$

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$A = 3.5, 4, 4.5, 5, 5.5, 6, 7, 8$

Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

Surface phase-transition

$A < A_c$ second order $P(t) \sim t^{-\delta(A)}$

decay exponent is a function of *A* $\delta(A)$

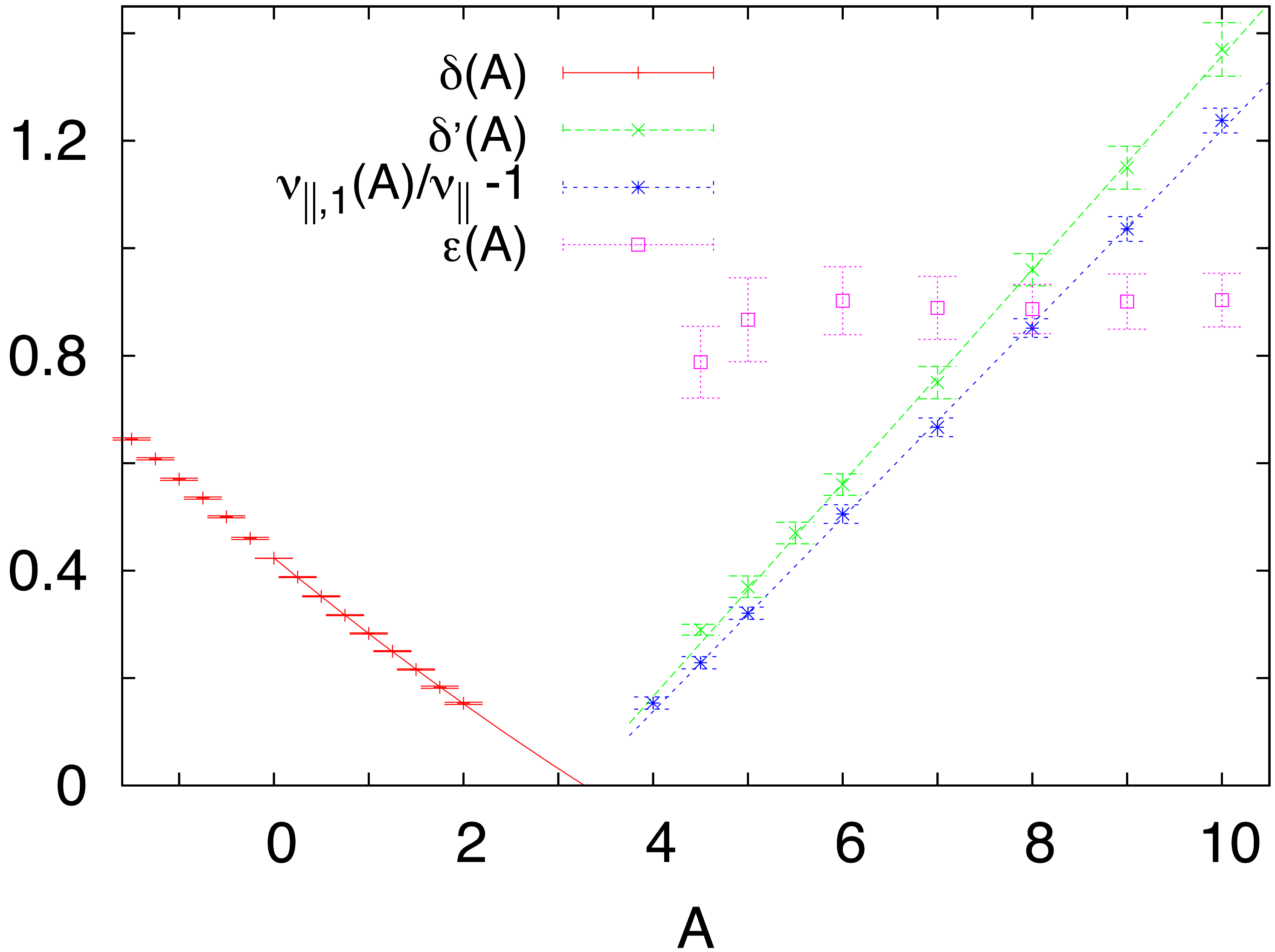
correlation length $\xi_\parallel \sim |\Delta|^{-\nu_{\parallel,1}(A)}$

which can be measured from $P(t,\Delta) = t^{-\delta(A)} f(\Delta t^{1/\nu_{\parallel,1}(A)})$

and does not depend on A: $\nu_{\parallel,1}(A) = \nu_\parallel$

Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

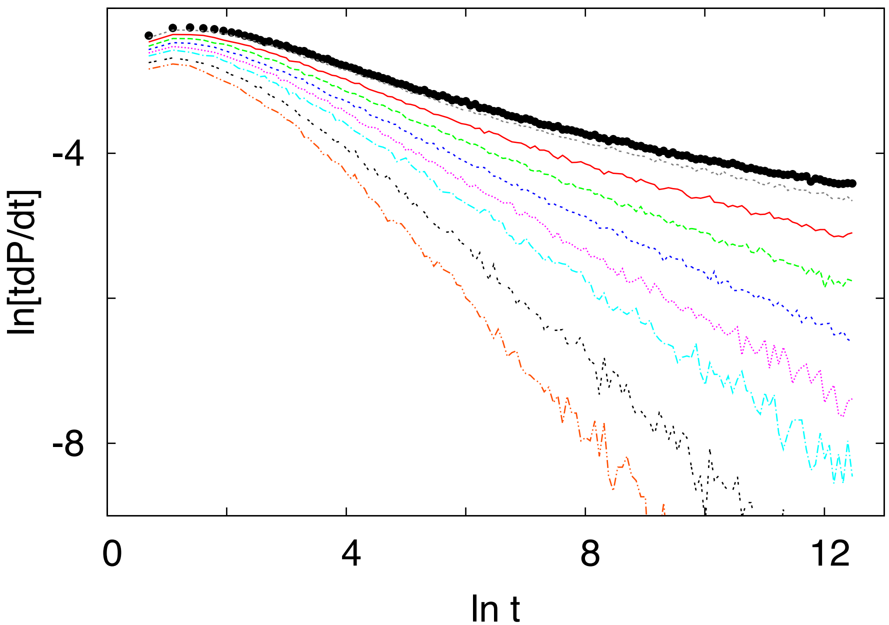
Surface critical exponents

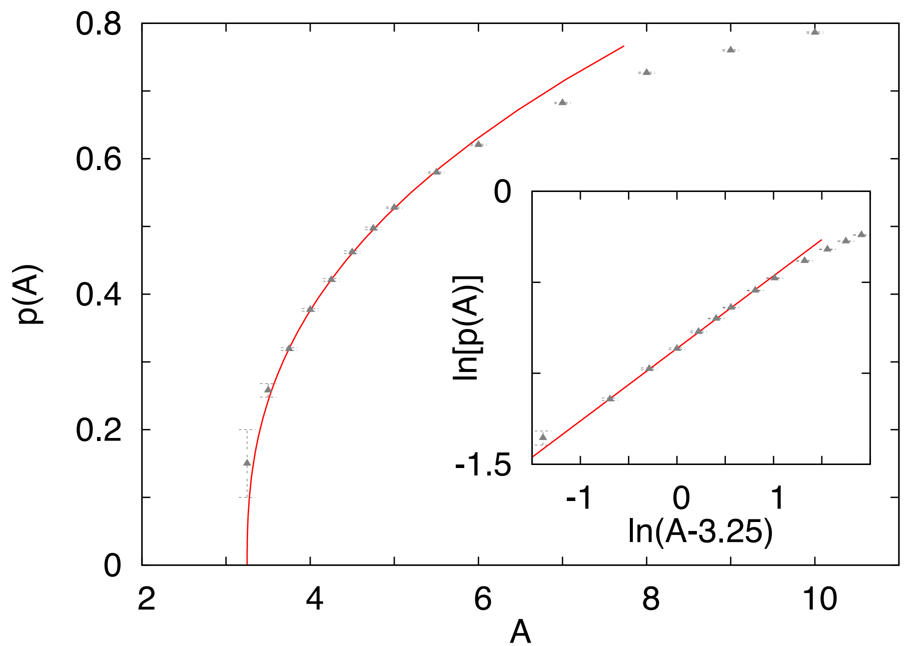


Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

Surface phase-transition

$A > A_c$ mixed-order $P(t) \simeq b(A)t^{-\delta'(A)} + p(A)$





$A = 3.5, 4, 4.5, 5, 5.5, 6, 7, 8$

$p(A) \sim (A - A_c)^{\beta_{tc}}, \quad \beta_{tc} \approx 0.40$

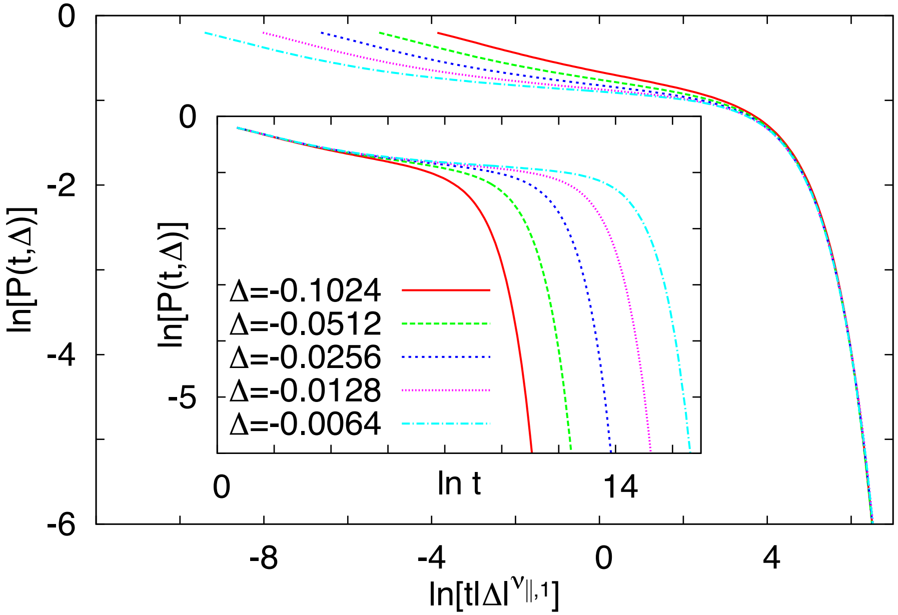
Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

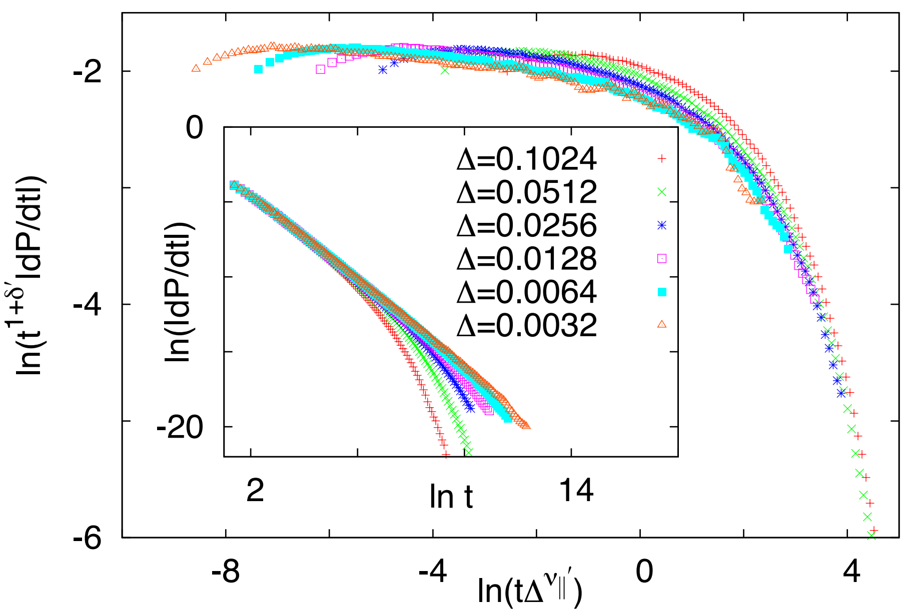
$A > A_c$ mixed order surface transition

<u>correlation-length scaling is asymmetric</u>

$\Delta < 0$ inactive phase

$\Delta > 0$ active phase



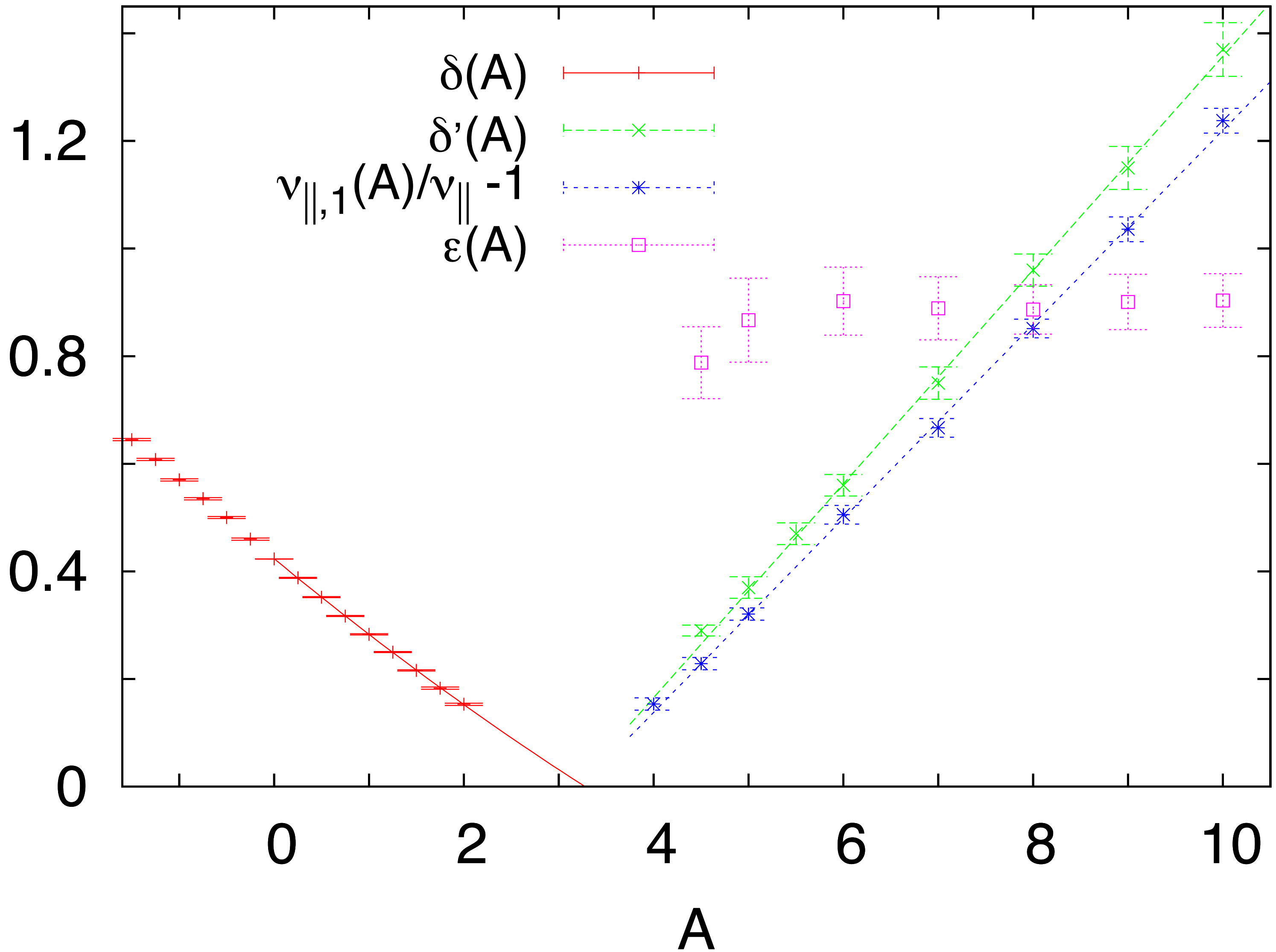


$\nu_{\|,1}(A)$ depends on *A*

$\nu_{\|,1}(A) = \nu_{\|}$

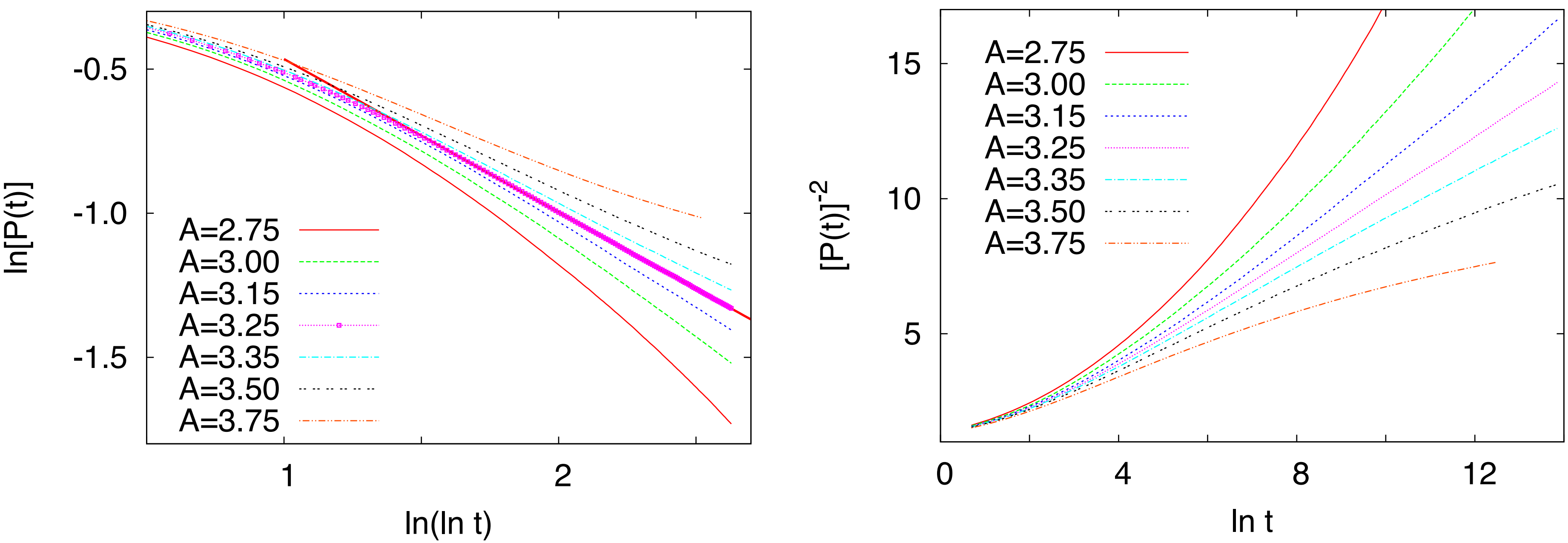
Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

Surface critical exponents



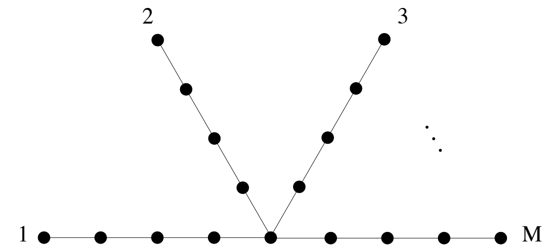
Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

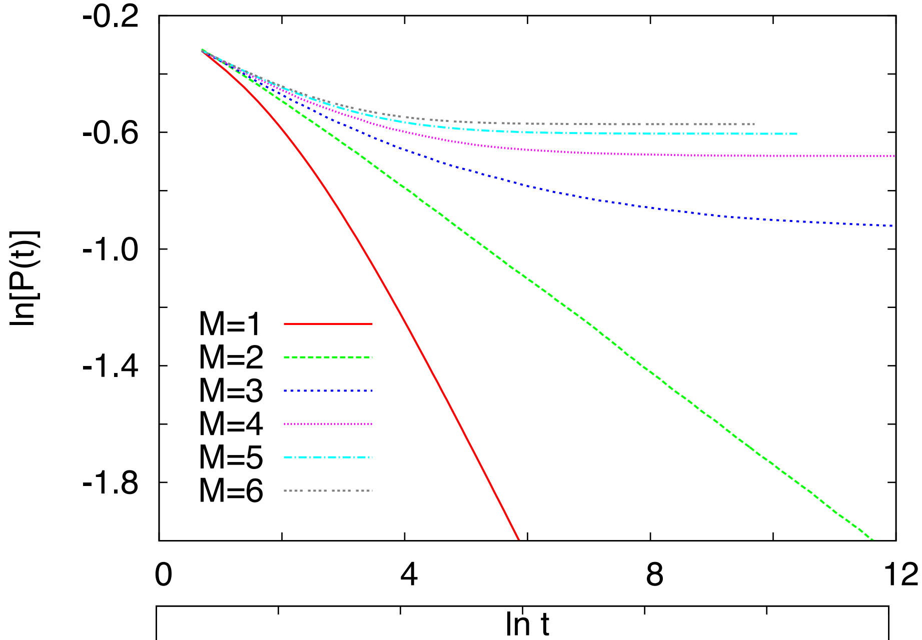
$A = A_c$ tricritical point, logarithmic decay

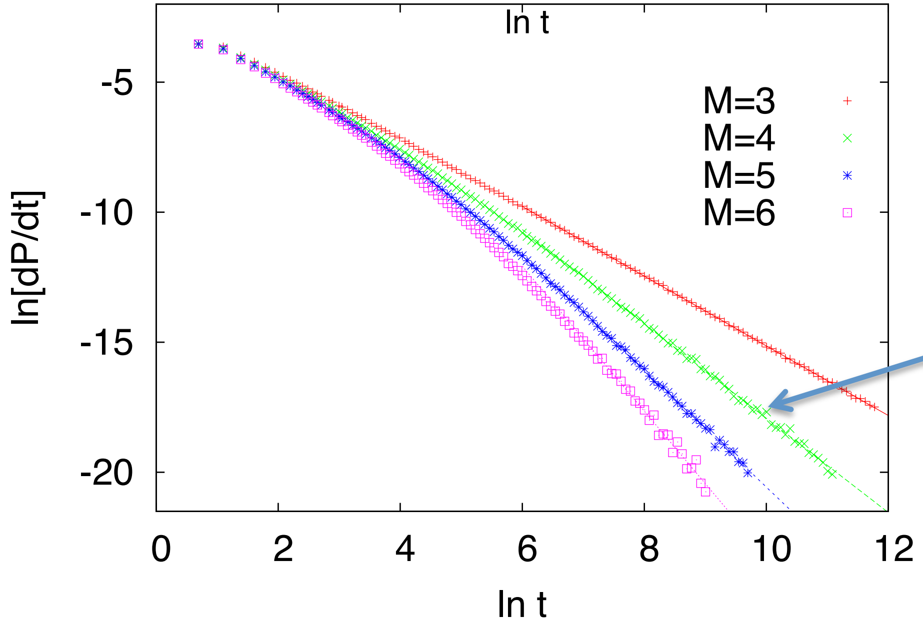


$$P(t) \sim (\ln t)^{-\gamma}, \quad \gamma \approx 0.53; \quad \nu_{\|,1}(A) = \nu_{\|}$$

Multiple junctions





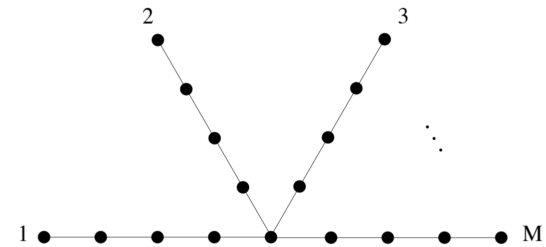


M>2 mixed-order transition

$$P(t) \simeq b_M t^{-\delta'_M} + p_M$$

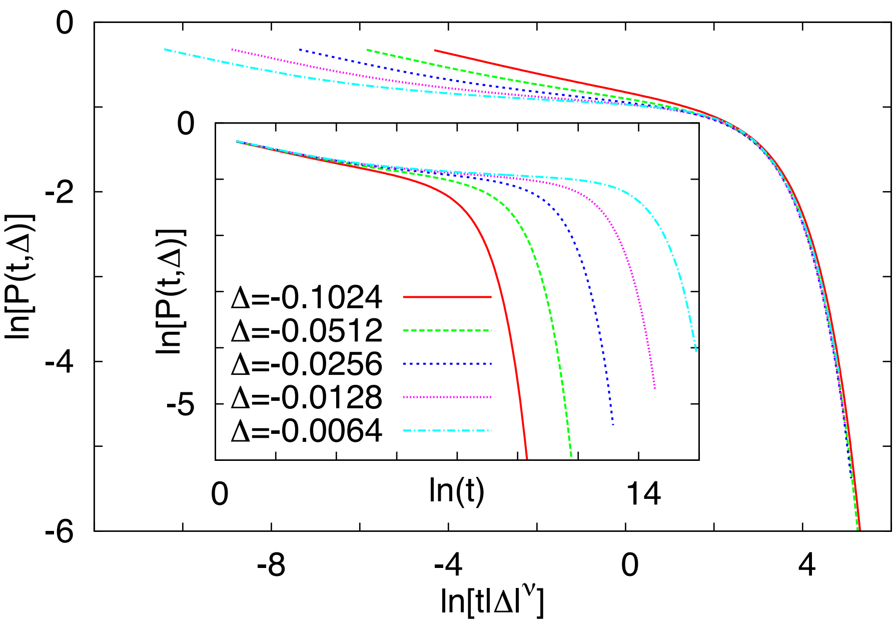
slope: $-\delta'_M - 1$

Multiple junctions



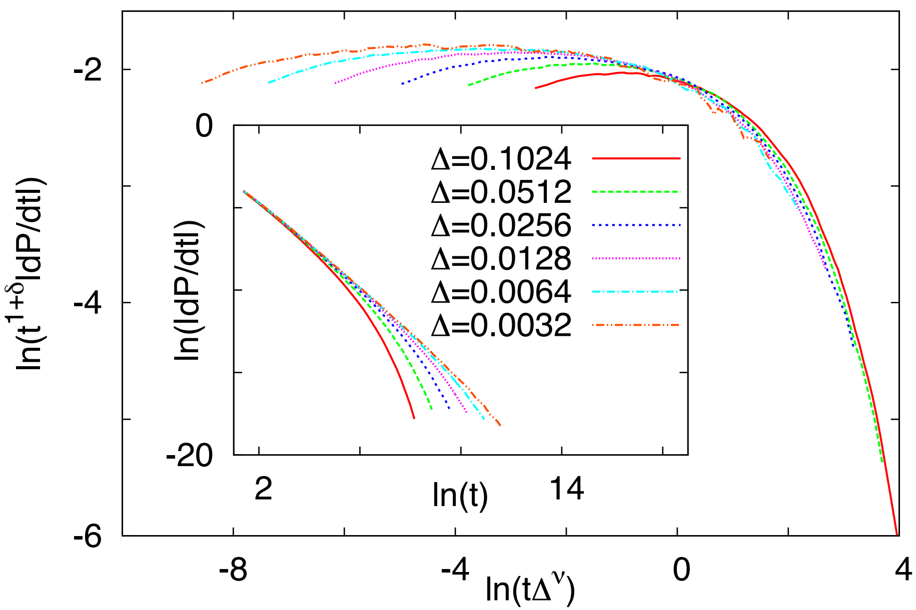
correlation-length scaling is asymmetric

$\Delta < 0$ inactive phase



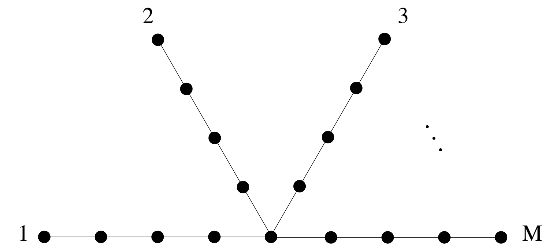
$\nu_{\parallel,M}$ depends on *M*

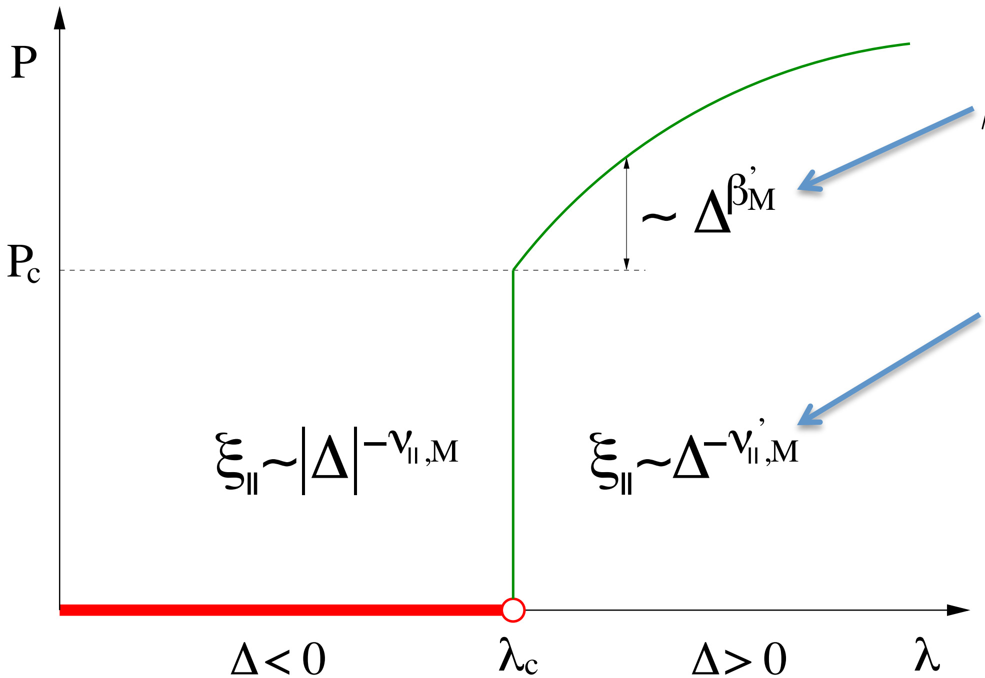
$\Delta > 0$ active phase



$\nu_{\parallel,M} = \nu_{\parallel}$

Multiple junctions





$$\beta'_M = \delta'_M \nu'_{\|,M}$$

$\nu_{\|}$

smoothly varying inhomogeneity

M	P_c	δ'_M	$\nu_{\|,M}$
3	0.391(2)	0.34(1)	2.20(3)
4	0.507(2)	0.81(3)	3.00(10)
5	0.546(2)	1.25(10)	3.7(1)
6	0.564(2)	1.8(1)	4.5(1)

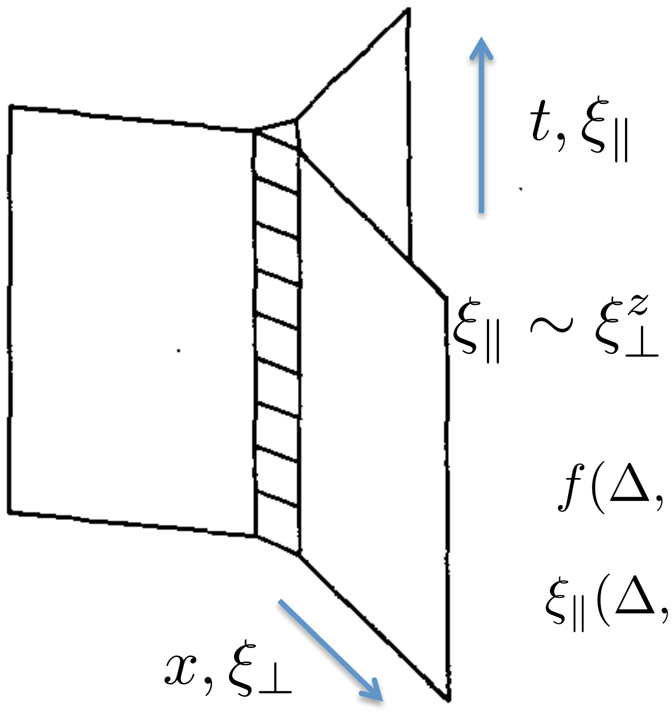
$A_3 \approx 4.75$

$A_4 \approx 7.3$

$A_5 \approx 9.5$

Mixed-order transition - Scaling considerations

Equivalent (1+1)-dimensional static problem



local free-energy per site

$$f(\Delta, h_1, \Delta_1, t)$$

local control parameter (Δ_1)

local ordering field (h_1)

scaling relations: $x \to x/b$

$$f(\Delta, h_1, \Delta_1, t) = b^{-z} f(\Delta b^{1/\nu_\perp}, h_1 b^{y_{h_1}}, \Delta_1 b^{y_{\Delta_1}}, t/b^z)$$

$$\xi_\parallel(\Delta, h_1, \Delta_1, t) = b^z \xi_\parallel(\Delta b^{1/\nu_\perp}, h_1 b^{y_{h_1}}, \Delta_1 b^{y_{\Delta_1}}, t/b^z)$$

scaling exponents (y_{h_1}, y_{Δ_1})

local order parameter (survival probability) $P = \partial f / \partial h_1$

$$P(\Delta, h_1, \Delta_1, t) = b^{-z+y_{h_1}} f(\Delta b^{1/\nu_\perp}, h_1 b^{y_{h_1}}, \Delta_1 b^{y_{\Delta_1}}, t/b^z), \quad \to \quad y_{h_1} = z$$

Mixed-order transition - Scaling considerations

Δ_1 irrelevant variable $y_{\Delta_1} < 0$

$\Delta > 0$ active phase – harmless variable – analytic in Δ_1

$$P(\Delta, h_1, \Delta_1, t) - P_c = b^{y_{\Delta_1}} \tilde{P}(\Delta b^{1/\nu_\perp}, h_1 b^{y_{h_1}}, t/b^z)$$

$$b = t^{1/z}, \quad \Delta = h_1 = 0, \quad P(t) - P_c \sim t^{y_{\Delta_1}/z}, \quad \delta'_M = -y_{\Delta_1}/z$$

$$b = \Delta^{-\nu_\perp}, \quad \beta'_M = -y_{\Delta_1}\nu_\perp$$

$\Delta < 0$ inactive phase – dangerous variable – non-analytic in Δ_1

assumption: $\xi_\parallel(\Delta, h_1, \Delta_1, x) = \Delta_1^{-\epsilon} \tilde{\xi}_\parallel(\Delta, h_1 \Delta_1^{-\epsilon}, x)$

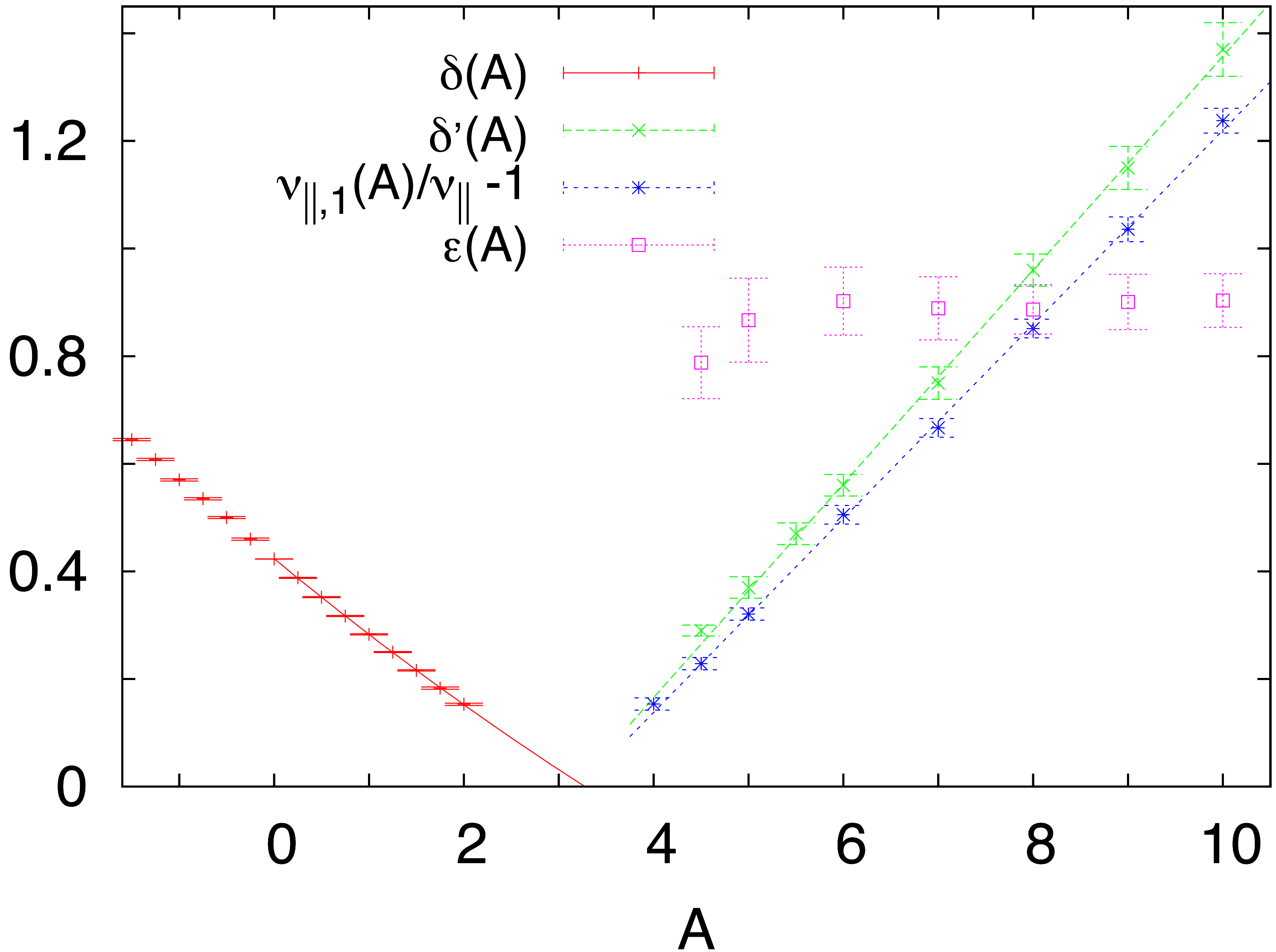
scaling relation: $\xi_\parallel(\Delta, h_1, \Delta_1, x) = b^{z-\epsilon y_{\Delta_1}} \tilde{\xi}_\parallel(\Delta b^{1/\nu_\perp}, h_1 b^{z-\epsilon y_{\Delta_1}}, x/b)$

$$b = x = \xi_\perp, \quad \xi_\parallel \sim \xi_\perp^{z_M}, \quad z_M = z - \epsilon y_{\Delta_1}$$

$$\nu_{\parallel,M} = \nu_\parallel - \epsilon y_{\Delta_1}\nu_\perp = \nu_\parallel(1 + \epsilon\delta'_M), \quad \epsilon = (\nu_{\parallel,M}/\nu_\parallel - 1)/\delta'_M$$

Smoothly varying inhomogeneity: $\Delta\lambda(l) = Al^{-s}$

Surface critical exponents

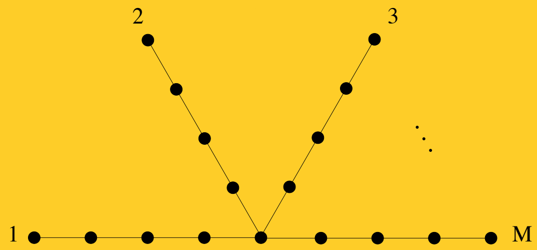


Summary

- Contact process

  at a smooth (surface) inhomogeneity $\Delta\lambda(l) = Al^{-s}$

  at a multiple junction



- Second-order transition for $A < A_c$ and $M \leq 2$
- Mixed-order transition for $A > A_c$ and $M > 2$

  the critical exponents are non-universal (*A* and *M* dependent)

  scaling theory with (dangerous) irrelevant variables

Thank you for your attention!