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Disassembling Casimir scaling functions at finite aspect ratios

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Casimir Interaction between two Colloids

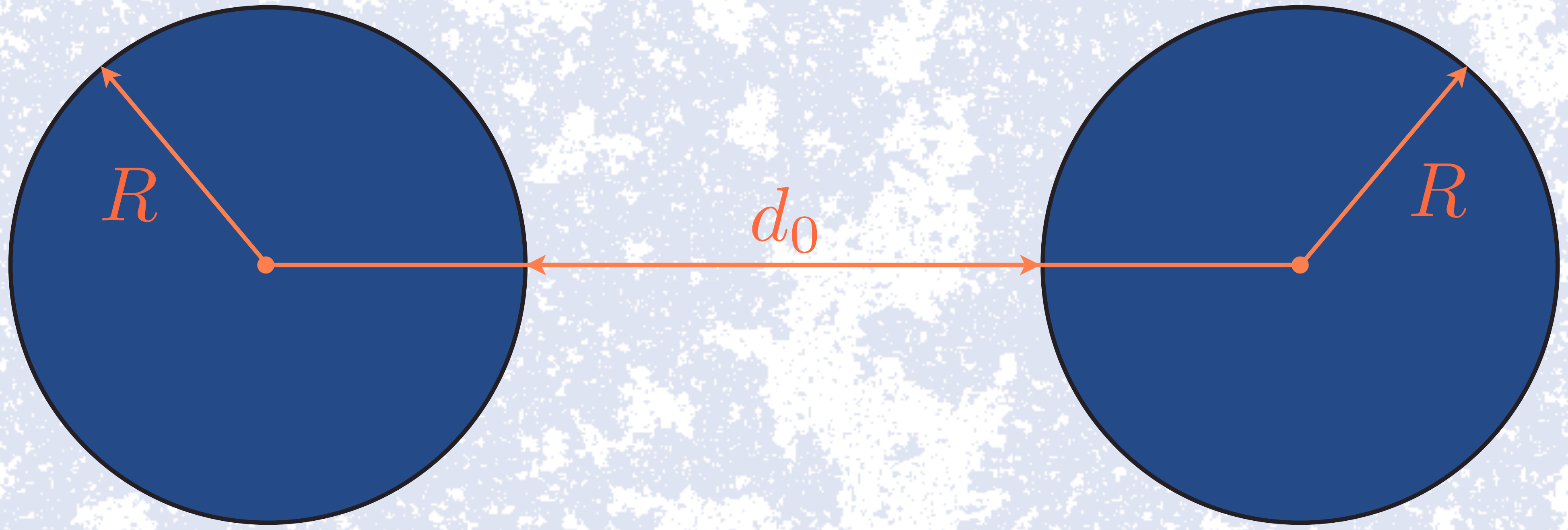
- Highly non-additive
- Exactly known at $T = T_{\mathrm{c}}$ due to conformal field theory

$$\Phi_{\mathrm{C}}^{(ab)}(\kappa)$$

with conformal scaling variable

$$\kappa = \left(\frac{d_0}{2R} + 1\right)^2 - 1$$

- Derjaguin approximation for small distances $d_0 \ll R$ from thin film geometry



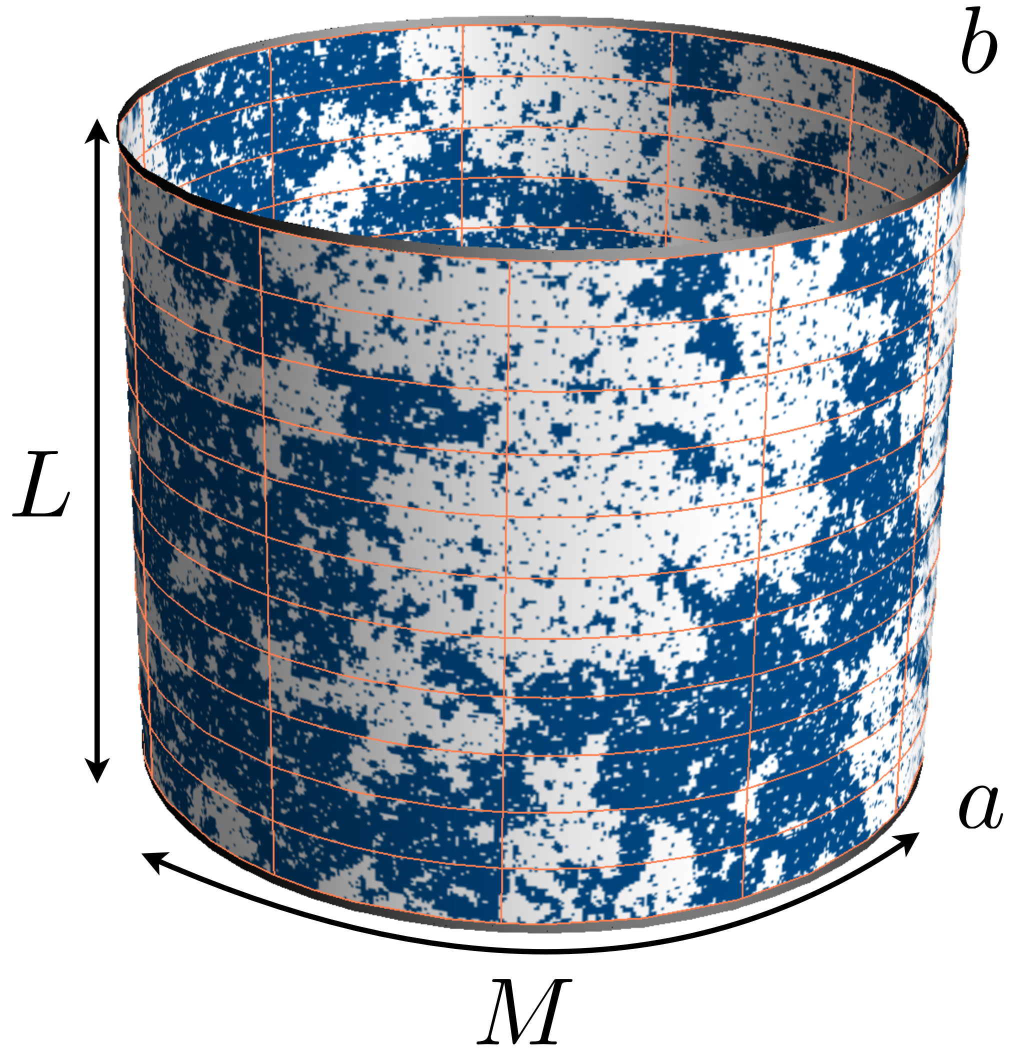
What do we really know about scaling functions?

Conformal Field Theory

$x_\circ = 0$ and ρ arbitrary

$$\Theta_\circ^{(\mathrm{p}/ab)}(x_\circ = 0, \rho) = -\ln\left[\sqrt{\frac{\vartheta_3\left(\mathrm{e}^{-2\pi\rho}\right)}{\eta\left(2\mathrm{i}\rho\right)}} + s_{ab}\sqrt{\frac{\vartheta_2\left(\mathrm{e}^{-2\pi\rho}\right)}{\eta\left(2\mathrm{i}\rho\right)}}\right]$$

with $s_{ab} = \{1, 0, -1\}$ for $ab = \{++, \mathrm{oo}, +-\}$



$$\Phi_\mathrm{C}^{(ab)}(\kappa) = \frac{\pi}{12}\rho + \Theta_\circ^{(ab/\mathrm{p})}(x_\circ = 0, \rho) \quad \text{with} \quad \kappa = \cosh(2\pi\rho)$$

VS

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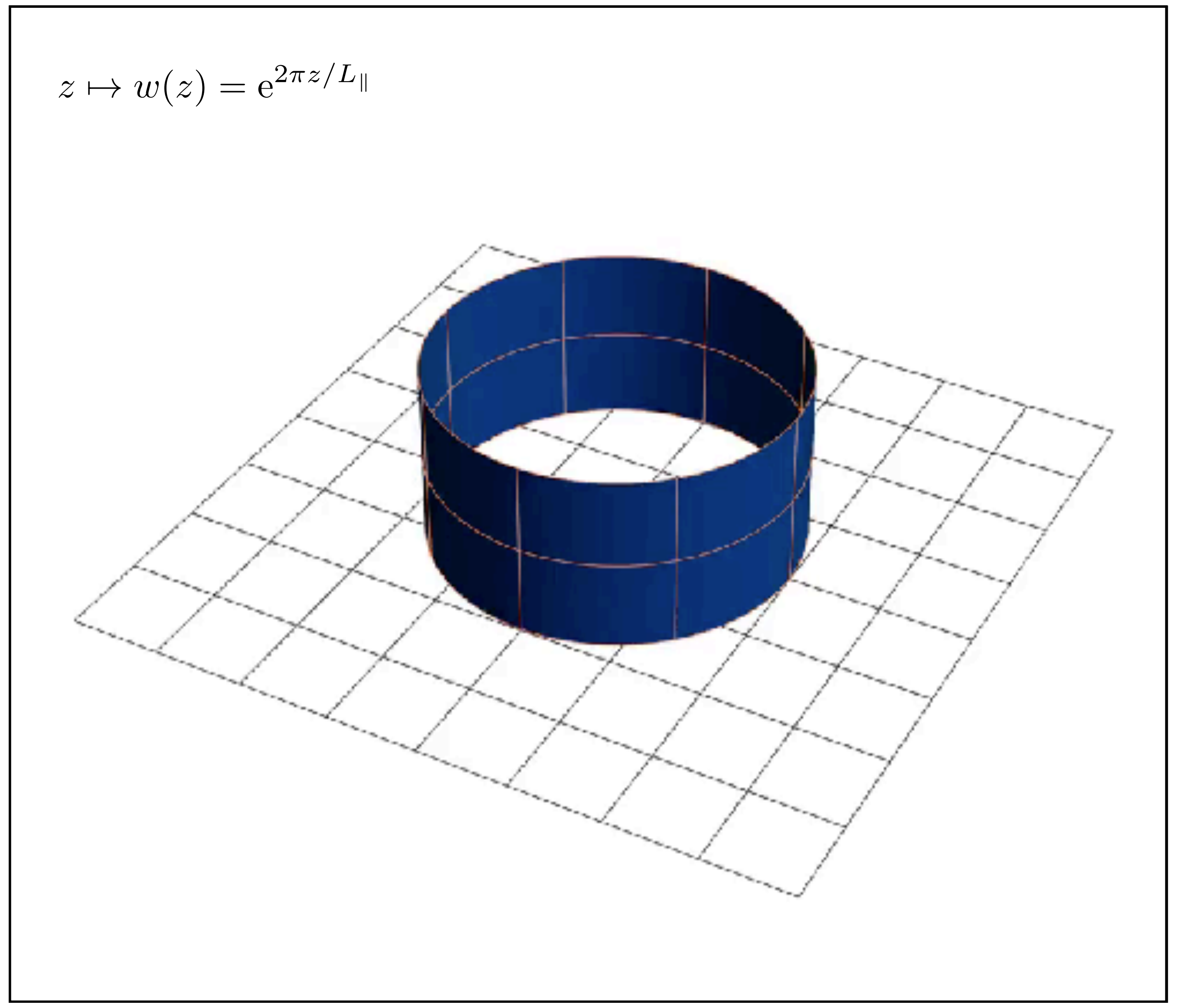

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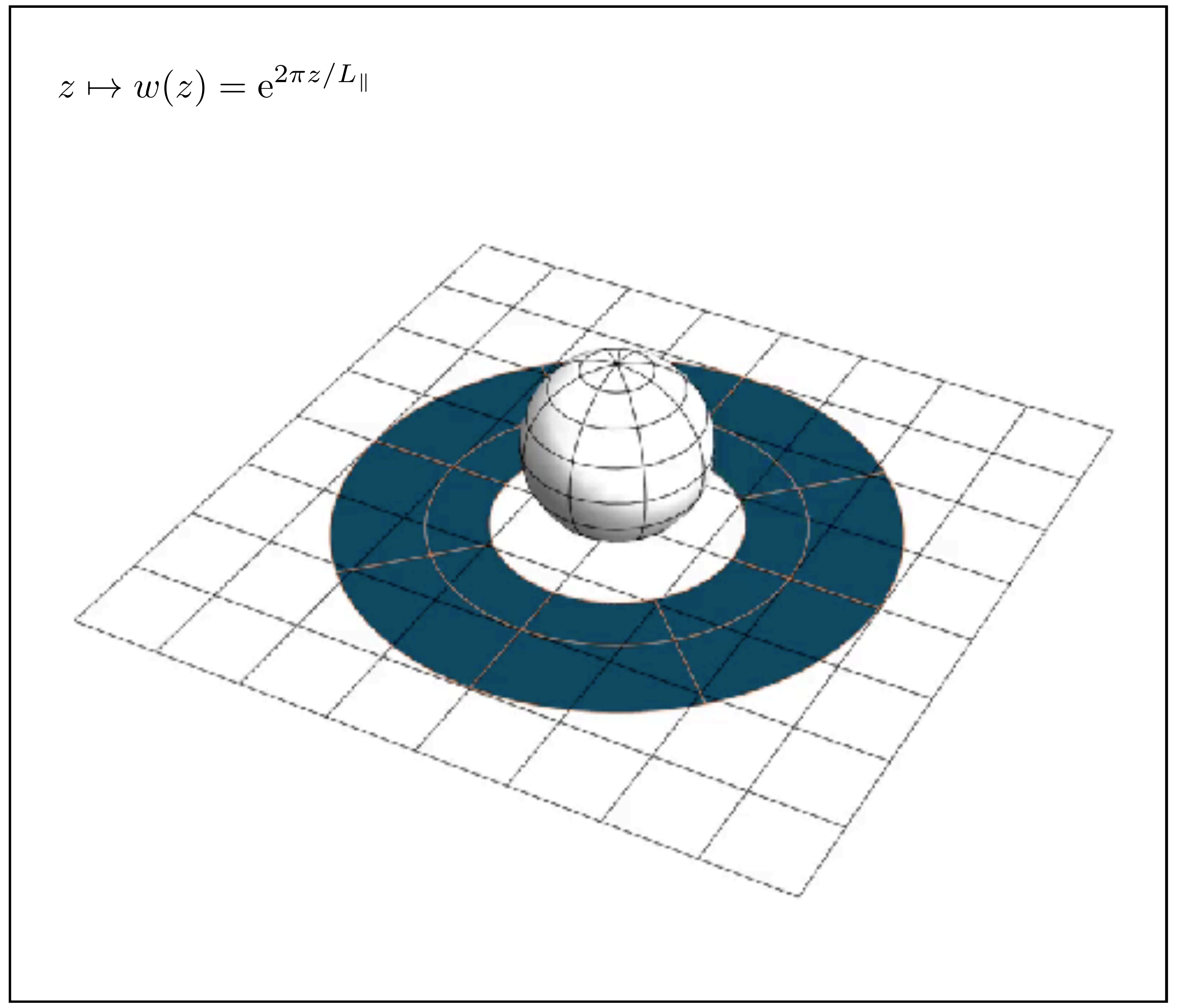

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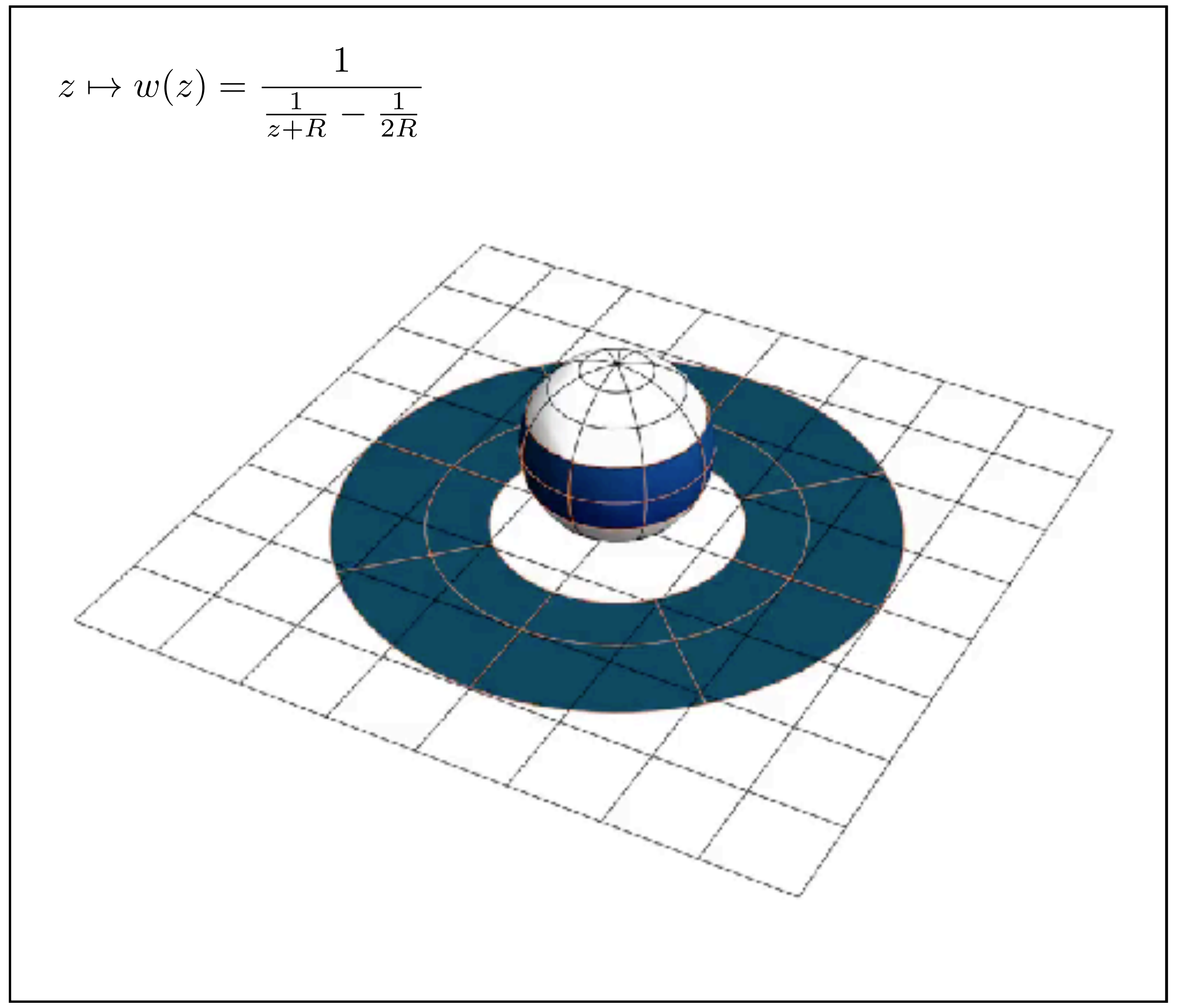

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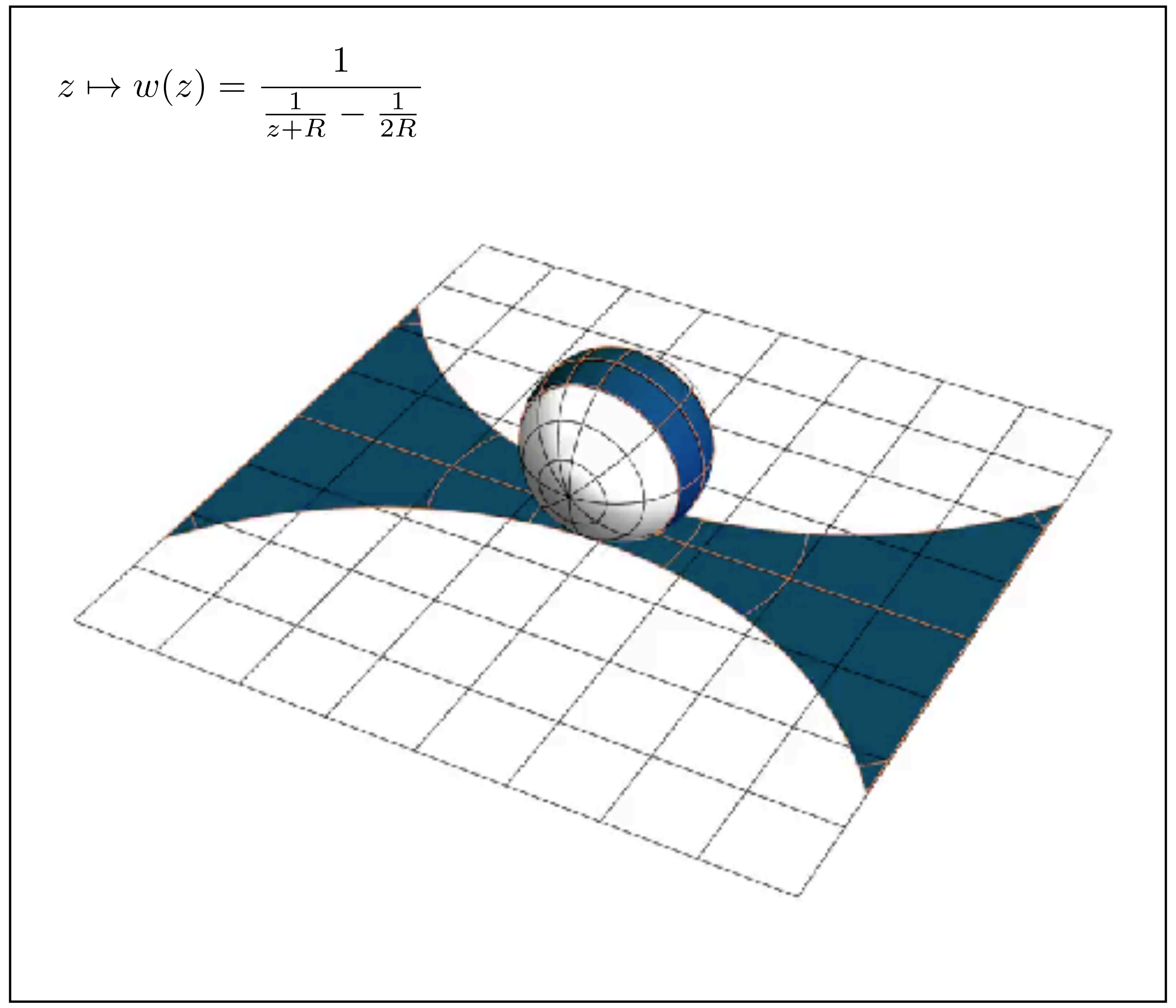

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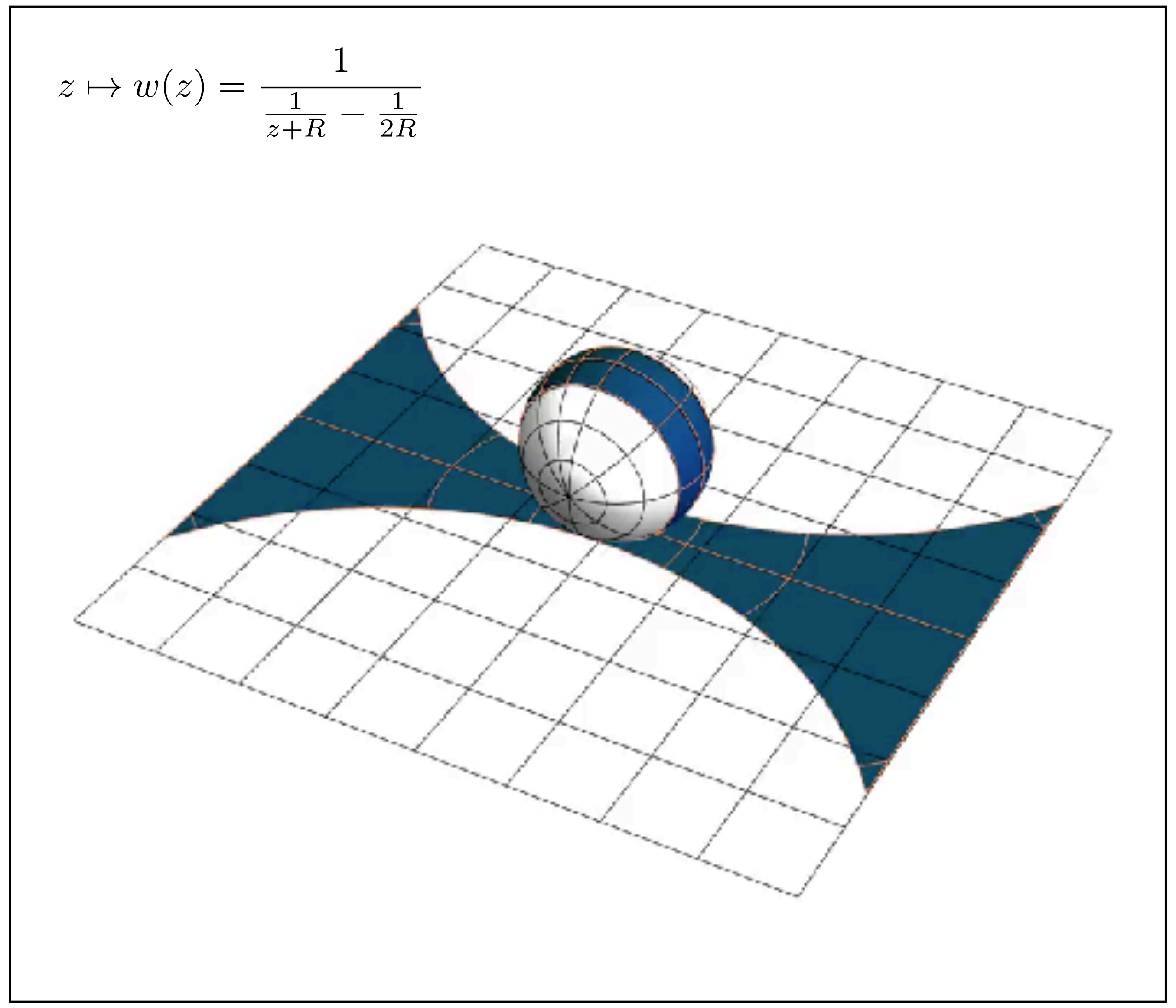

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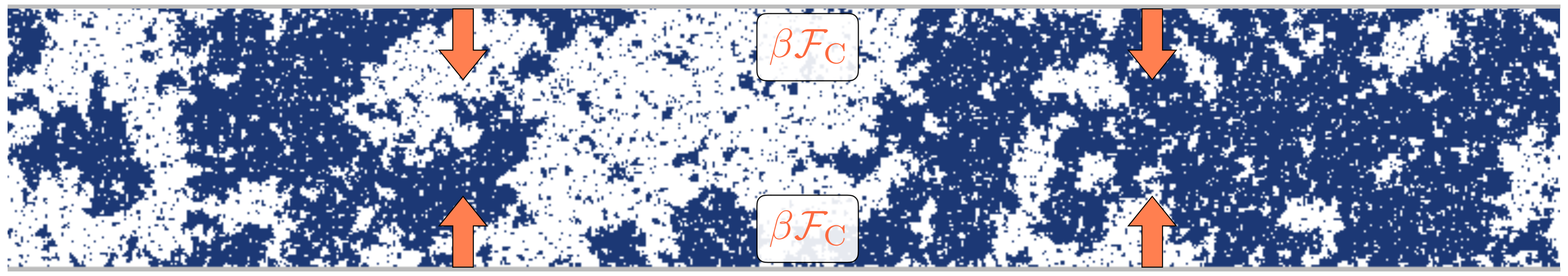

$$\Theta_\perp^{(\mathrm{oo})}(x_\perp) = -\frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi \, \ln\left[1 + \frac{\sqrt{x_\perp^2 + \Phi^2} - x_\perp}{\sqrt{x_\perp^2 + \Phi^2} + x_\perp}\mathrm{e}^{-2\sqrt{x_\perp^2 + \Phi^2}}\right]$$

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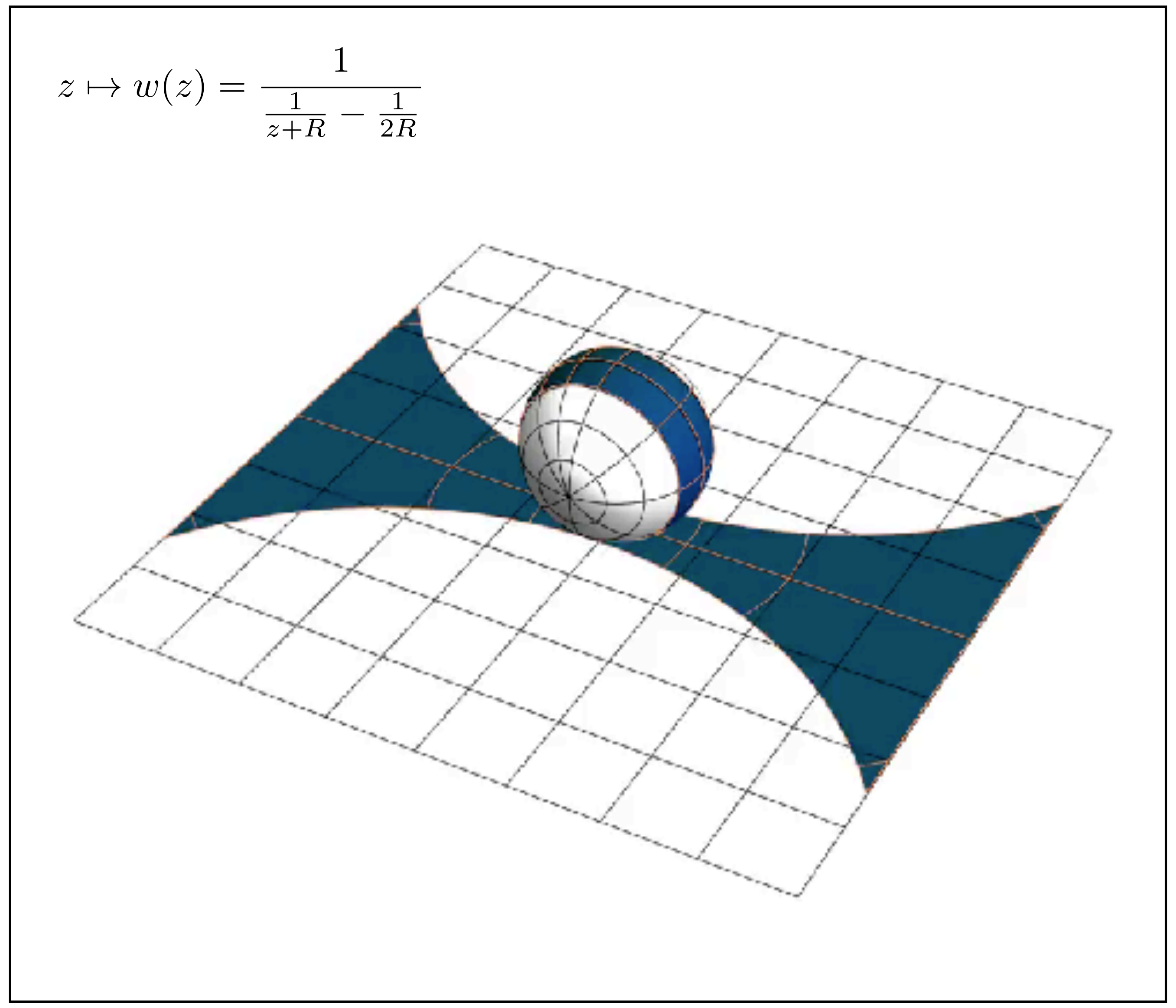

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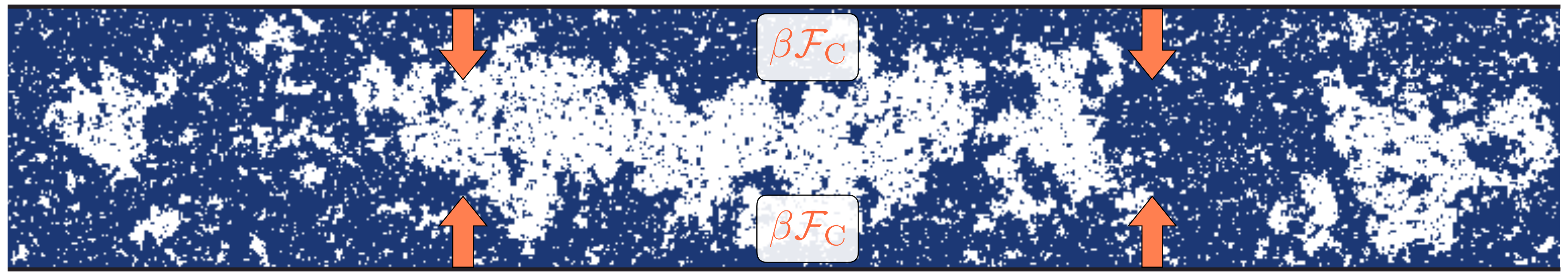

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exact duality symmetry: $\Theta_\perp^{(++)}(x_\perp) = \Theta_\perp^{(\mathrm{oo})}(-x_\perp)$

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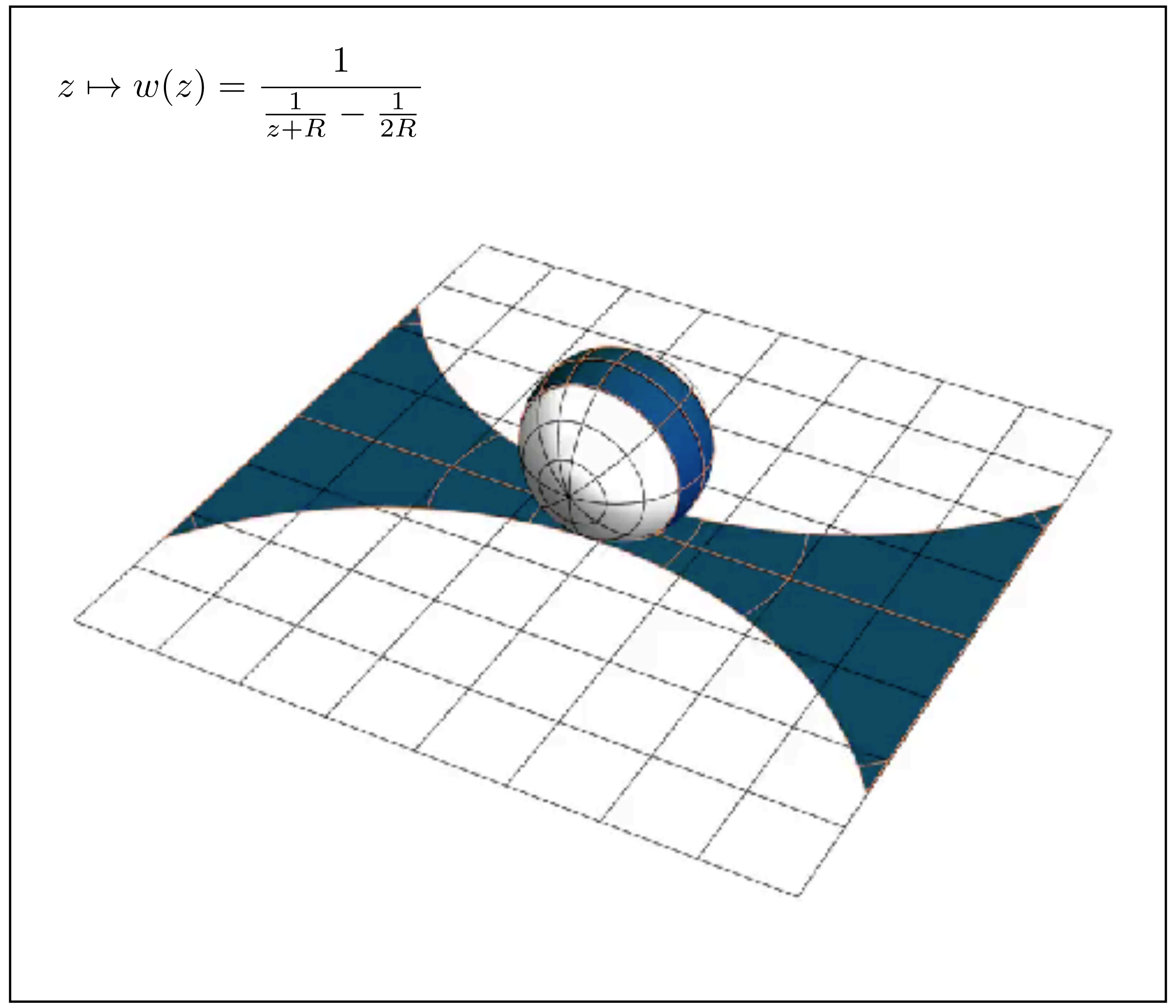

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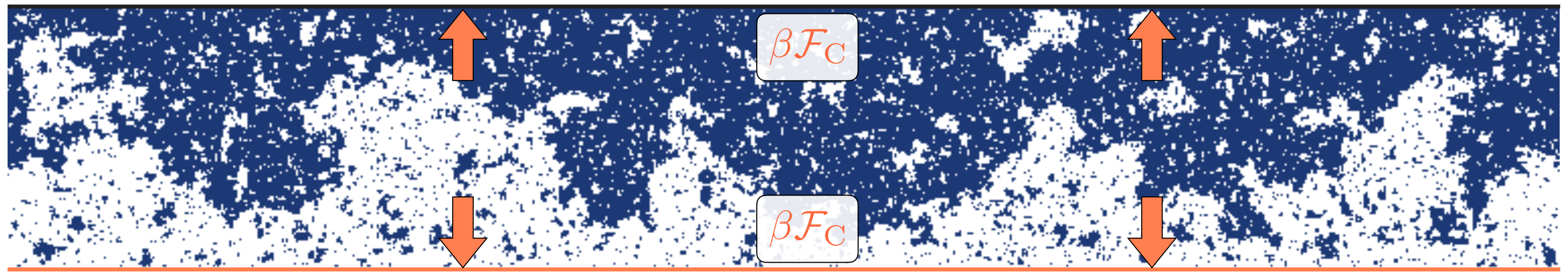

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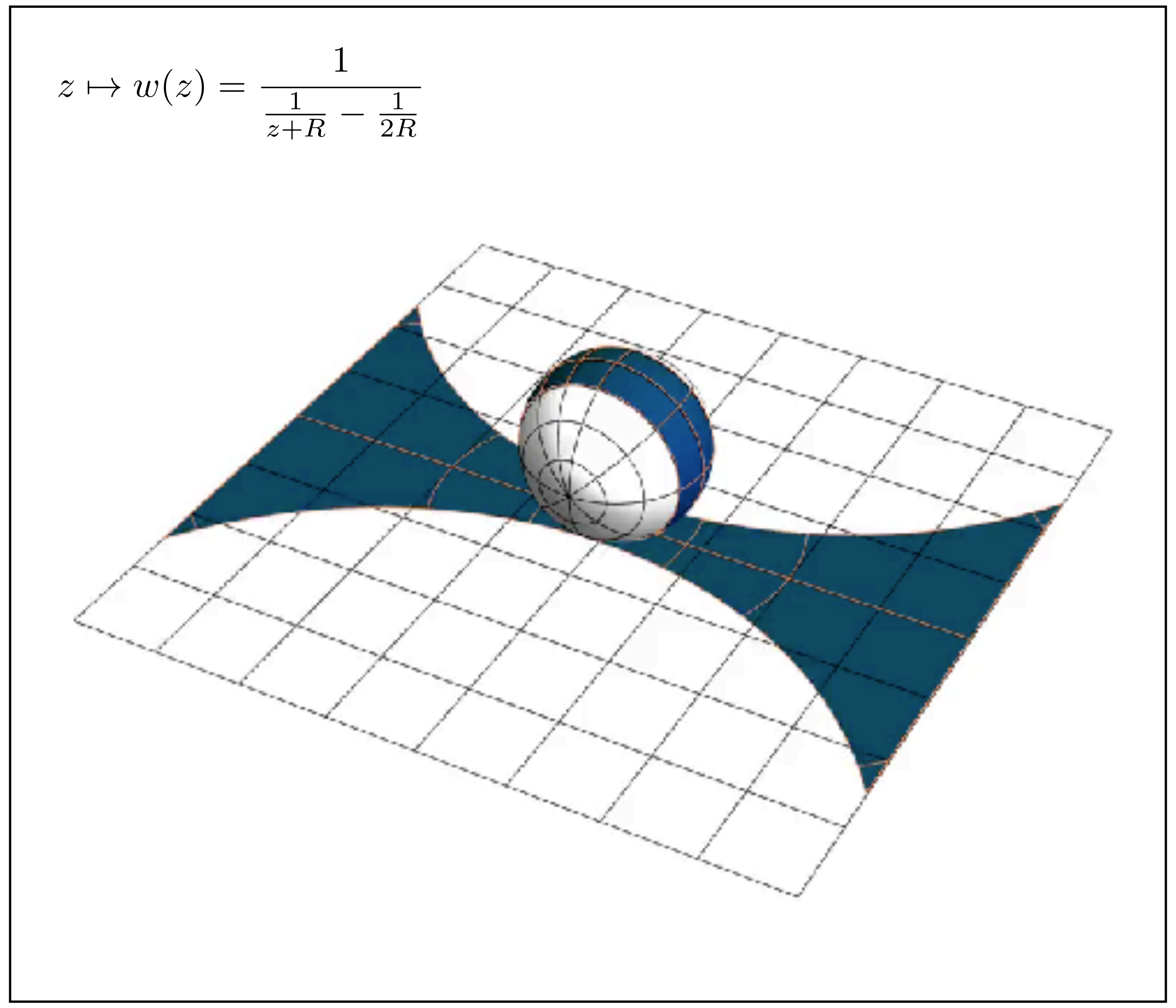

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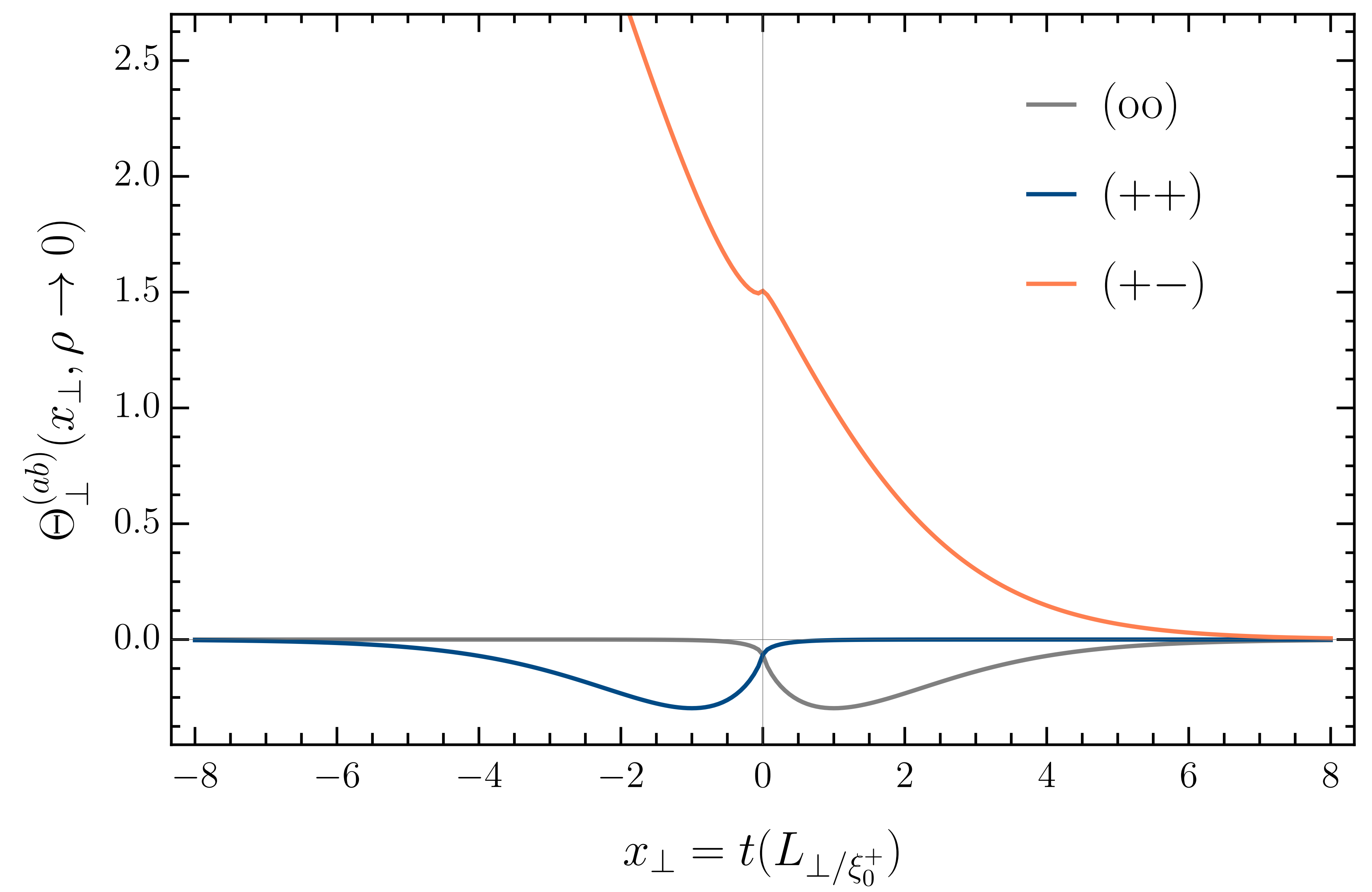

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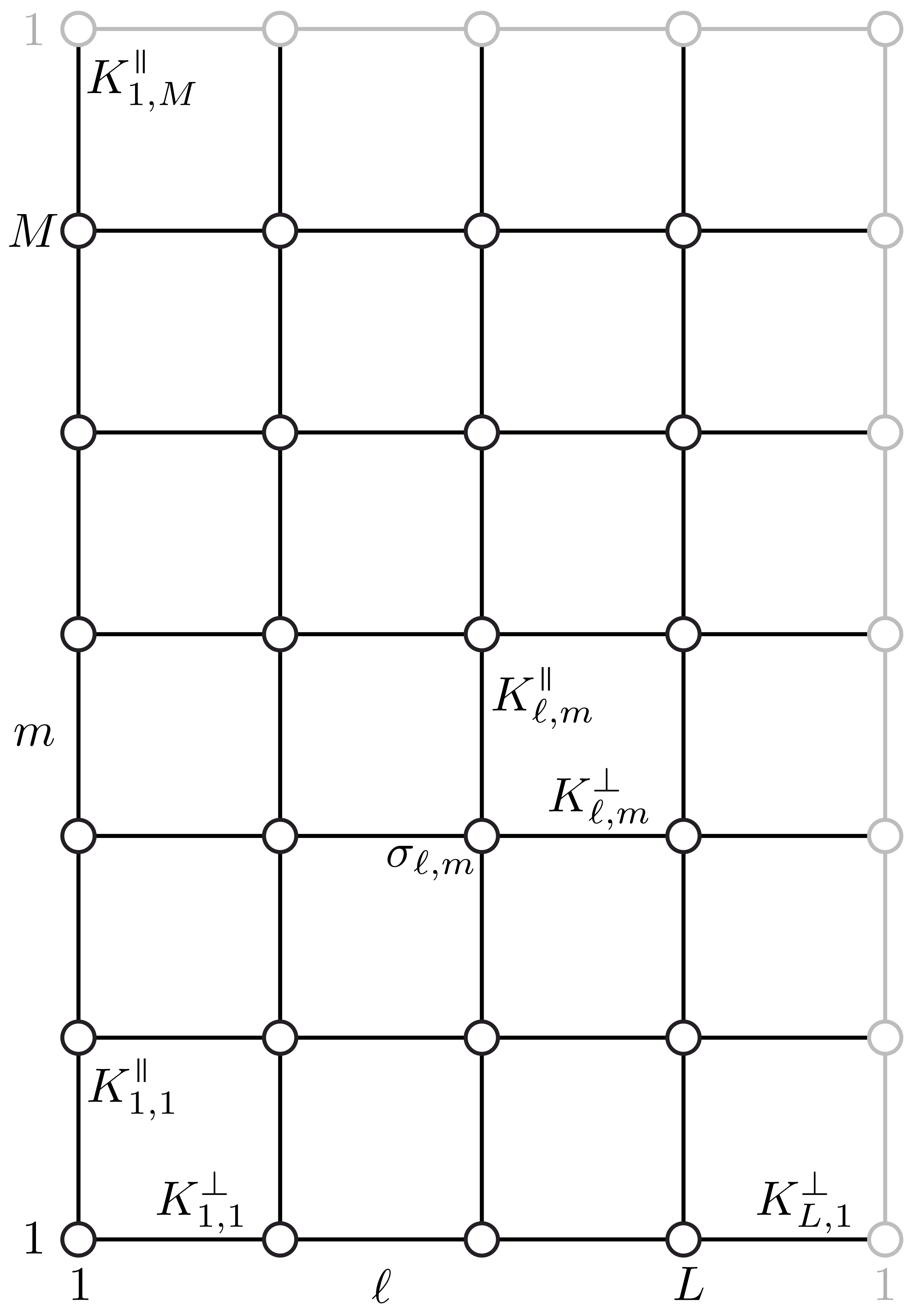
1 Calculation of the Free Energy

- 2d Ising model Hamiltonian on torus

$$\mathcal{H} = -\sum_{\ell=1}^{L}\sum_{m=1}^{M} K^{\perp}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell+1,m} - \sum_{\ell=1}^{L}\sum_{m=1}^{M} K^{\parallel}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell,m+1}$$



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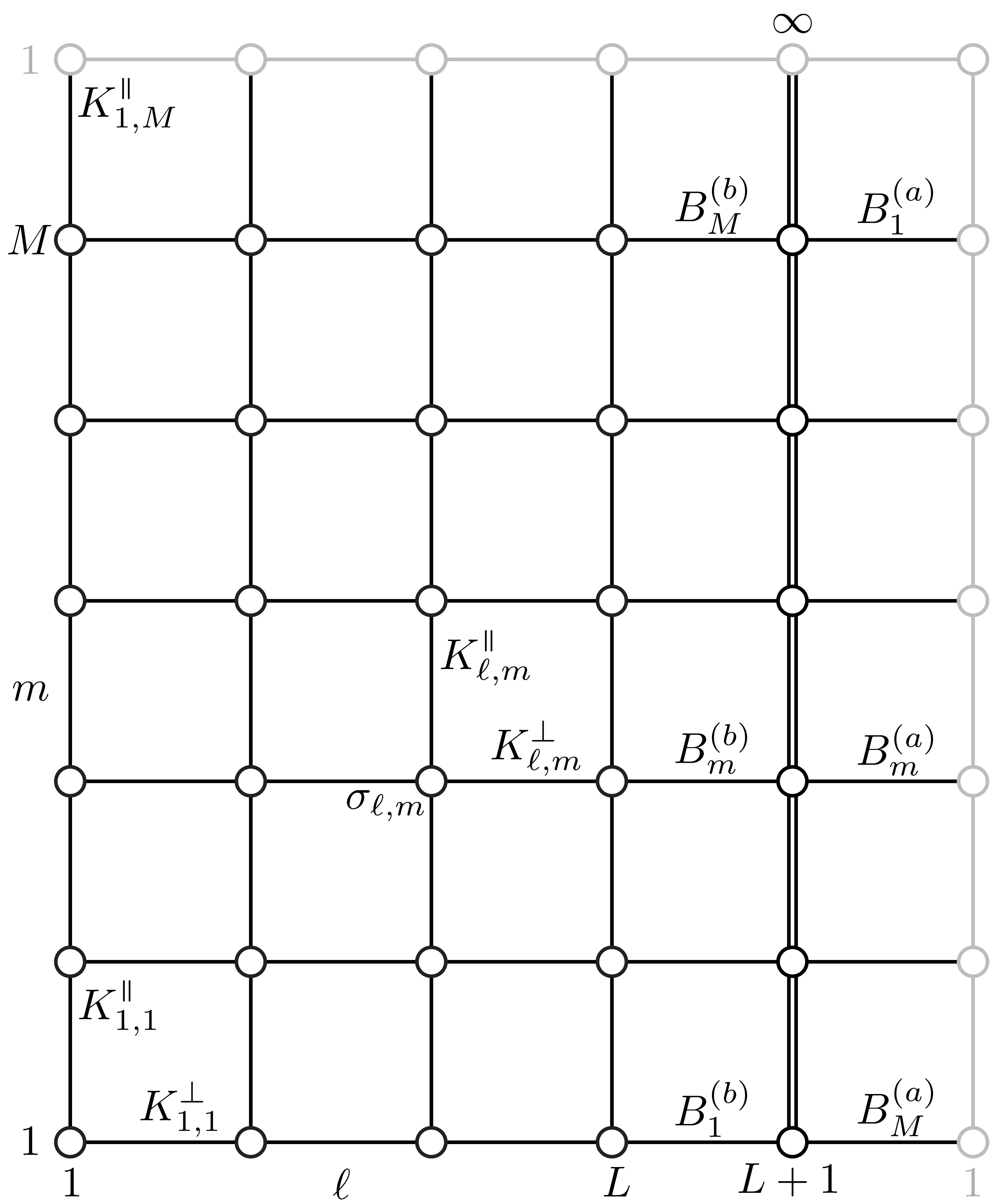
$$\mathcal{H}^{(a/b)} = -\sum_{\ell=1}^{L}\sum_{m=1}^{M} K^{\perp}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell+1,m} - \sum_{\ell=1}^{L}\sum_{m=1}^{M} K^{\parallel}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell,m+1} - \sum_{m=1}^{M}\left[B^{(a)}_{m}\sigma_{1,m} + B^{(b)}_{m}\sigma_{L-1,m}\right]$$



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Abbreviations

$$z^{\perp}_{\ell,m} = \tanh K^{\perp}_{\ell,m} \qquad z^{(a)}_{m} = \tanh B^{(a)}_{m}$$

$$z^{\parallel}_{\ell,m} = \tanh K^{\parallel}_{\ell,m} \qquad z^{(b)}_{m} = \tanh B^{(b)}_{m}$$

$$Z^{(a/b)} = \overbrace{\prod_{\ell=1}^{L-1}\prod_{m=1}^{M}\cosh K^{\perp}_{\ell,m}\prod_{\ell=1}^{L}\prod_{m=1}^{M}\cosh K^{\parallel}_{\ell,m} \times \prod_{m=1}^{M}\cosh B^{(a)}_{m}\prod_{m=1}^{M}\cosh B^{(b)}_{m}}^{\equiv Z_0^{(a/b)}}$$
$$\underbrace{\times \sum_{\{\sigma\}}\left[\prod_{\ell=1}^{L-1}\prod_{m=1}^{M}\left(1+z^{\perp}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell+1,m}\right)\right] \times \left[\prod_{\ell=1}^{L}\prod_{m=1}^{M}\left(1+z^{\parallel}_{\ell,m}\sigma_{\ell,m}\sigma_{\ell,m+1}\right)\right] \times \left[\prod_{m=1}^{M}\left(1+z^{(a)}_{m}\sigma_{1,m}\right)\right] \times \left[\prod_{m=1}^{M}\left(1+z^{(b)}_{m}\sigma_{L,m}\right)\right]}_{\equiv Z^{(a/b)}_{\mathrm{sing}}}$$

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- Add surface field line
- High-temperature expansion of partition function
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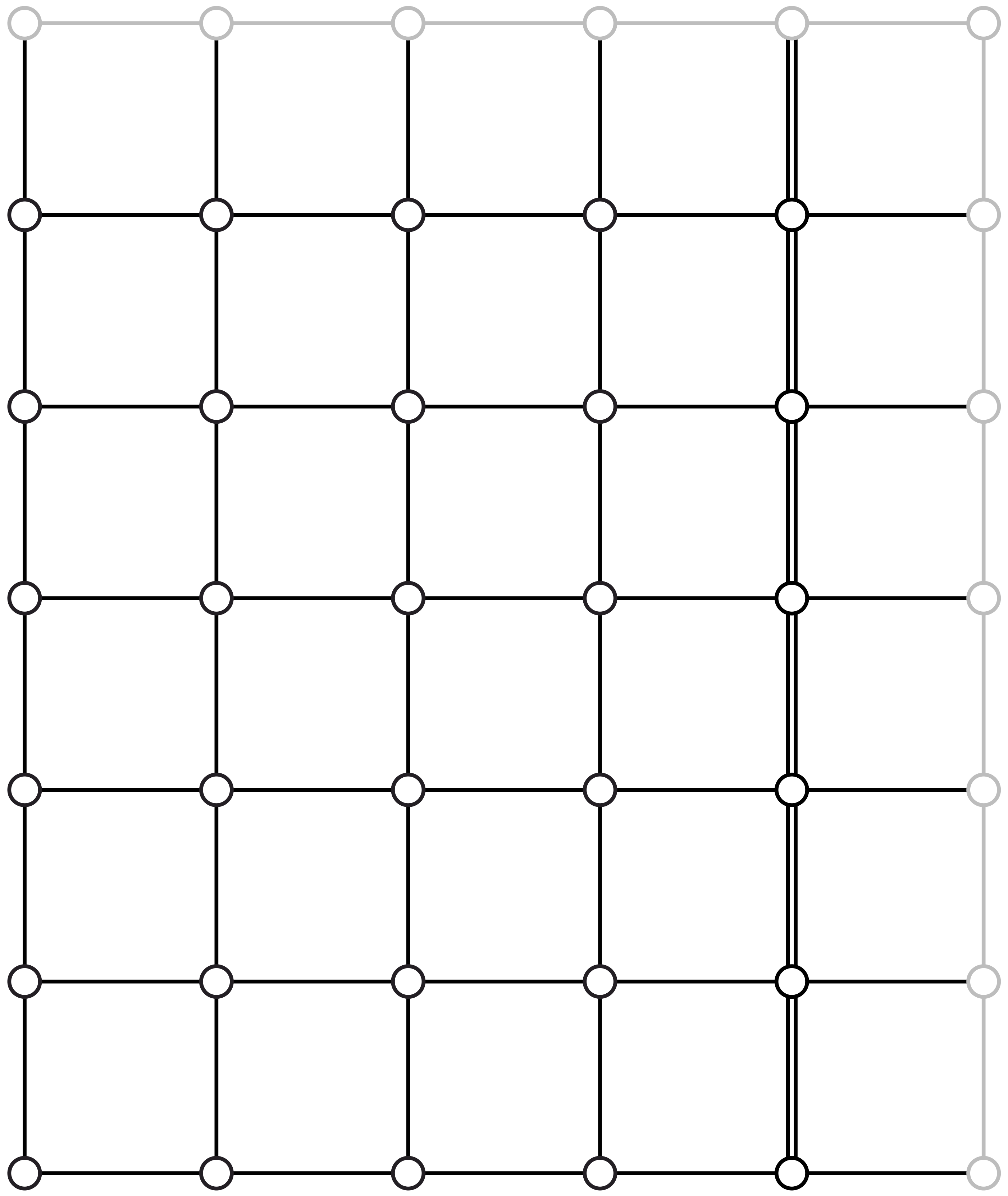

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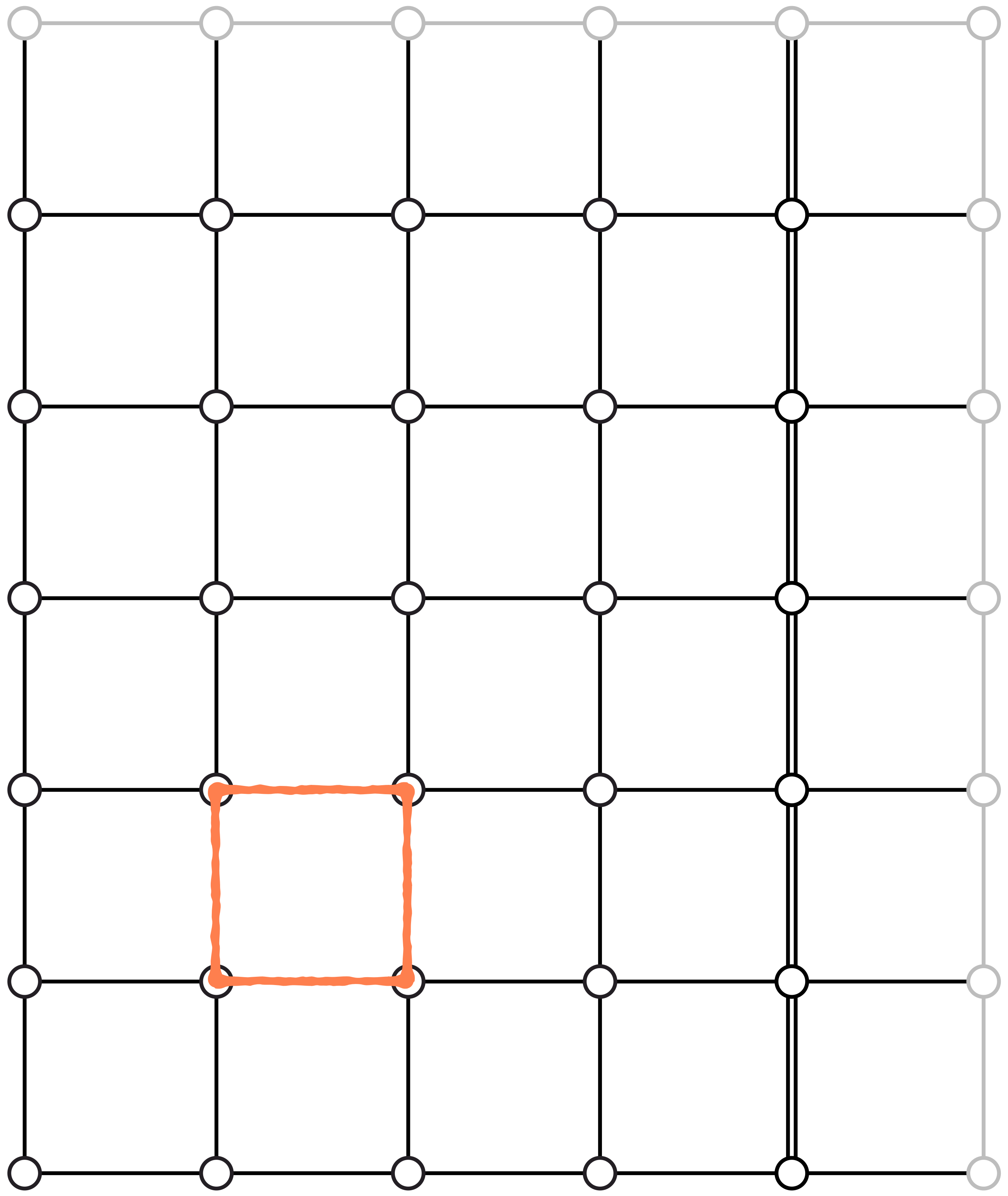

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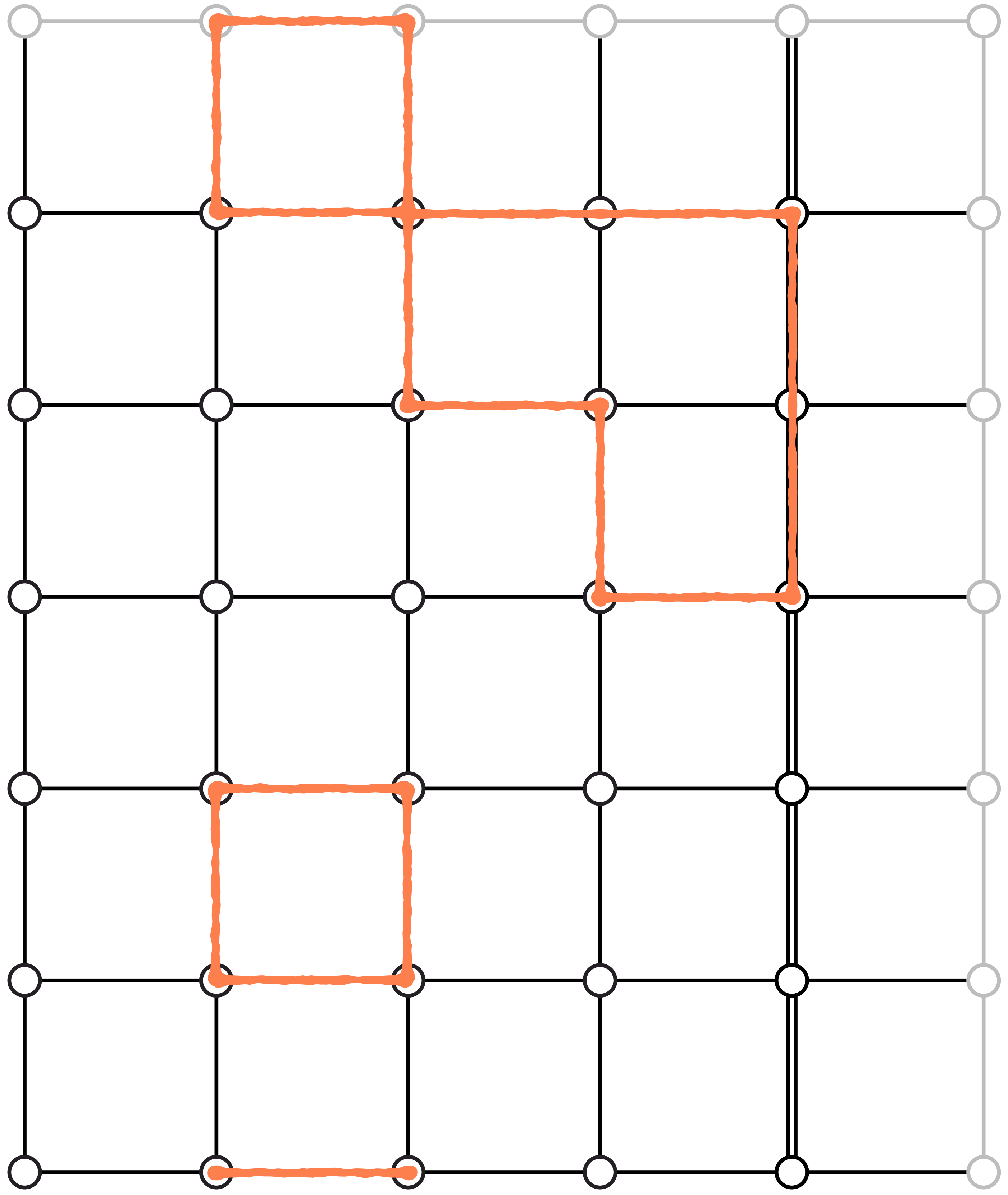

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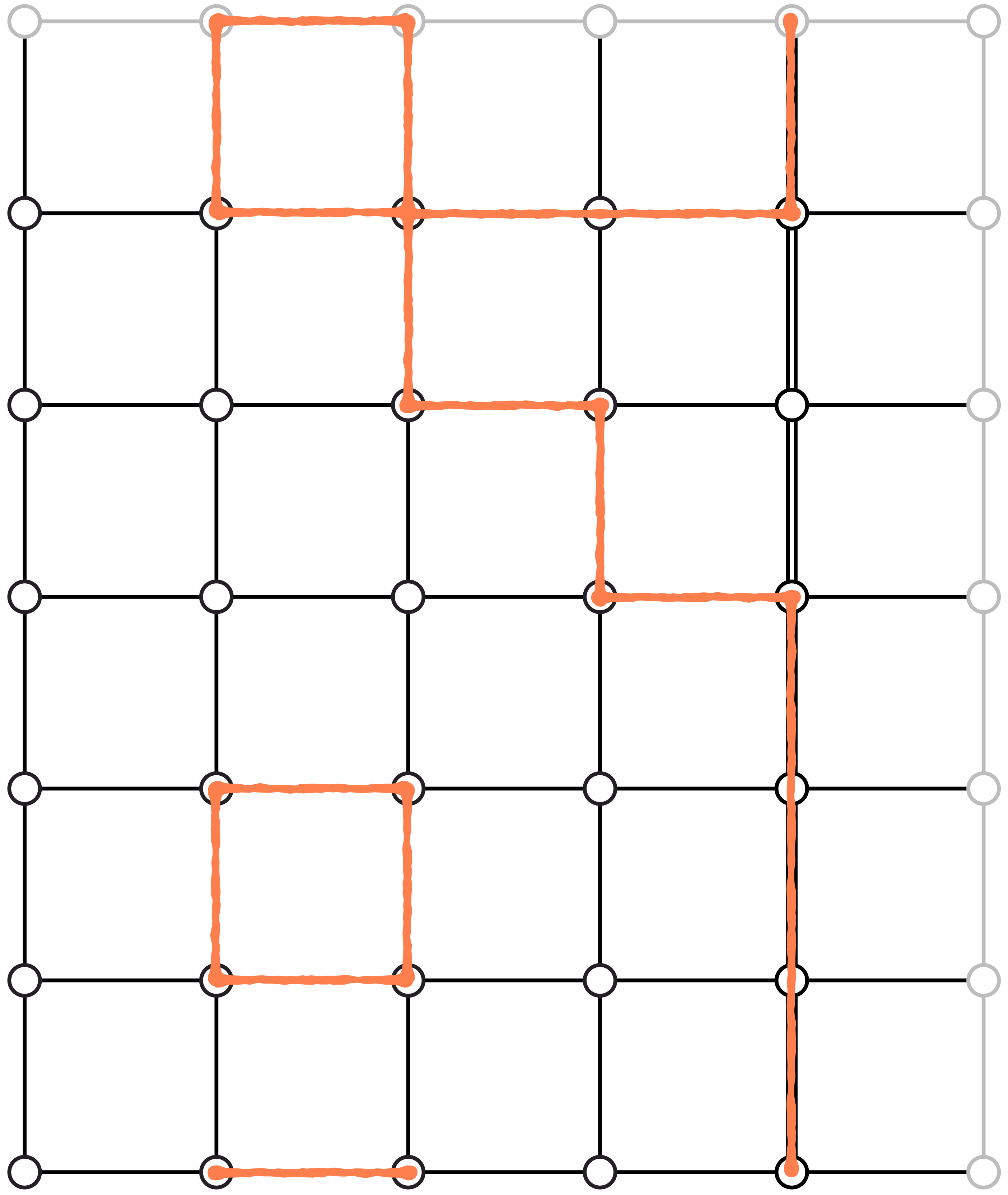

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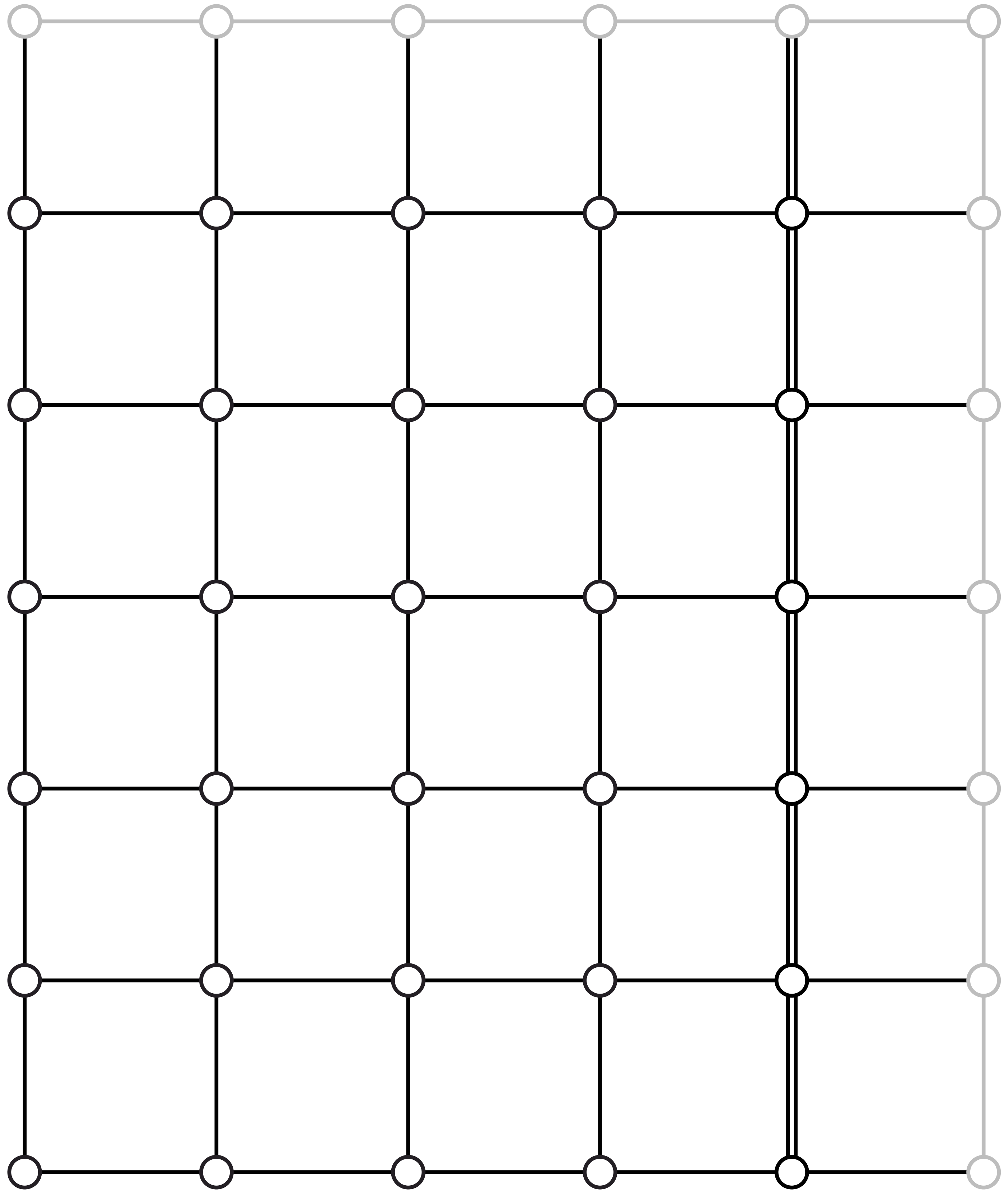

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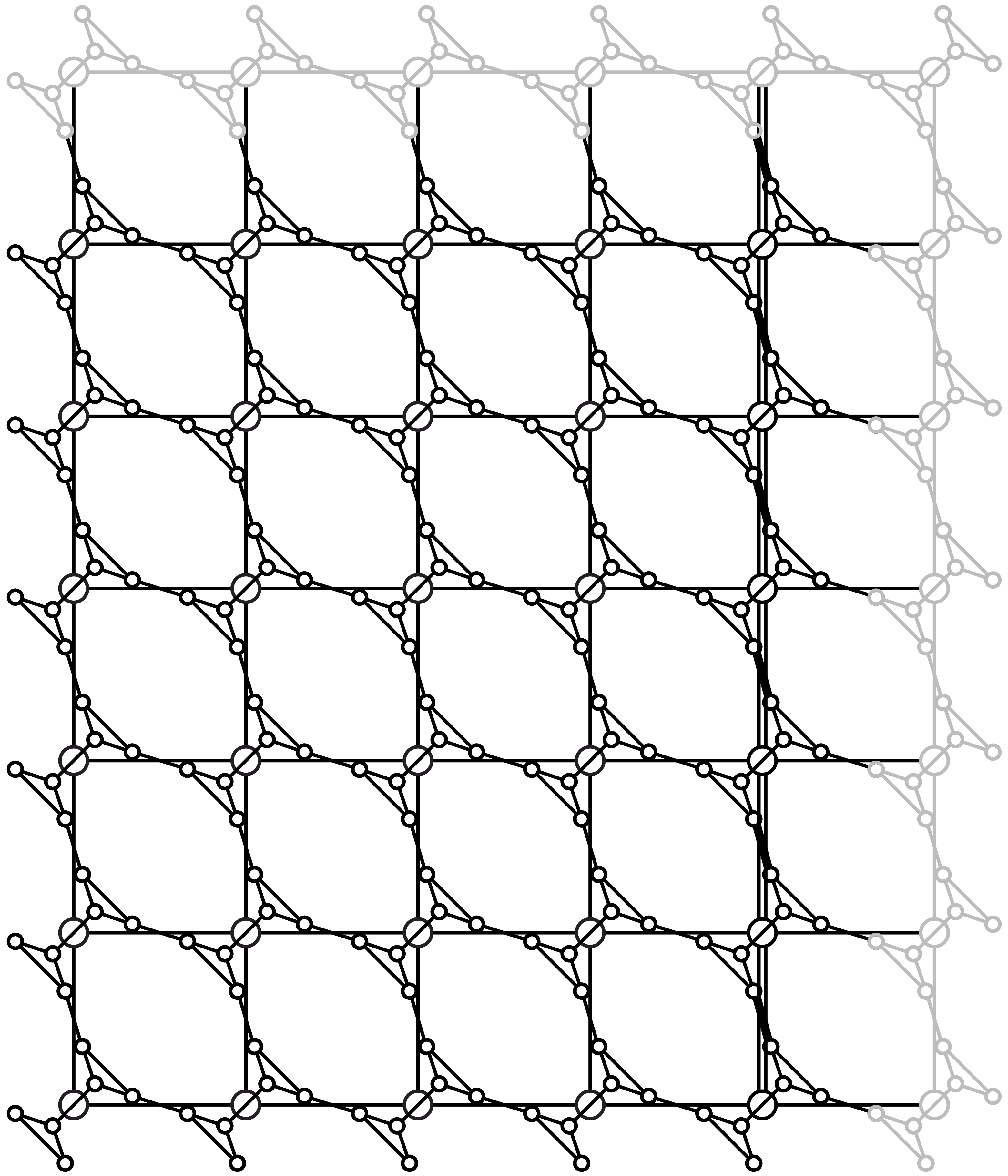

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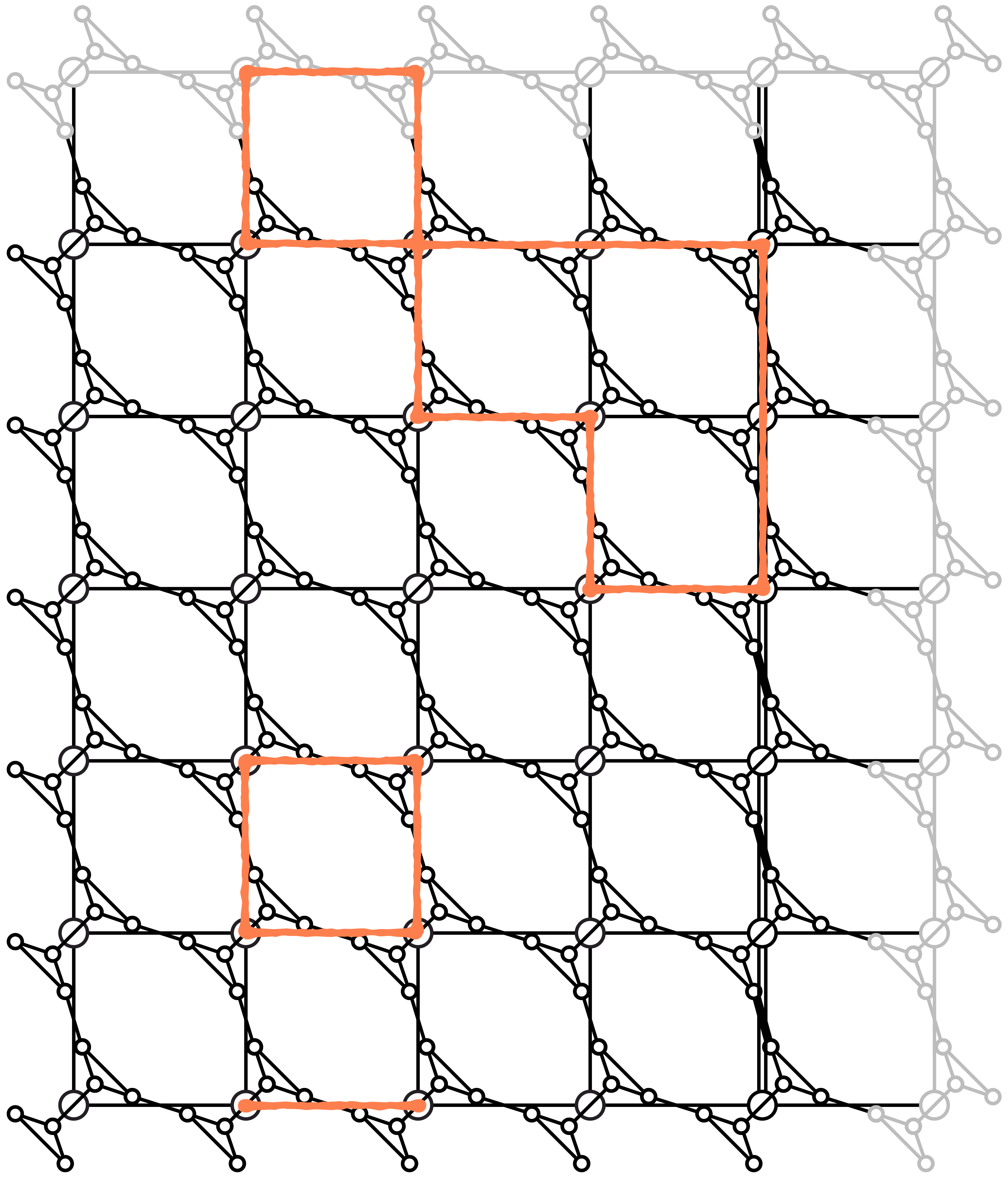

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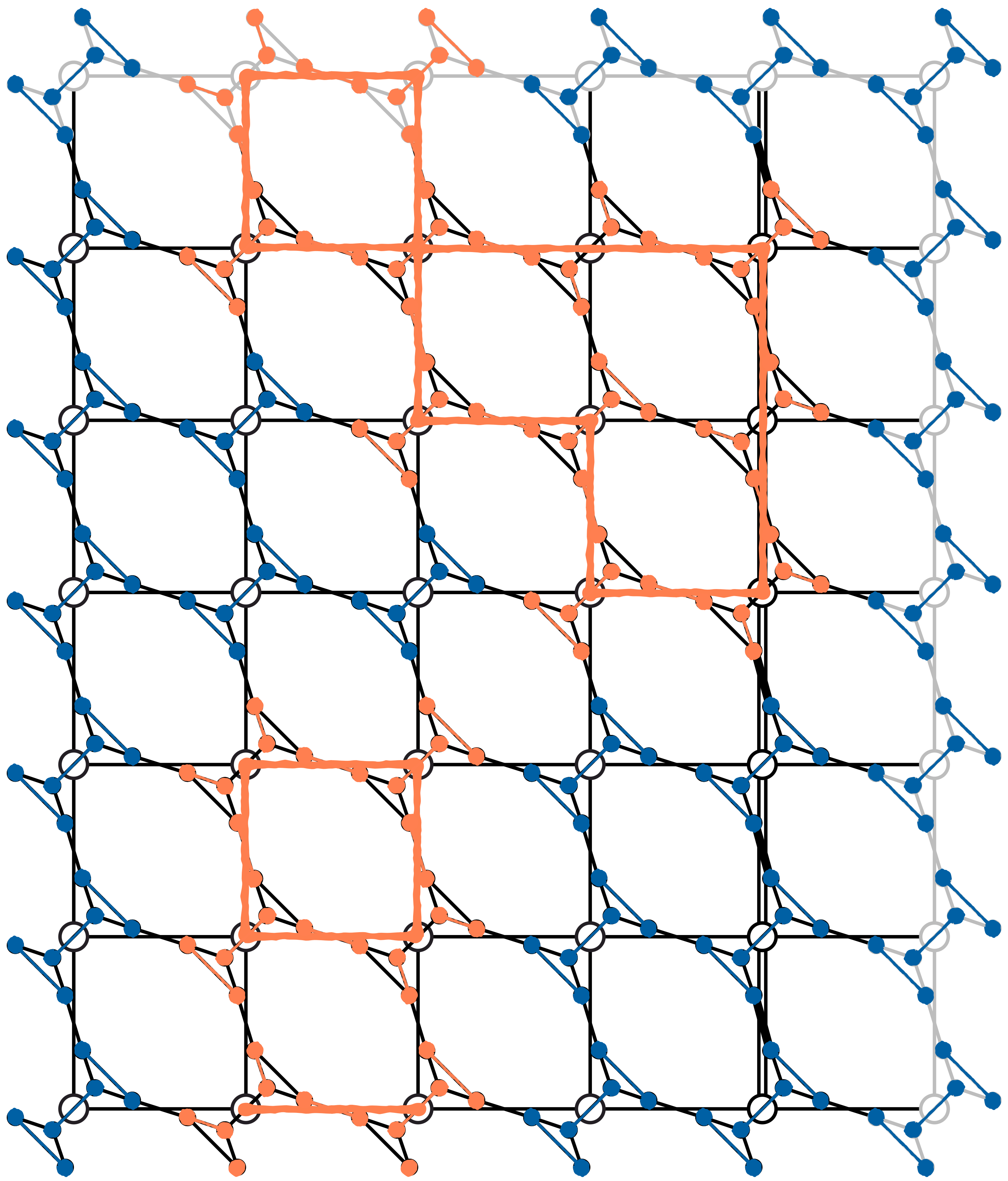

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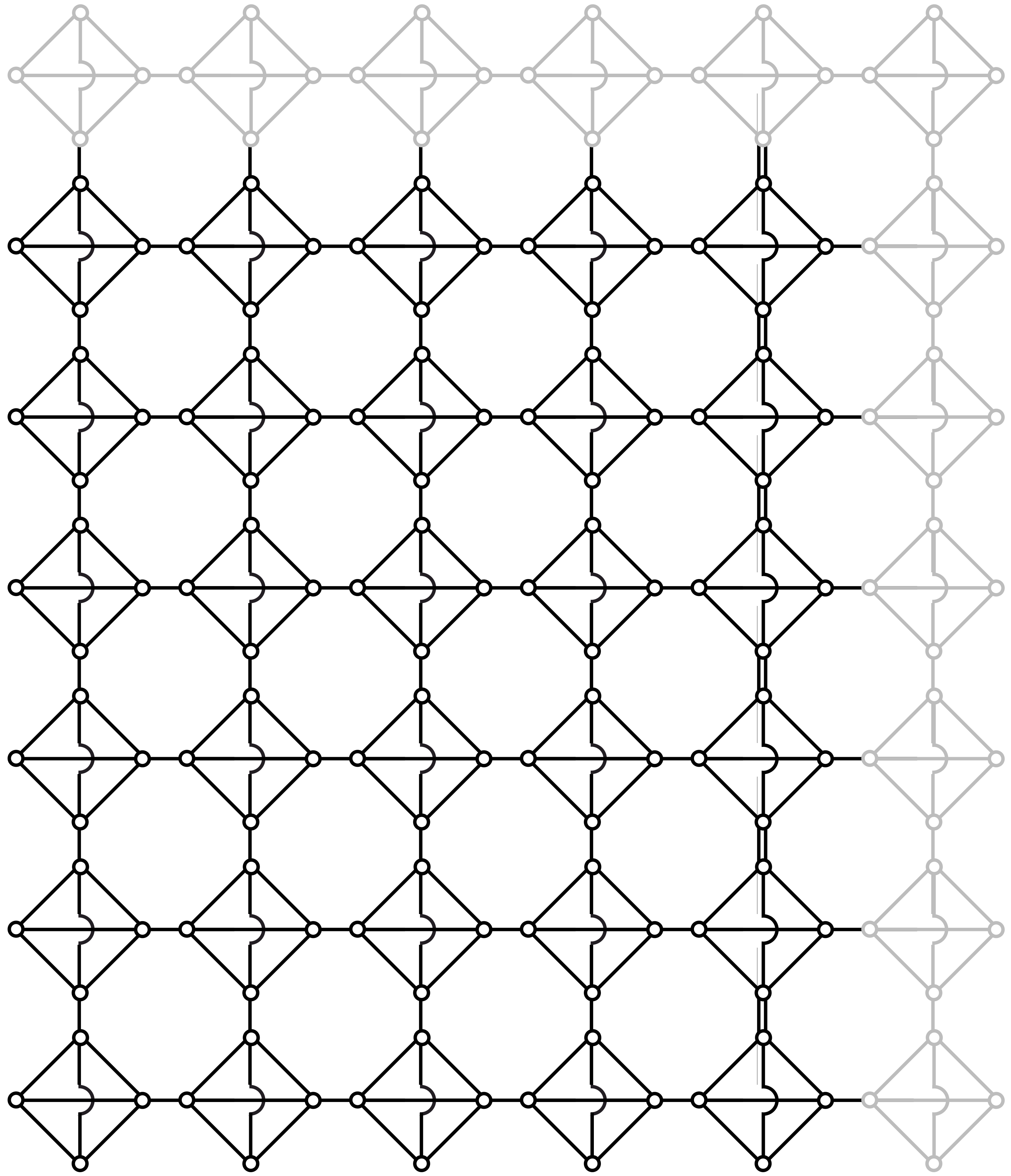

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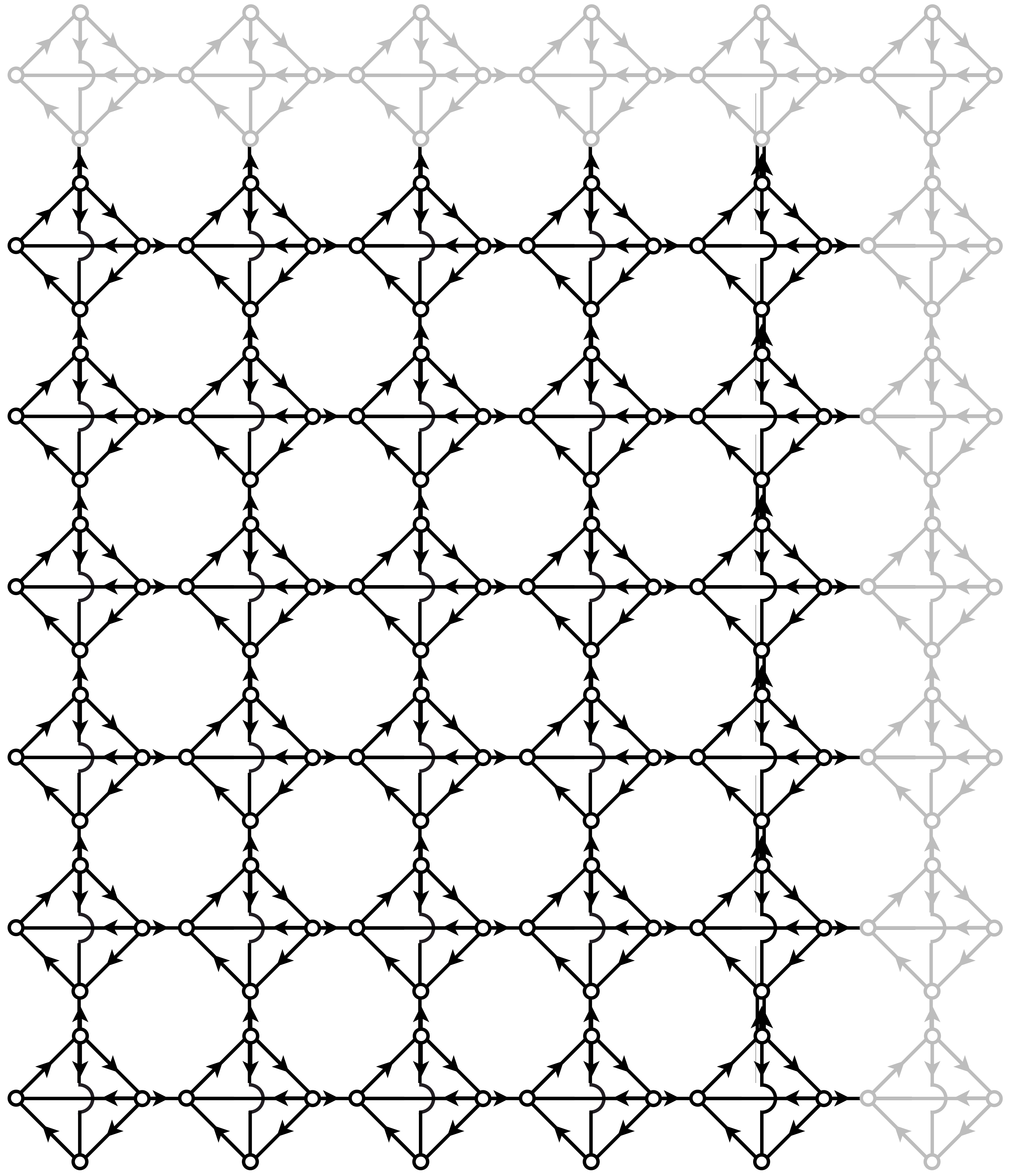

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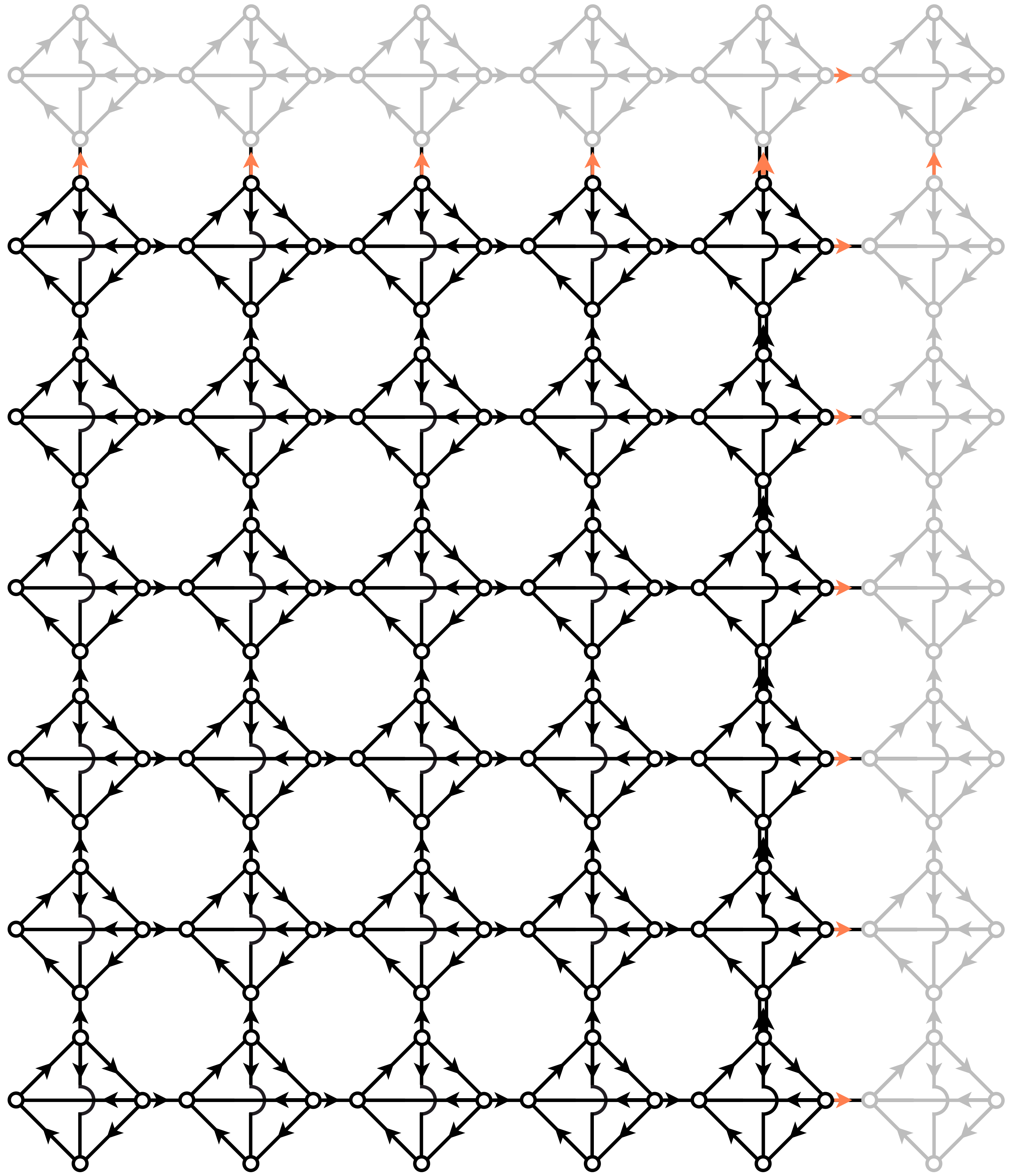
$$\frac{Z^{(\mathrm{p/p})}}{Z_0^{(\mathrm{p/p})}} = \frac{1}{2}\left[\pm\mathrm{Pf}\left(\square\right) + \mathrm{Pf}\left(\square\right) + \mathrm{Pf}\left(\square\right) + \mathrm{Pf}\left(\square\right)\right]$$



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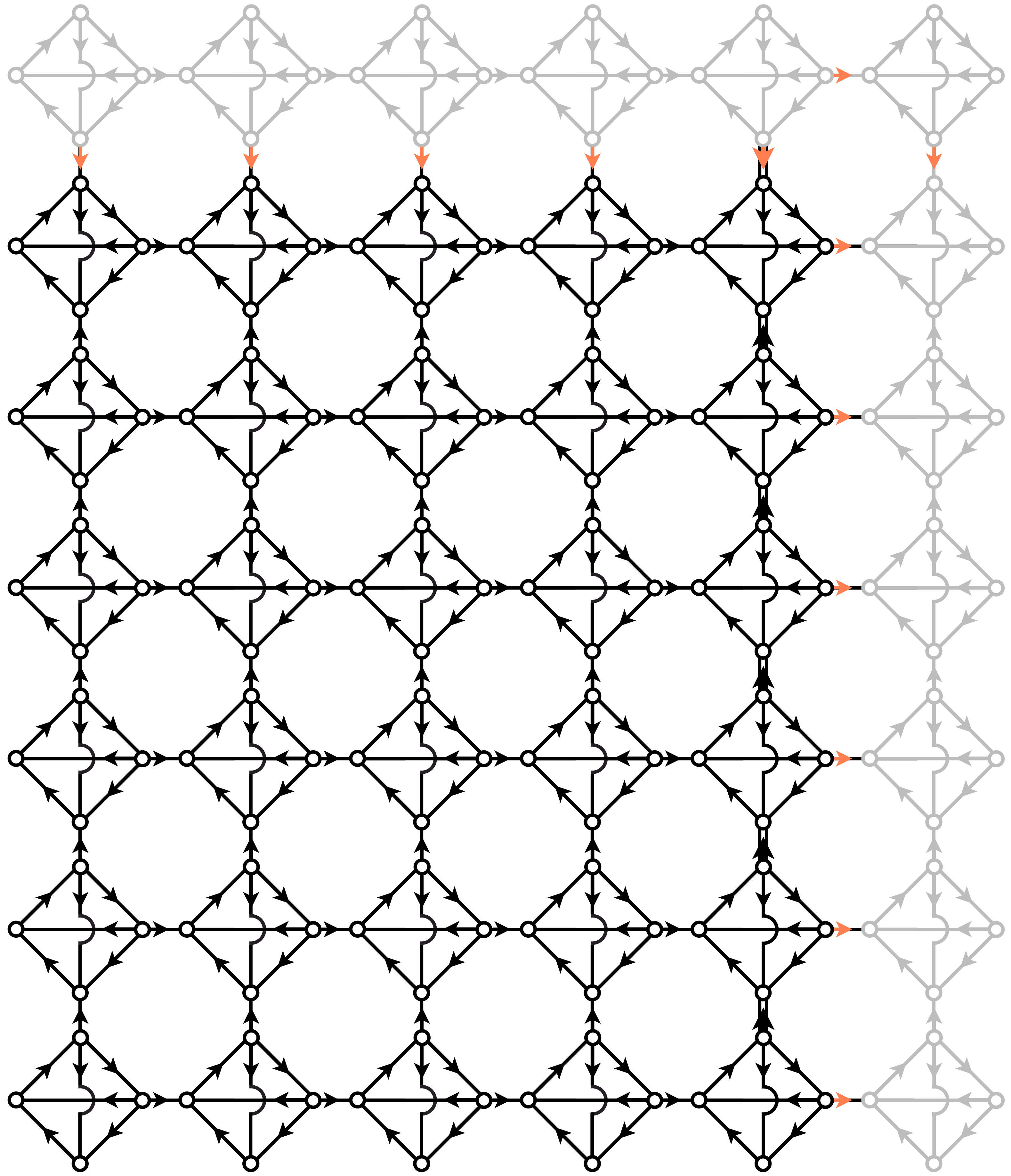

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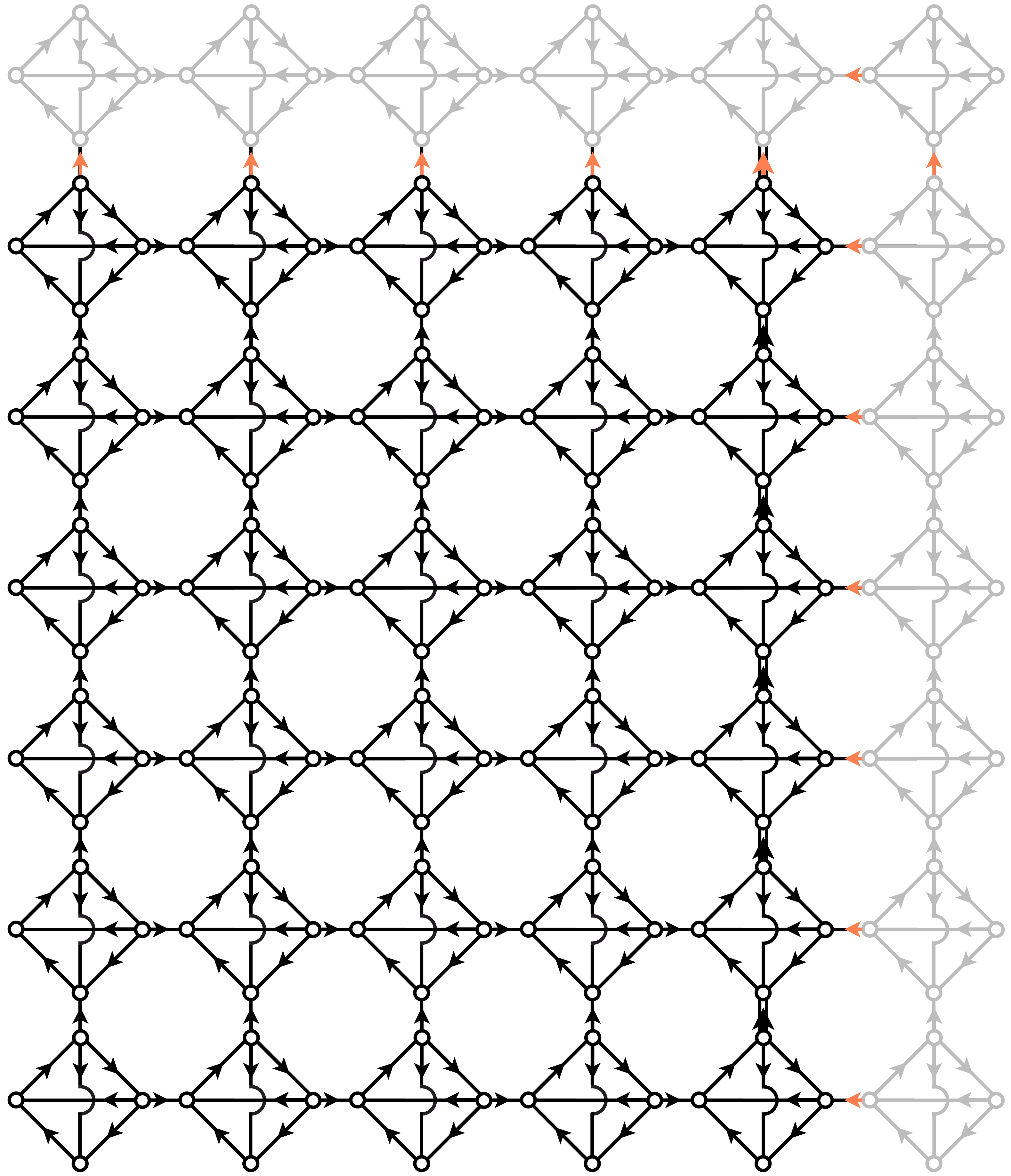

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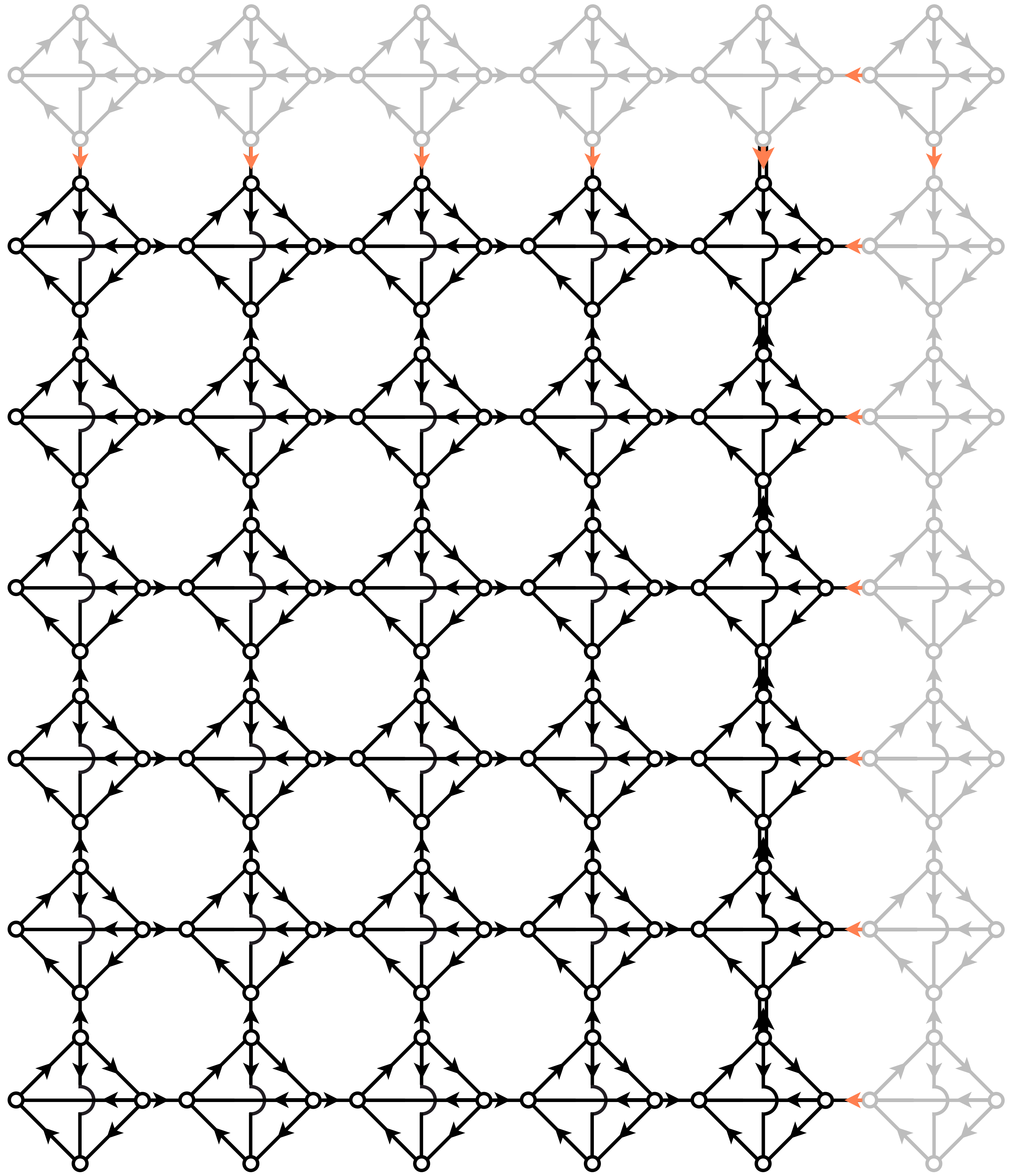

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$$\left[\mathrm{Pf}\boldsymbol{\mathcal{A}}_{\mathrm{e/o}}^{(ab)}\right]^2 = \det\boldsymbol{\mathcal{A}}_{\mathrm{e/o}}^{(ab)}$$

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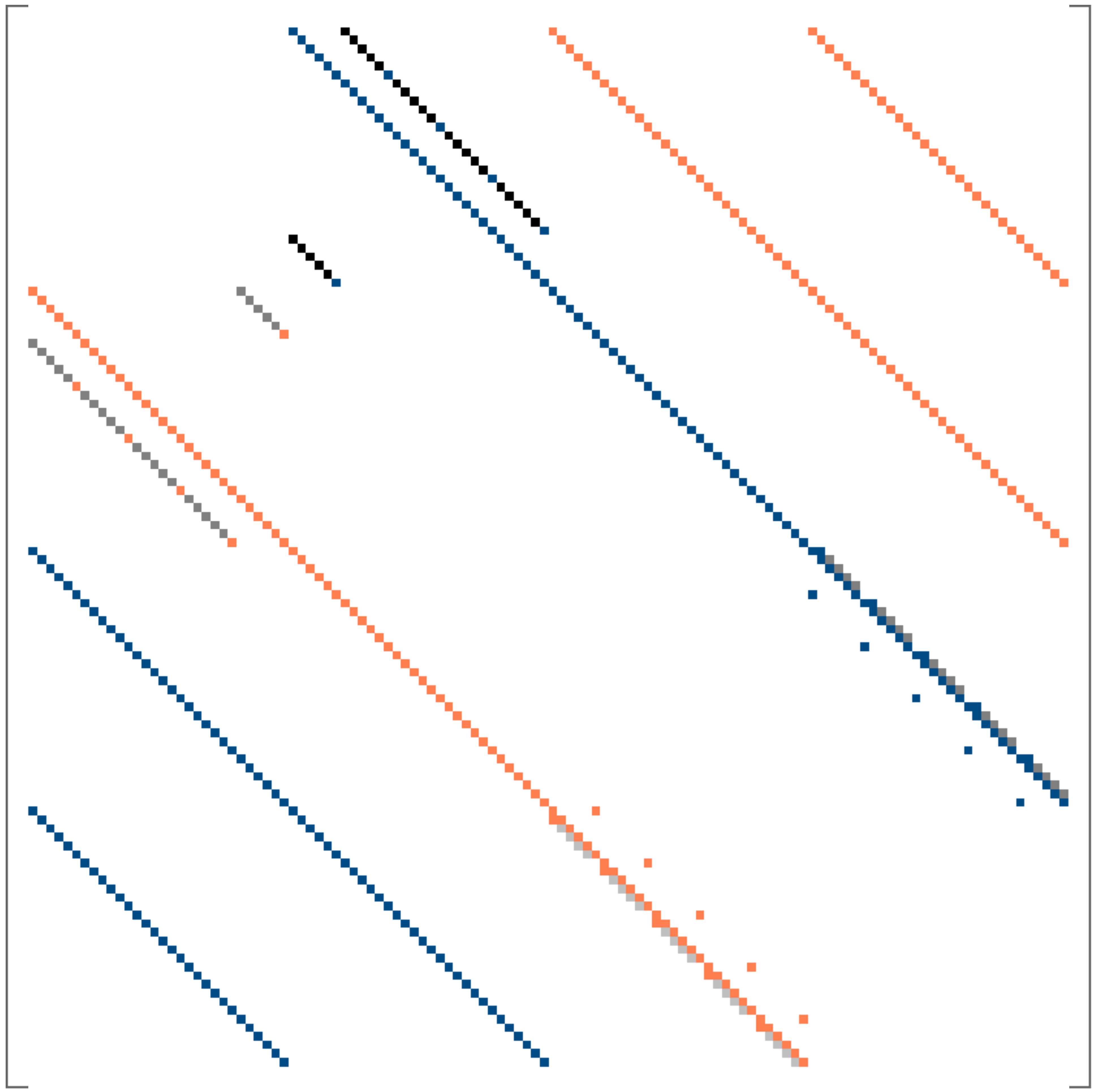

$$\det \boldsymbol{\mathcal{A}}^{(ab)}_{\mathrm{e/o}}$$

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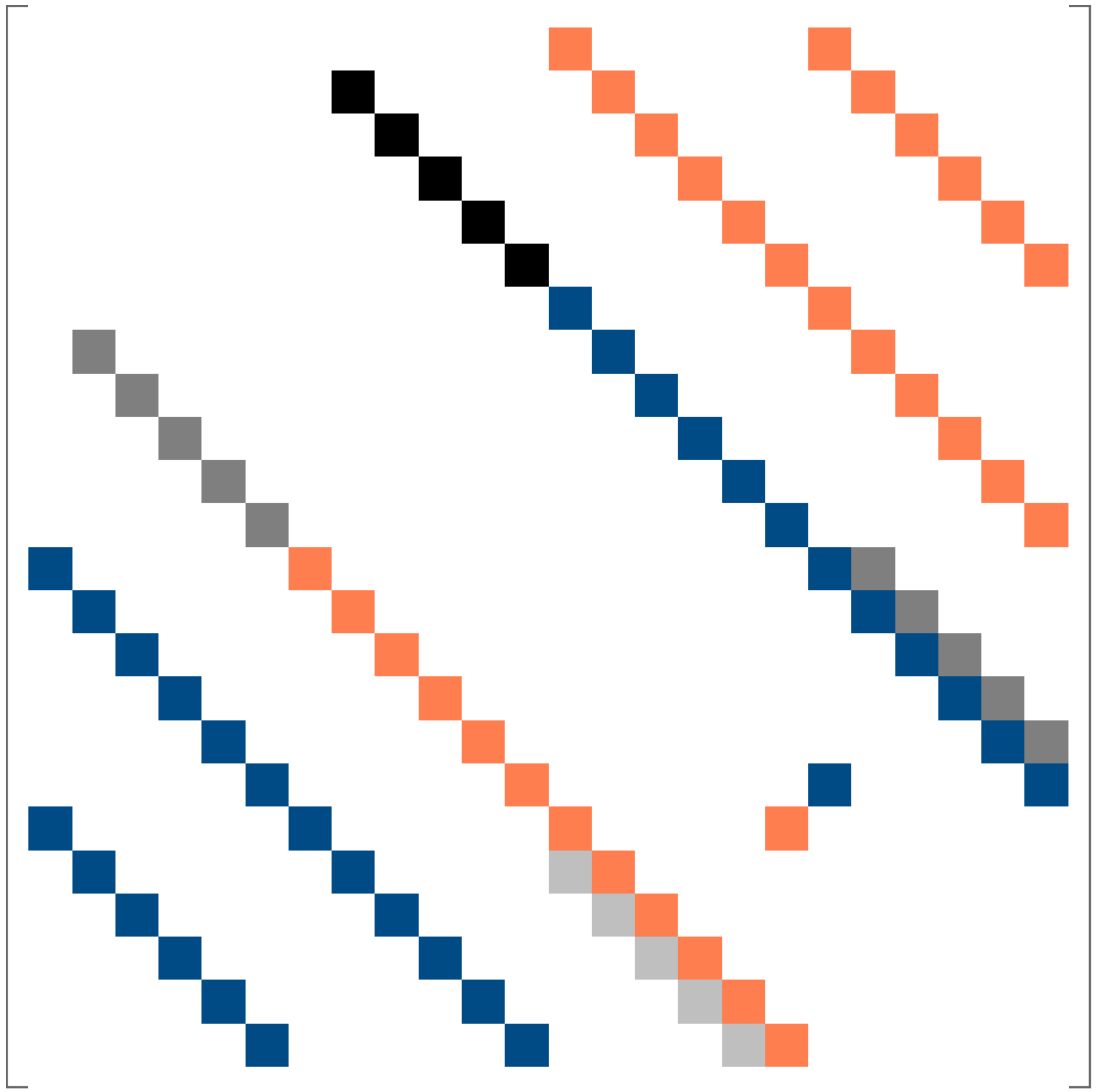
$$\boldsymbol{\mathcal{B}}^{(ab)}(\varphi_m^{\mathrm{e/o}}) =$$



$$\det \boldsymbol{\mathcal{A}}^{(ab)}_{\mathrm{e/o}} = \prod_{m=1}^{M} \det \boldsymbol{\mathcal{B}}^{(ab)}(\varphi_m^{\mathrm{e/o}})$$

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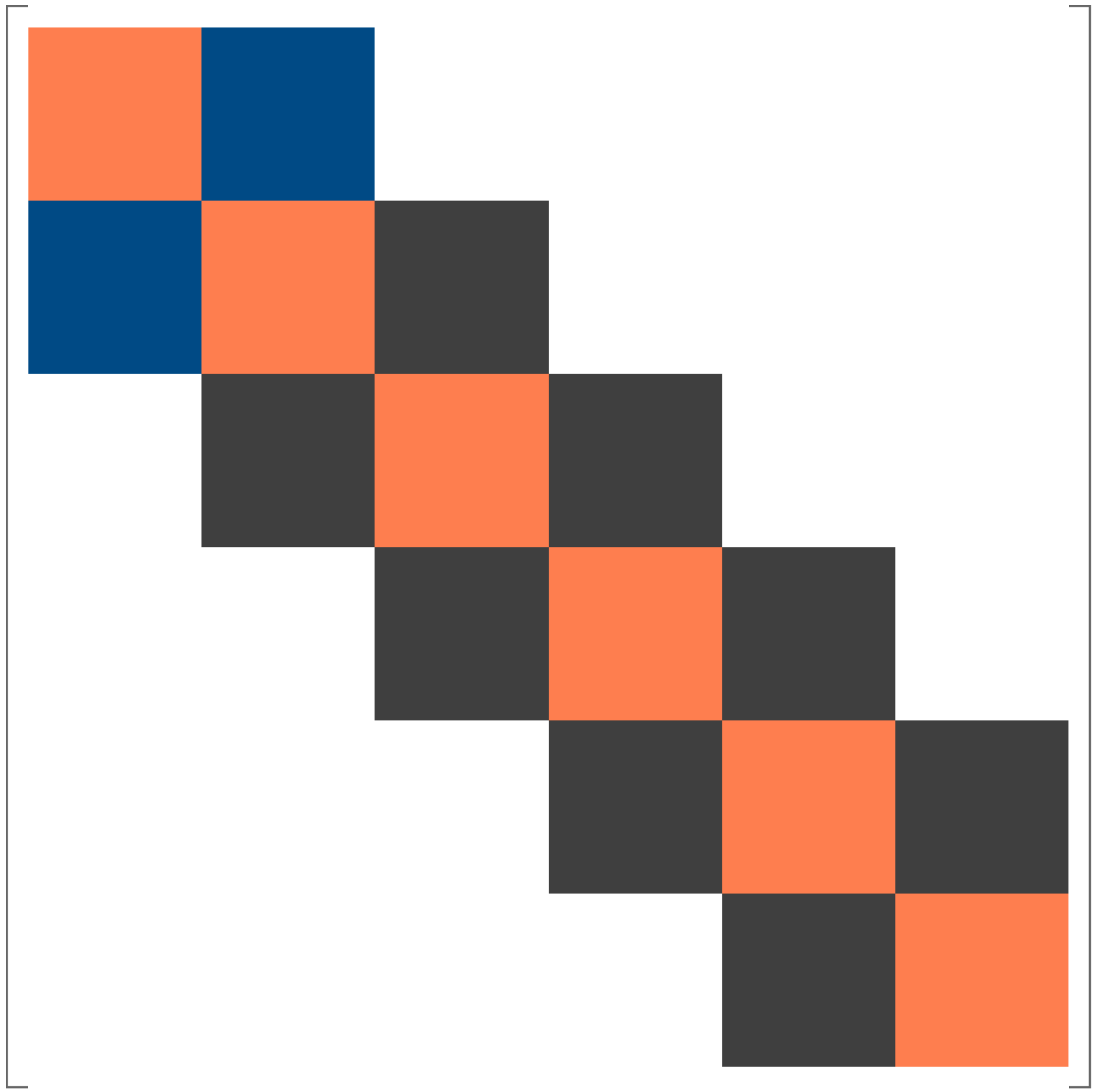
- Schur reduction

$$\boldsymbol{\mathcal{C}}^{(ab)}(\varphi_m^{\mathrm{e/o}}) =$$



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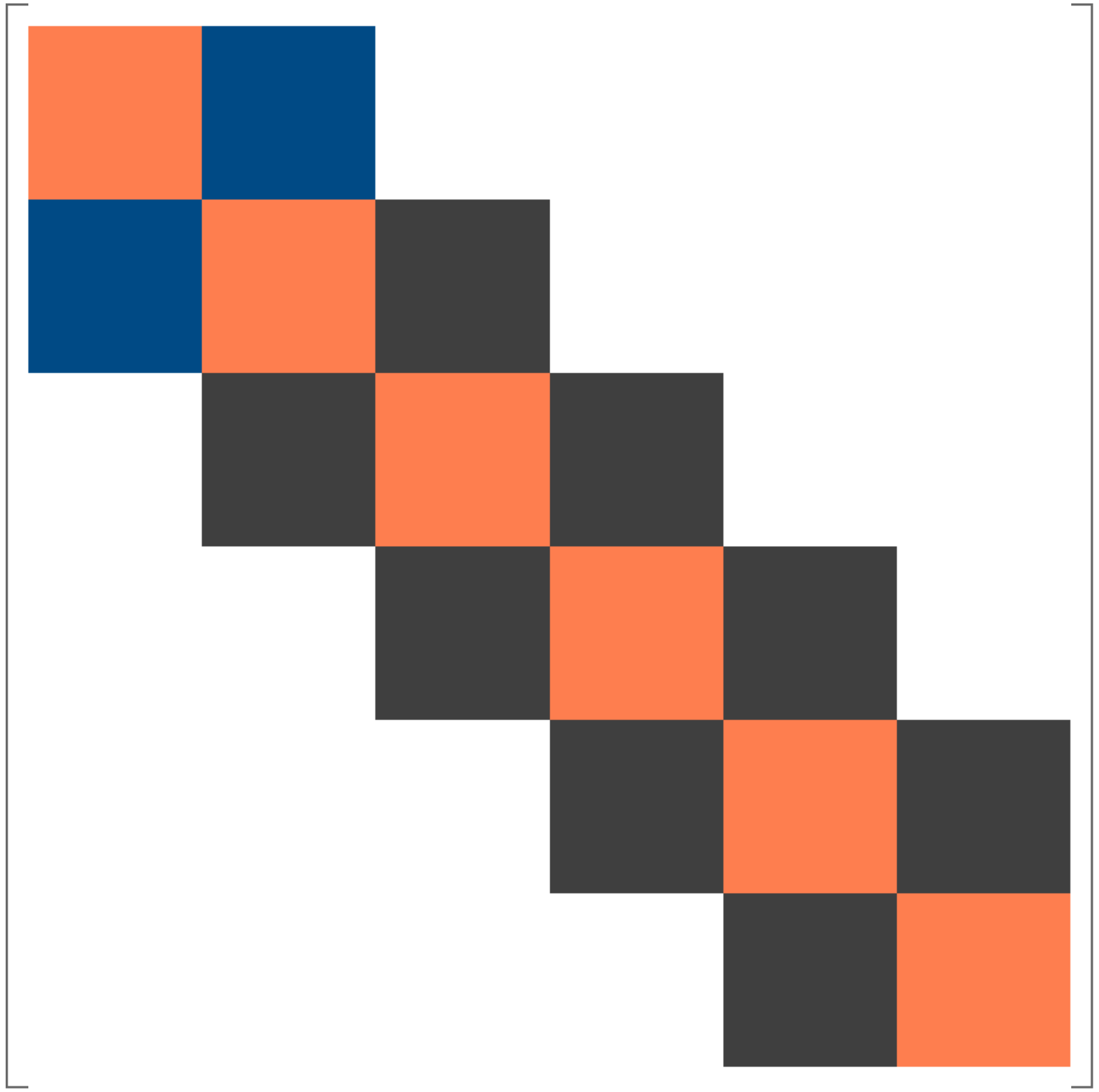

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$$\boxed{\cosh\gamma_m = t_+ z_+ - t_- z_- \cos\varphi_m}$$

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$$\det \boldsymbol{\mathcal{C}}^{(ab)}_{L+1}(\varphi_m) = \frac{t_-}{z}\mu_0^- \frac{\eta^-(p,q,r)\mathrm{e}^{+L\gamma_m} + \eta^+(p,q,r)\mathrm{e}^{-L\gamma_m}}{2\sinh\gamma_m}$$

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- Free energy $F^{(a/b)}(z,t;L,M) = -\ln Z^{(a/b)}$

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2 Disassembling the Free Energy

$$F^{(a/b)}(z, t; L, M)$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = -\frac{LM}{2}\ln\left[\frac{2zt^*}{t_-}\right] - \frac{L}{2}\sum_{\substack{0<m\leq M \\ m\,\mathrm{odd}}}\gamma_m$$

$$F^{(a/b)}(z,t;L,M) \quad = \quad \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}}$$

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Thermodynamic Limit

$$LM f_{\mathrm{b}}(z,t) = -\frac{LM}{2}\ln\left[\frac{2zt^*}{t_-}\right] - \frac{LM}{4\pi}\int_{-\pi/2}^{\pi/2}\mathrm{d}\varphi\,\gamma(\varphi)$$

bulk

$$F^{(a/b)}(z,t;L,M) \quad = \quad \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}}$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM\,f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\,\rho\,\Theta_{\mathrm{b}}(x_{\parallel})}$$

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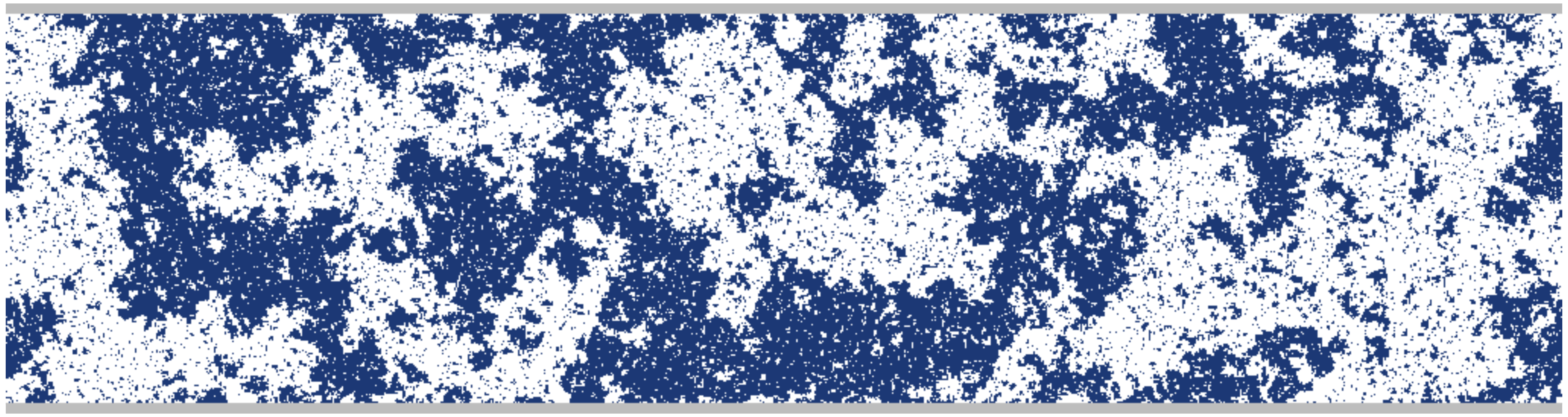

$$F^{(a/b)}(z,t;L,M)$$

$$F_{\mathrm{s}}^{(\mathrm{oo/p})}(z,t;M) = M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + \underbrace{F_{\mathrm{res,s}}^{(\mathrm{oo/p})}(z,t;M)}_{\simeq\, \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel})}$$

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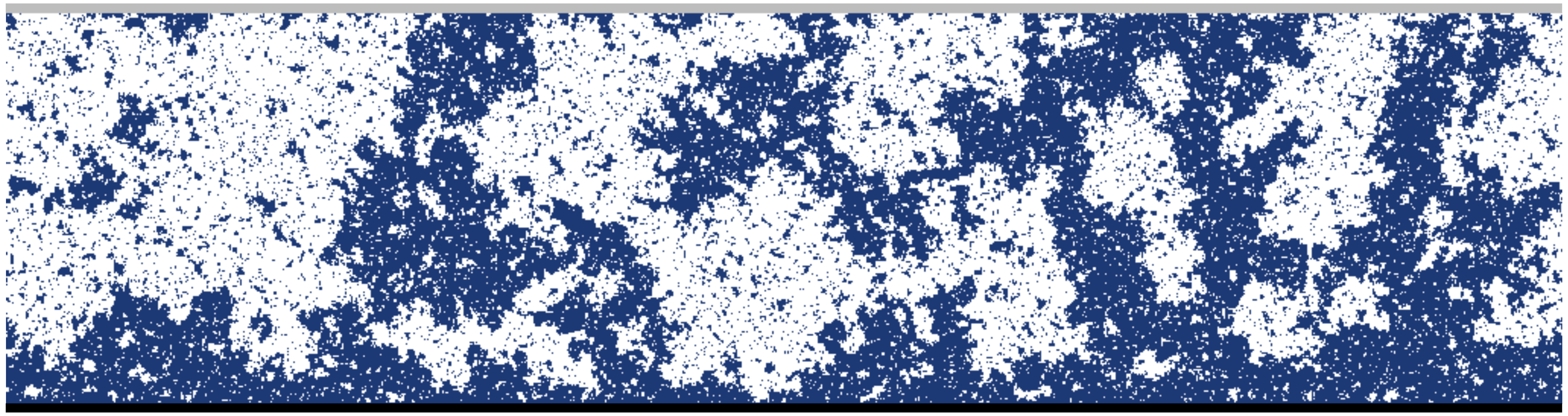
$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + F_{\mathrm{res,b}}(z,t;L,M)$$

$$F^{(a/b)}(z,t;L,M)$$



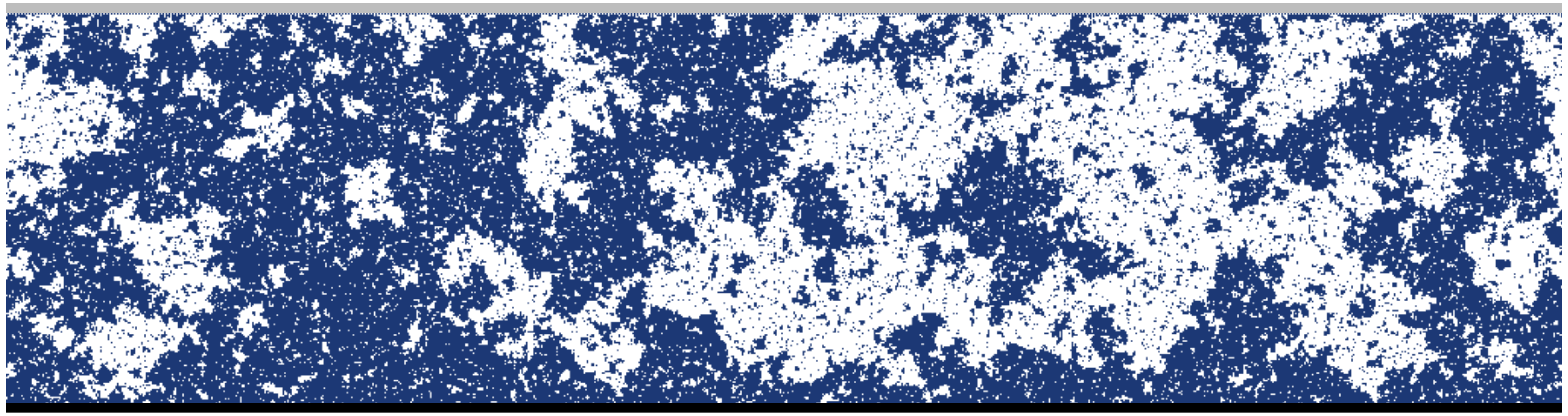
$$F_{\mathrm{s}}^{(+\mathrm{o}/\mathrm{p})}(z_{\mathrm{h}};z,t;M) = M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) + \underbrace{F_{\mathrm{res,s}}^{(+\mathrm{o}/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}_{}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + \Theta_{\mathrm{h}}(x_{\parallel})$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z, t; L, M) = LM f_{\mathrm{b}}(z, t) + F_{\mathrm{res,b}}(z, t; L, M)$$

$$F^{(a/b)}(z, t; L, M)$$



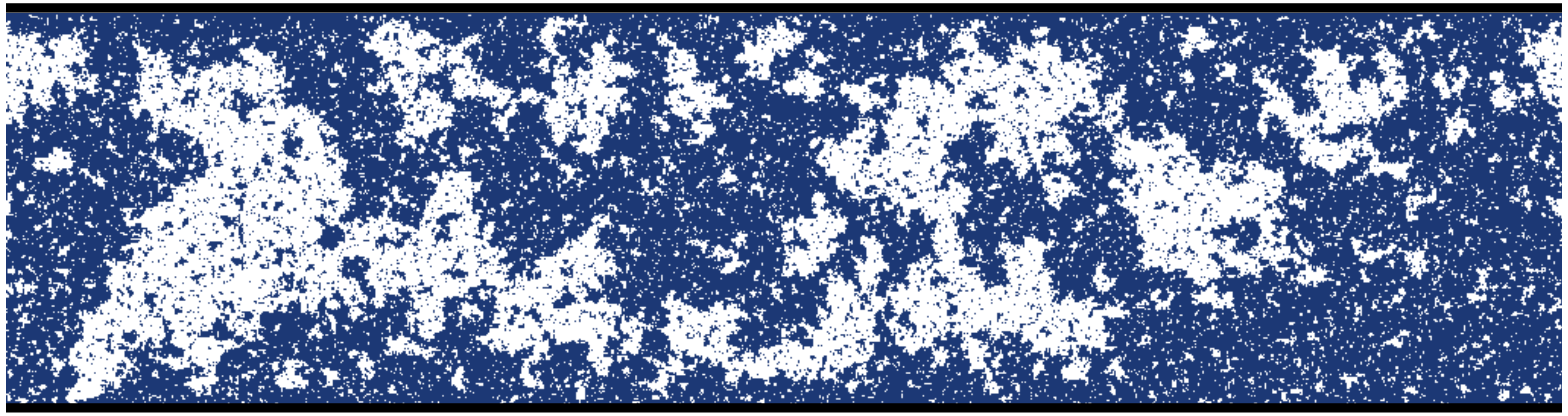
$$F_{\mathrm{s}}^{(+\mathrm{s}/\mathrm{p})}(z_{\mathrm{h}}; z, t; M) = M f_{\mathrm{s}}^{(\mathrm{oo})}(z, t) + M f_{\mathrm{h}}(z_{\mathrm{h}}; z, t) + M f_{\mathrm{st}}(z_{\mathrm{h}}; z, t) + \underbrace{F_{\mathrm{res,s}}^{(+\mathrm{s}/\mathrm{p})}(z_{\mathrm{h}}; z, t; M)}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + \Theta_{\mathrm{h}}(x_{\parallel})$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + F_{\mathrm{res,b}}(z,t;L,M)$$

$$F^{(a/b)}(z,t;L,M)$$



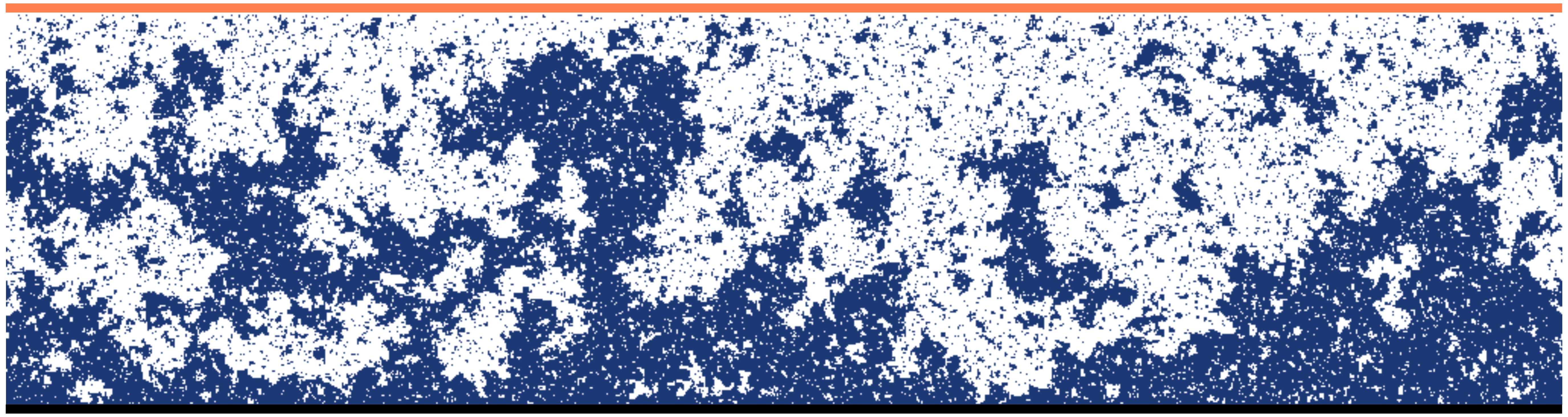
$$F_{\mathrm{s}}^{(++/\mathrm{p})}(z_{\mathrm{h}};z,t;M) = M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) + \underbrace{F_{\mathrm{res,s}}^{(++/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + F_{\mathrm{res,b}}(z,t;L,M)$$

$$F^{(a/b)}(z,t;L,M)$$



$$F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M) = M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) + \underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM\,f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\, \rho\, \Theta_{\mathrm{b}}(x_{\parallel})}$$

$$F^{(a/b)}(z,t;L,M) \quad = \quad \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}} \quad + \quad \underbrace{F_{\mathrm{s}}^{(a/b)}(z,t;M)}_{\text{surface}}$$

$$\begin{aligned} F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M) &= M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) \\ &\quad + \underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}_{\overset{z_{\mathrm{h}}=1}{\simeq}\ \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})} \end{aligned}$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\, \rho\,\Theta_{\mathrm{b}}(x_\parallel)}$$

$$F_{\mathrm{strip}}^{(\mathrm{oo/p})}(z,t;L,M) = -\sum_{m=1}^{M} \ln\left[1 + \frac{\eta_{\mathrm{o}}^{+}(\varphi_m^{\mathrm{o}})}{\eta_{\mathrm{o}}^{-}(\varphi_m^{\mathrm{o}})} \mathrm{e}^{-2L\gamma_m}\right]$$

$$F^{(a/b)}(z,t;L,M) = \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}} + \underbrace{F_{\mathrm{s}}^{(a/b)}(z,t;M)}_{\text{surface}} + \overbrace{F_{\mathrm{strip}}^{(a/b)}(z,t;L,M)}^{\text{finiteness}}$$

$$\begin{aligned} F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M) &= M f_{\mathrm{s}}^{(\mathrm{oo})}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) \\ &\quad + \underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}_{\overset{z_{\mathrm{h}}=1}{\simeq}\ \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) + 2\Theta_{\mathrm{h}}(x_\parallel)} \end{aligned}$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\, \rho\,\Theta_{\mathrm{b}}(x_{\parallel})}$$

$$F^{(a/b)}_{\mathrm{strip}}(z,t;L,M) \simeq \Psi^{(a/b)}(x_{\parallel},\rho)$$

$$F^{(a/b)}(z,t;L,M) = \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}} + \underbrace{F^{(a/b)}_{\mathrm{s}}(z,t;M)}_{\text{surface}} + \overbrace{F^{(a/b)}_{\mathrm{strip}}(z,t;L,M)}^{\text{finiteness}}$$

$$\begin{aligned} F^{(+-/\mathrm{p})}_{\mathrm{s}}(z_{\mathrm{h}};z,t;M) &= M f^{(\mathrm{oo})}_{\mathrm{s}}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) \\ &\quad + \underbrace{F^{(+-/\mathrm{p})}_{\mathrm{res,s}}(z_{\mathrm{h}};z,t;M)}_{\overset{z_{\mathrm{h}}=1}{\simeq}\ \Theta^{(\mathrm{oo})}_{\mathrm{s}}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})} \end{aligned}$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\, \rho\, \Theta_{\mathrm{b}}(x_\parallel)}$$

$$F^{(a/b)}_{\mathrm{strip}}(z,t;L,M) \simeq \Psi^{(a/b)}(x_\parallel,\rho)$$

$$F^{(a/b)}(z,t;L,M) = \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}} + \underbrace{F^{(a/b)}_{\mathrm{s}}(z,t;M)}_{\text{surface}} + \overbrace{F^{(a/b)}_{\mathrm{strip}}(z,t;L,M)}^{\text{finiteness}} + \underbrace{\sigma^{(a/b)}(z,t;L,M)}_{\text{interface tension}}$$

$$\begin{aligned} F^{(+-/\mathrm{p})}_{\mathrm{s}}(z_{\mathrm{h}};z,t;M) &= M f^{(\mathrm{oo})}_{\mathrm{s}}(z,t) + 2M f_{\mathrm{h}}(z_{\mathrm{h}};z,t) \\ &\quad + \underbrace{F^{(+-/\mathrm{p})}_{\mathrm{res,s}}(z_{\mathrm{h}};z,t;M)}_{\overset{z_{\mathrm{h}}=1}{\simeq}\ \Theta^{(\mathrm{oo})}_{\mathrm{s}}(x_\parallel) + 2\Theta_{\mathrm{h}}(x_\parallel)} \end{aligned}$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = L \ldots$$

$$F^{(a/b)}(z,t;L,M)$$

$$\underbrace{\sigma^{(a/b)}(z,t;L,M)}_{\text{interface tension}}$$

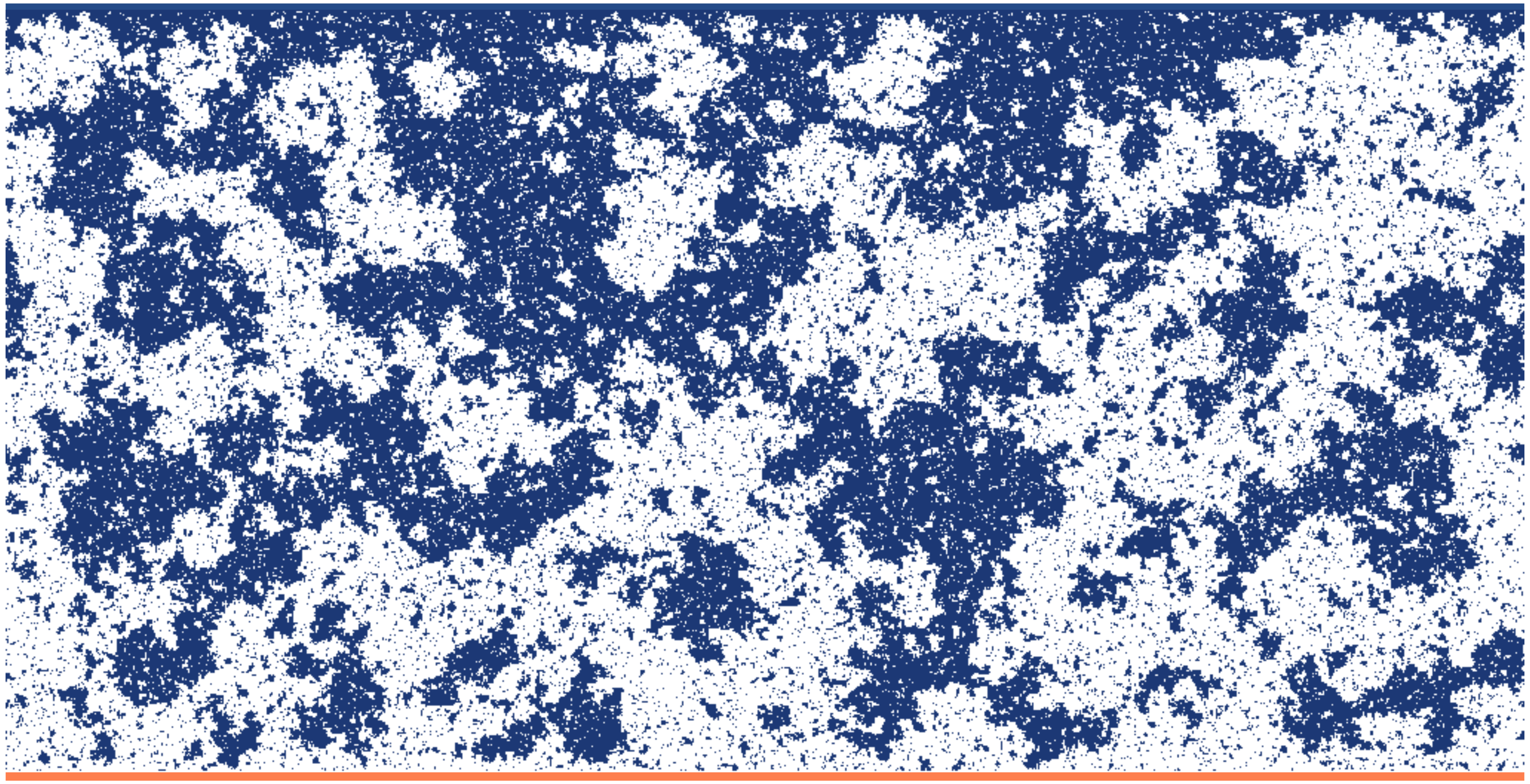
$$F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)$$

$$+ \underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})$$

2 Disassembling the Free Energy

$F_{\mathrm{b}}(z,t;L,M) = L$



$F^{(a/b)}(z,t;L,M)$

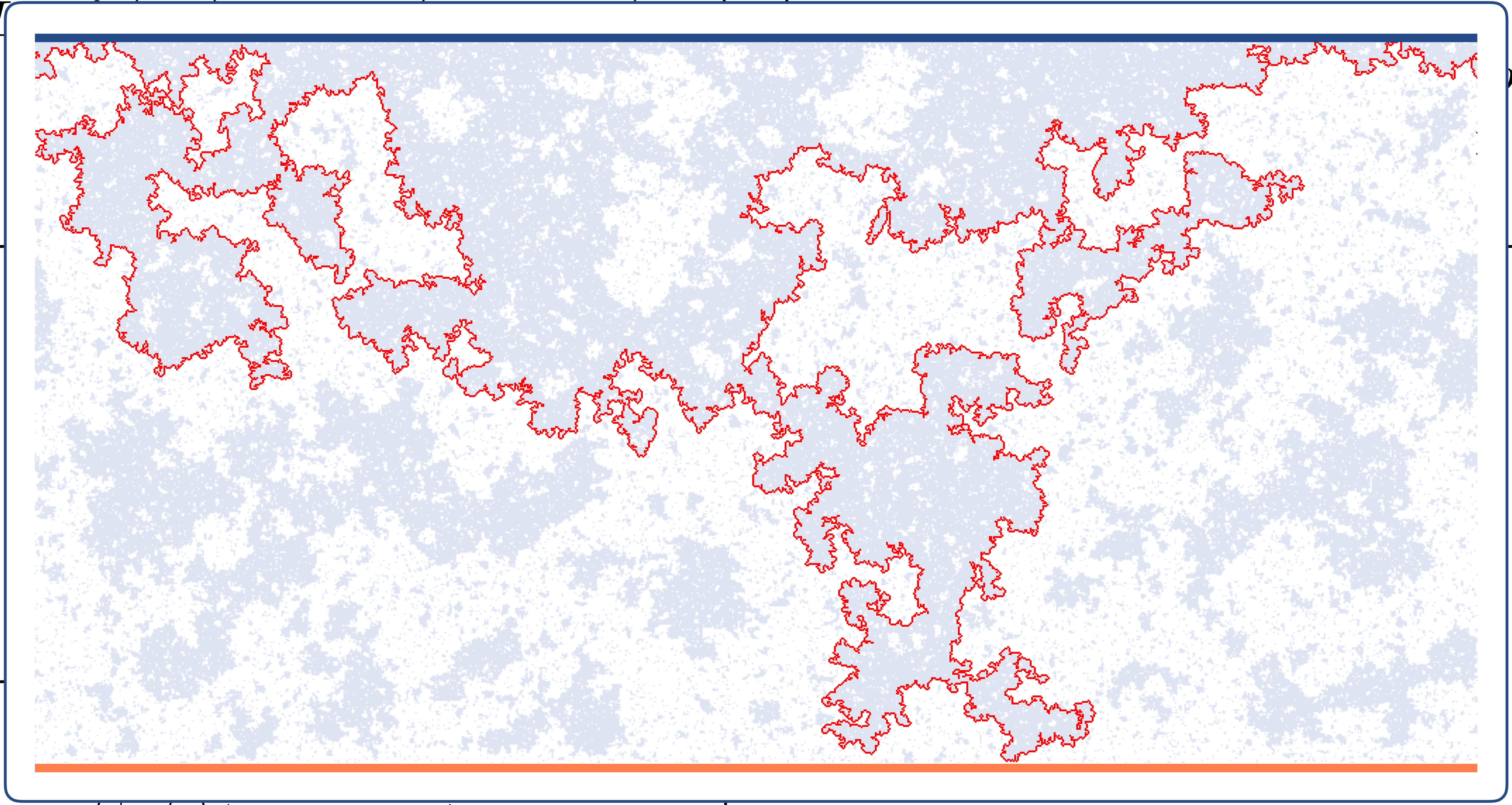
$\underbrace{\sigma^{(a/b)}(z,t;L,M)}_{\text{interface tension}}$

$F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)$

$$+ \underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)} \overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) + 2\Theta_{\mathrm{h}}(x_\parallel)$$

2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = L$$



$$F^{(a/b)}(z,t;L,M)$$

$$\underbrace{\sigma^{(a/b)}(z,t;L,M)}_{\text{interface tension}}$$

$$F_{\mathrm{s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)$$

$$+\underbrace{F_{\mathrm{res,s}}^{(+-/\mathrm{p})}(z_{\mathrm{h}};z,t;M)}$$

$$\overset{z_{\mathrm{h}}=1}{\simeq} \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) + 2\Theta_{\mathrm{h}}(x_{\parallel})$$

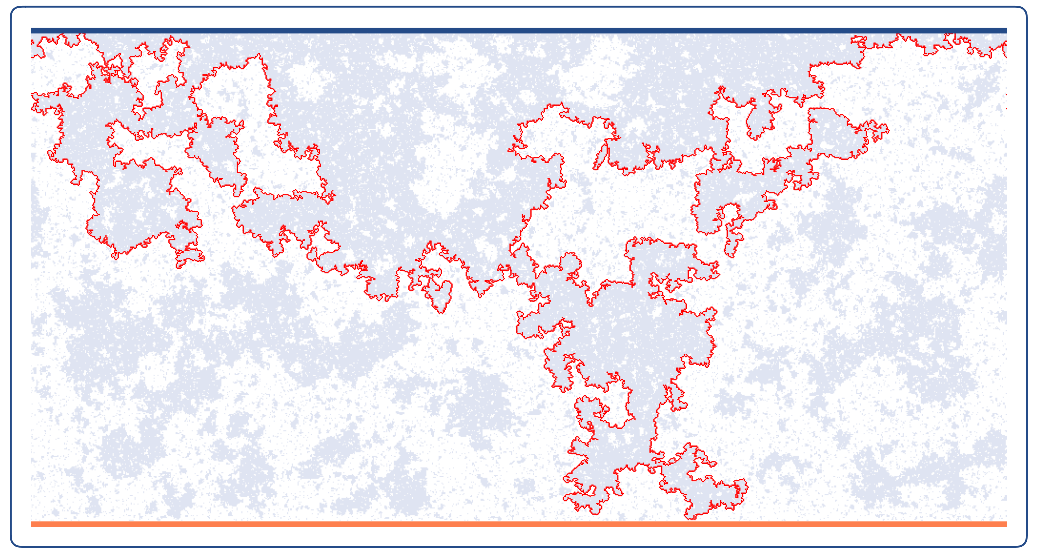
2 Disassembling the Free Energy

$$F_{\mathrm{b}}(z,t;L,M) = LM\,f_{\mathrm{b}}(z,t) + \underbrace{F_{\mathrm{res,b}}(z,t;L,M)}_{\simeq\,\rho\,\Theta_{\mathrm{b}}(x_\parallel)}$$

$$F^{(a/b)}_{\mathrm{strip}}(z,t;L,M) \simeq \Psi^{(a/b)}(x_\parallel,\rho)$$

$$F^{(a/b)}(z,t;L,M) = \overbrace{F_{\mathrm{b}}(z,t;L,M)}^{\text{bulk}} + \underbrace{F^{(a/b)}_{\mathrm{s}}(z,t;M)}_{\text{surface}} + \overbrace{F^{(a/b)}_{\mathrm{strip}}(z,t;L,M)}^{\text{finiteness}} + \underbrace{\sigma^{(a/b)}(z,t;L,M)}_{\text{interface tension}}$$

$$\begin{aligned} F^{(+-/\mathrm{p})}_{\mathrm{s}}(z_{\mathrm{h}};z,t;M) &= M\,f^{(\mathrm{oo})}_{\mathrm{s}}(z,t) + 2M\,f_{\mathrm{h}}(z_{\mathrm{h}};z,t) \\ &\quad + \underbrace{F^{(+-/\mathrm{p})}_{\mathrm{res,s}}(z_{\mathrm{h}};z,t;M)}_{\overset{z_{\mathrm{h}}=1}{\simeq}\ \Theta^{(\mathrm{oo})}_{\mathrm{s}}(x_\parallel)+2\Theta_{\mathrm{h}}(x_\parallel)} \end{aligned}$$



$\sigma^{(a/b)}(T;L,M) \simeq \rho\,\Sigma^{(a/b)}_{\parallel}(x_\parallel,\rho)$

IT scaling function decomposes, too:

$$\begin{aligned} \rho\,\Sigma^{(a/b)}_{\parallel}(x_\parallel,\rho) &= \rho\,\Sigma_{\mathrm{b}}(x_\parallel) && \text{bulk} \\ &\quad + \Sigma^{(a/b)}_{\mathrm{s}}(x_\parallel) && \text{surface} \\ &\quad + \Sigma^{(a/b)}_{\mathrm{strip}}(x_\parallel,\rho) && \text{strip} \end{aligned}$$

3 Disassembling the Free Energy Scaling Functions

$$F^{(a/b)}(T;L,M) \simeq \boldsymbol{\rho\,\Theta_{\parallel}^{(a/b)}(x_{\parallel},\rho)} \quad = \quad \rho\,\Theta_{\mathrm{b}}(x_{\parallel}) \quad + \quad \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) \quad + \quad n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_{\parallel}) \quad + \quad \Psi_{\mathrm{strip}}^{(a/b)}(x_{\parallel},\rho)$$

3 Disassembling the Free Energy Scaling Functions

$$\rho\,\Theta_{\parallel}^{(a/b)}(x_{\parallel},\rho) \quad = \quad \rho\,\Theta_{\mathrm{b}}(x_{\parallel}) \quad + \quad \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) \quad + \quad n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_{\parallel}) \quad + \quad \Psi_{\mathrm{strip}}^{(a/b)}(x_{\parallel},\rho)$$

3 Disassembling the Free Energy Scaling Functions

$$\Theta_{\mathrm{b}}(x_{\parallel}) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\, \frac{\Phi^2}{\Gamma}\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

with scaling form of Onsager dispersion

$$\Gamma = \sqrt{x_{\parallel}^2 + \Phi^2}$$

$$\rho\,\Theta_{\parallel}^{(a/b)}(x_{\parallel},\rho) \quad = \quad \rho\,\Theta_{\mathrm{b}}(x_{\parallel}) \quad + \quad \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_{\parallel}) \quad + \quad n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_{\parallel}) \quad + \quad \Psi_{\mathrm{strip}}^{(a/b)}(x_{\parallel},\rho)$$

3 Disassembling the Free Energy Scaling Functions

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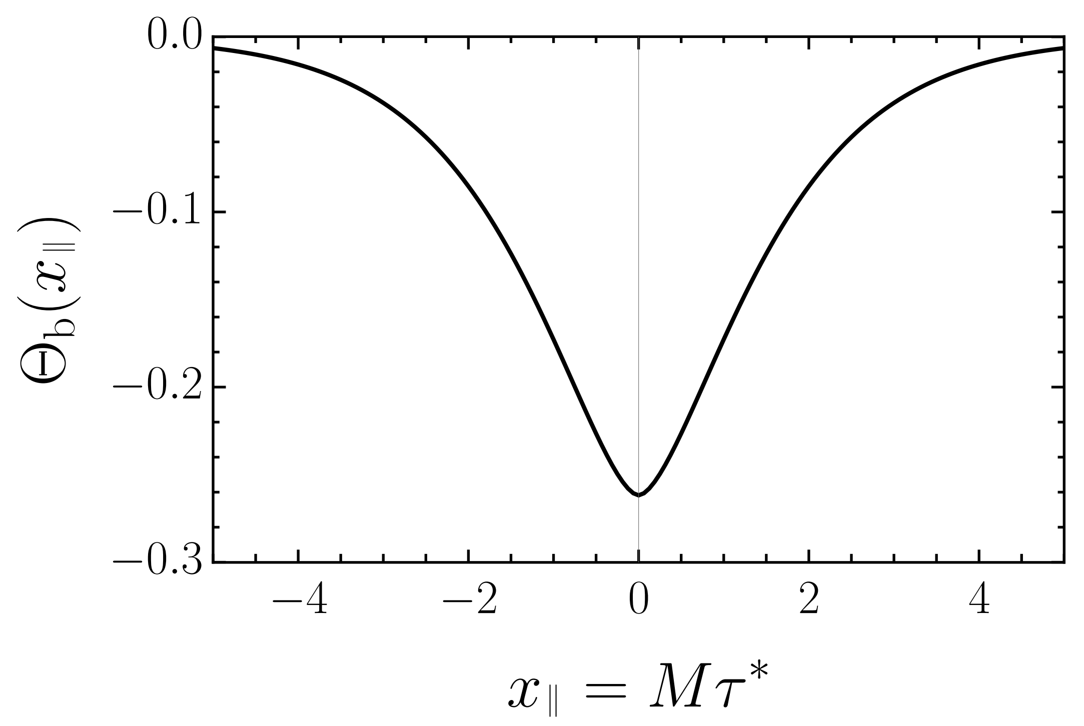

$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel,\rho) \quad = \quad \rho\,\Theta_{\mathrm{b}}(x_\parallel) \quad + \quad \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) \quad + \quad n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_\parallel) \quad + \quad \Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel,\rho)$$

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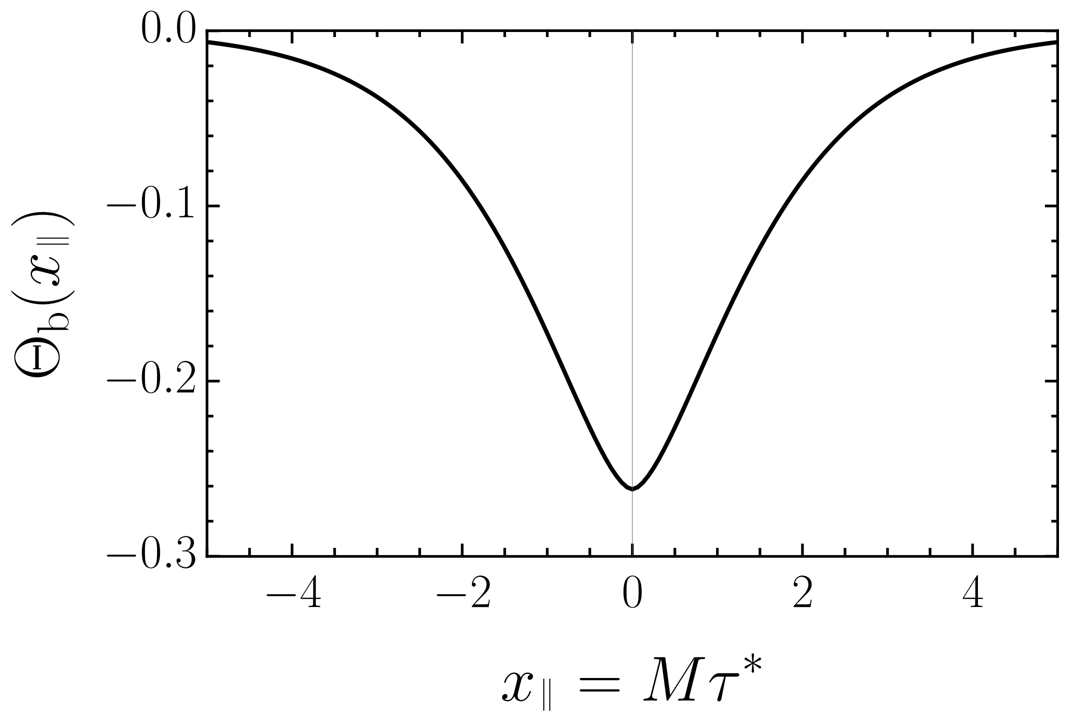

$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel,\rho) = \rho\,\Theta_\mathrm{b}(x_\parallel) + \Theta_\mathrm{s}^{(\mathrm{oo})}(x_\parallel) + n_\mathrm{h}\Theta_\mathrm{h}(x_\parallel) + \Psi_\mathrm{strip}^{(a/b)}(x_\parallel,\rho)$$

$$\Theta_\mathrm{s}^{(\mathrm{oo})}(x_\parallel) = H(-x_\parallel)\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\Phi\,\frac{\Phi}{\Gamma}\arctan\left(\frac{x}{\Phi}\right)\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

3 Disassembling the Free Energy Scaling Functions

$$\Theta_{\mathrm{b}}(x_\parallel) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\, \frac{\Phi^2}{\Gamma}\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

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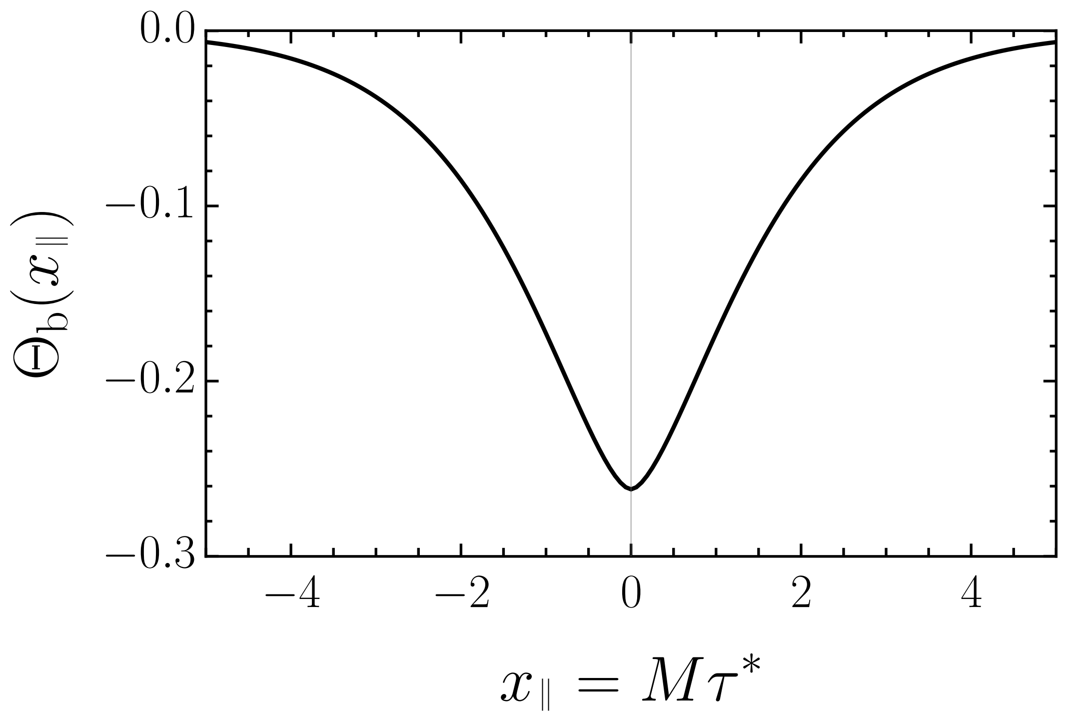
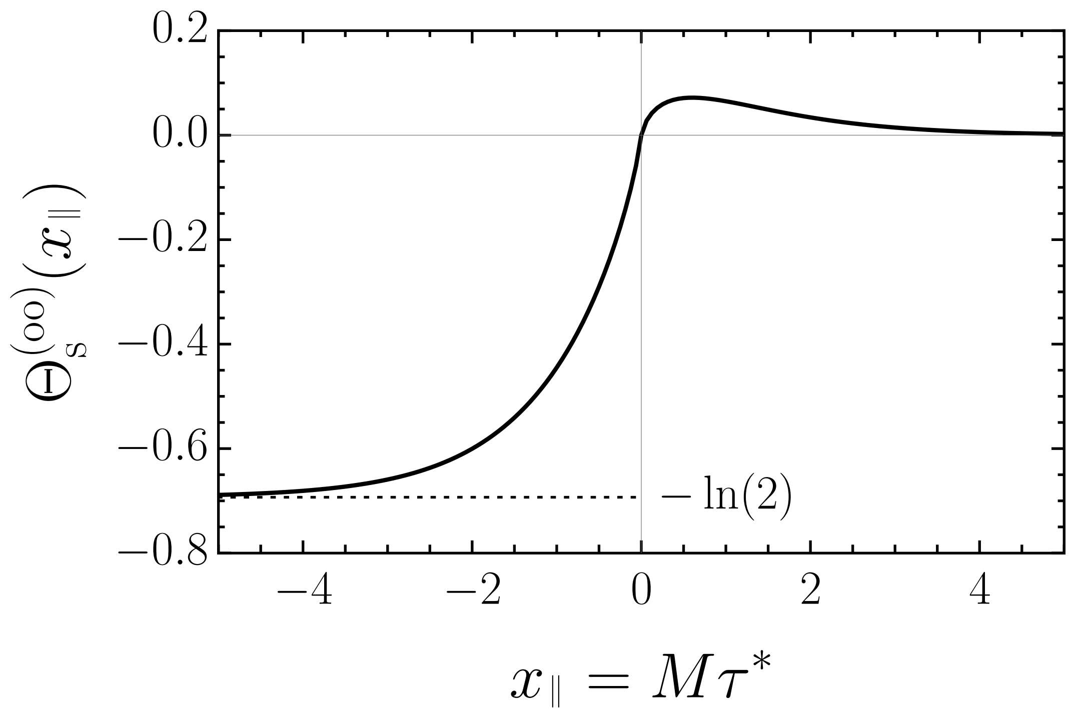
$$\Gamma = \sqrt{x_\parallel^2 + \Phi^2}$$



$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel,\rho) \quad = \quad \rho\,\Theta_{\mathrm{b}}(x_\parallel) \quad + \quad \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) \quad + \quad n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_\parallel) \quad + \quad \Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel,\rho)$$

$$\Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) = H(-x_\parallel)\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right]$$

$$-\frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\,\frac{\Phi}{\Gamma}\arctan\left(\frac{x}{\Phi}\right)\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$



3 Disassembling the Free Energy Scaling Functions

$$\Theta_{\mathrm{b}}(x_\parallel) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\, \frac{\Phi^2}{\Gamma}\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

with scaling form of Onsager dispersion

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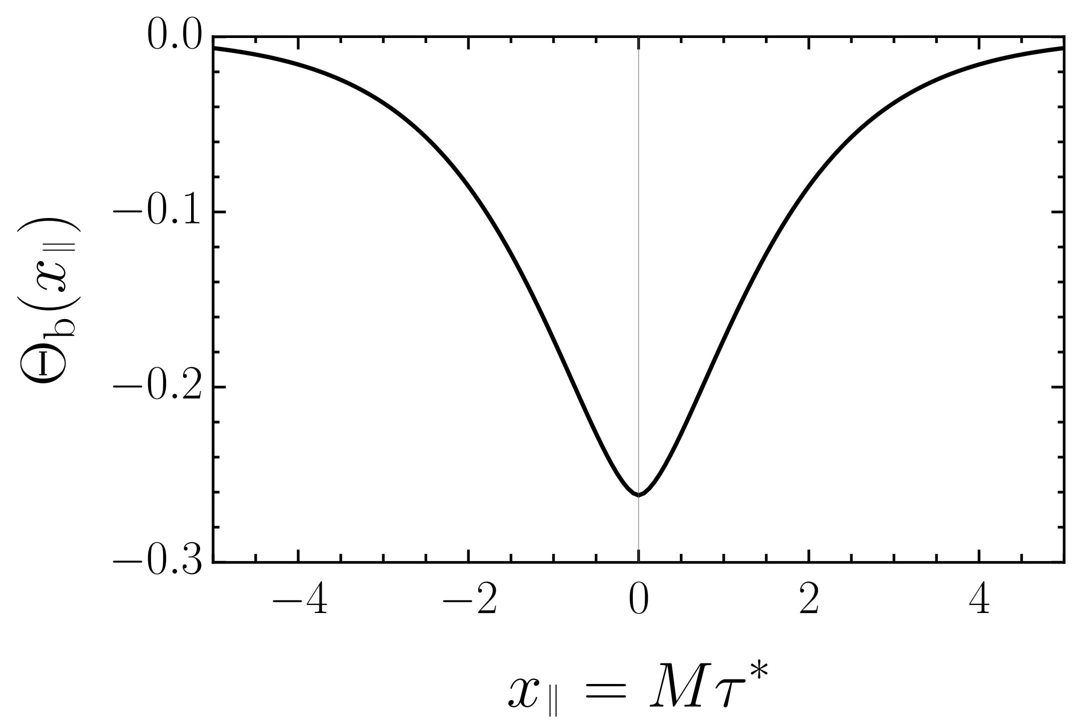
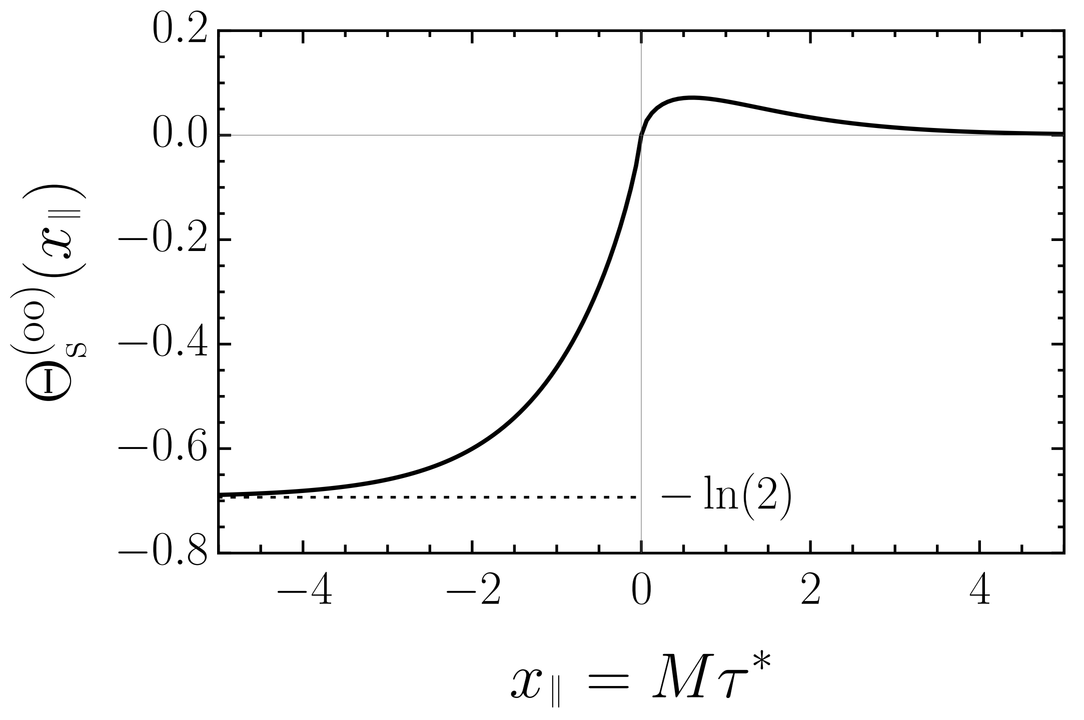


Infinitely strong surface field:

$$\Theta_{\mathrm{h}}(x_\parallel) = \frac{1}{2}\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel)$$

$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel,\rho) \;=\; \rho\,\Theta_{\mathrm{b}}(x_\parallel) \;+\; \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) \;+\; n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_\parallel) \;+\; \Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel,\rho)$$

$$\Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) = H(-x_\parallel)\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\Phi\,\frac{\Phi}{\Gamma}\arctan\left(\frac{x}{\Phi}\right)\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

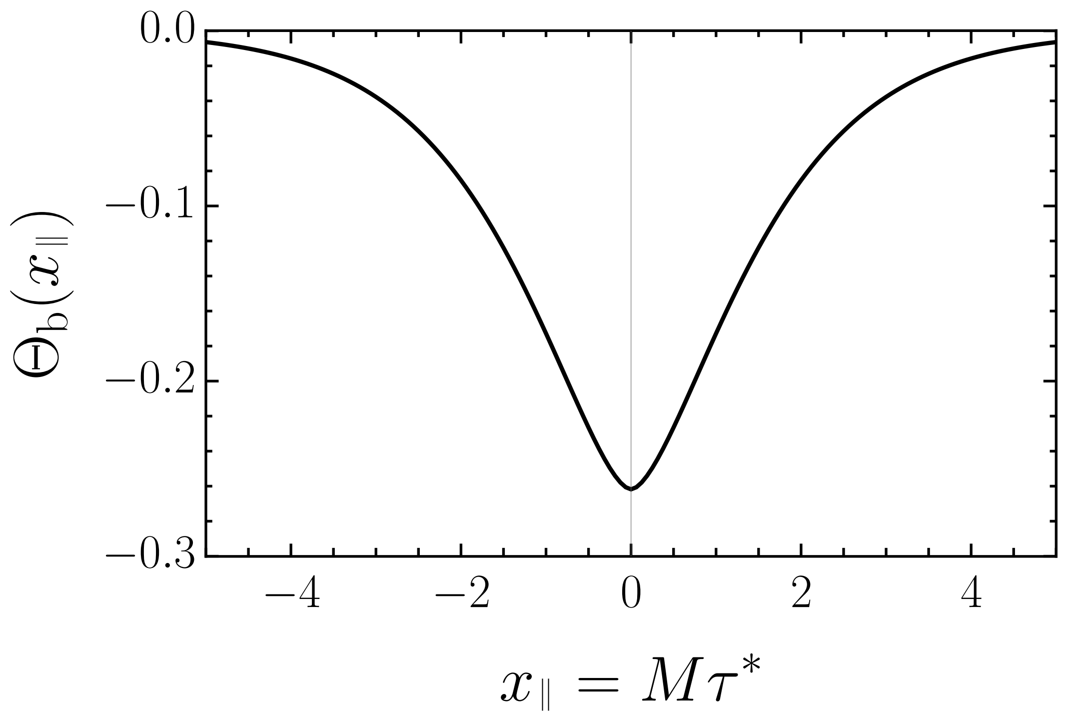


3 Disassembling the Free Energy Scaling Functions

$$\Theta_{\mathrm{b}}(x_\parallel) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\, \frac{\Phi^2}{\Gamma}\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

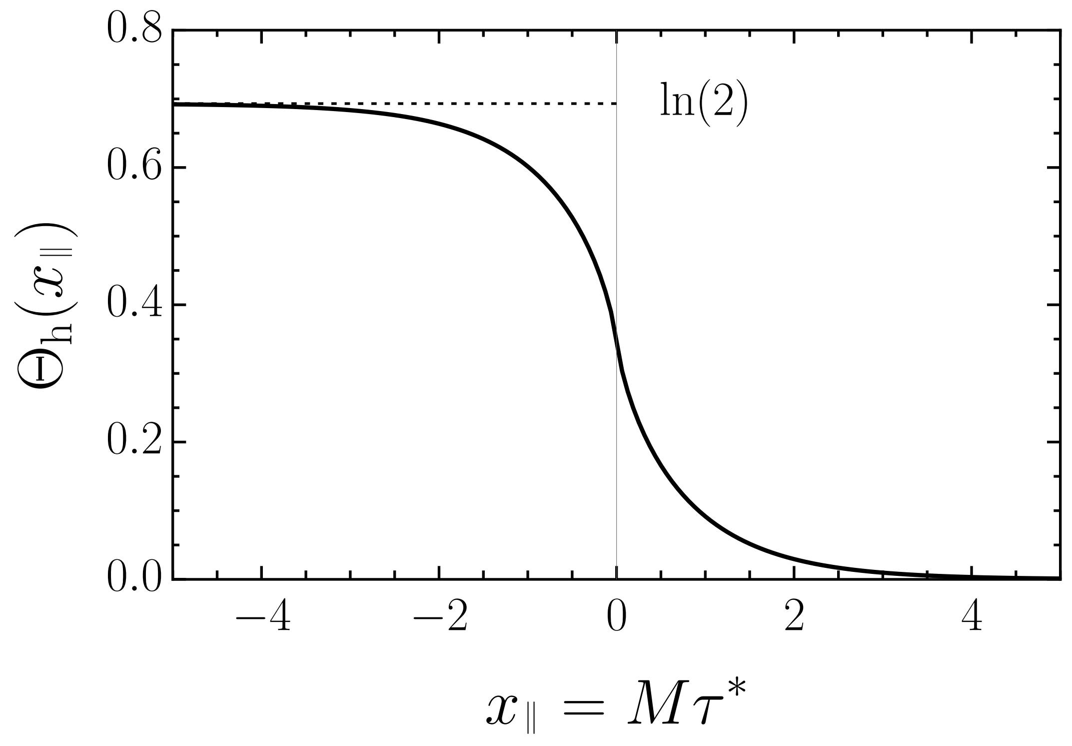
with scaling form of Onsager dispersion

$$\Gamma = \sqrt{x_\parallel^2 + \Phi^2}$$



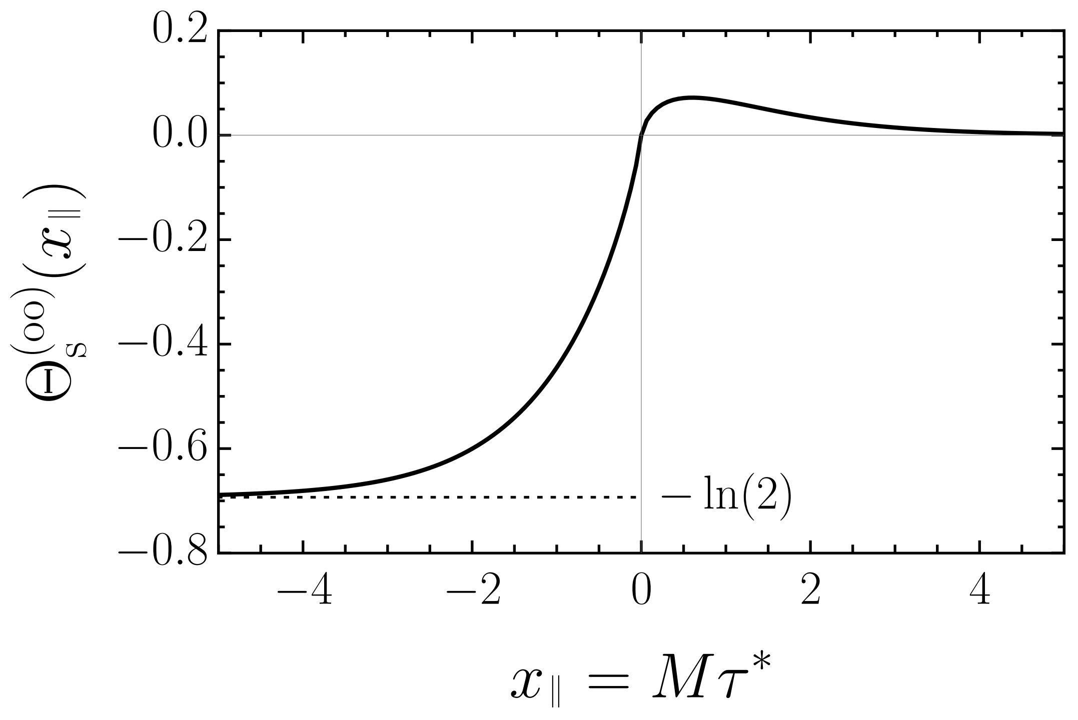
Infinitely strong surface field:

$$\Theta_{\mathrm{h}}(x_\parallel) = \frac{1}{2}\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel)$$



$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel,\rho) \;=\; \rho\,\Theta_{\mathrm{b}}(x_\parallel) \;+\; \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) \;+\; n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_\parallel) \;+\; \Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel,\rho)$$

$$\Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) = H(-x_\parallel)\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\Phi\,\frac{\Phi}{\Gamma}\arctan\left(\frac{x}{\Phi}\right)\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

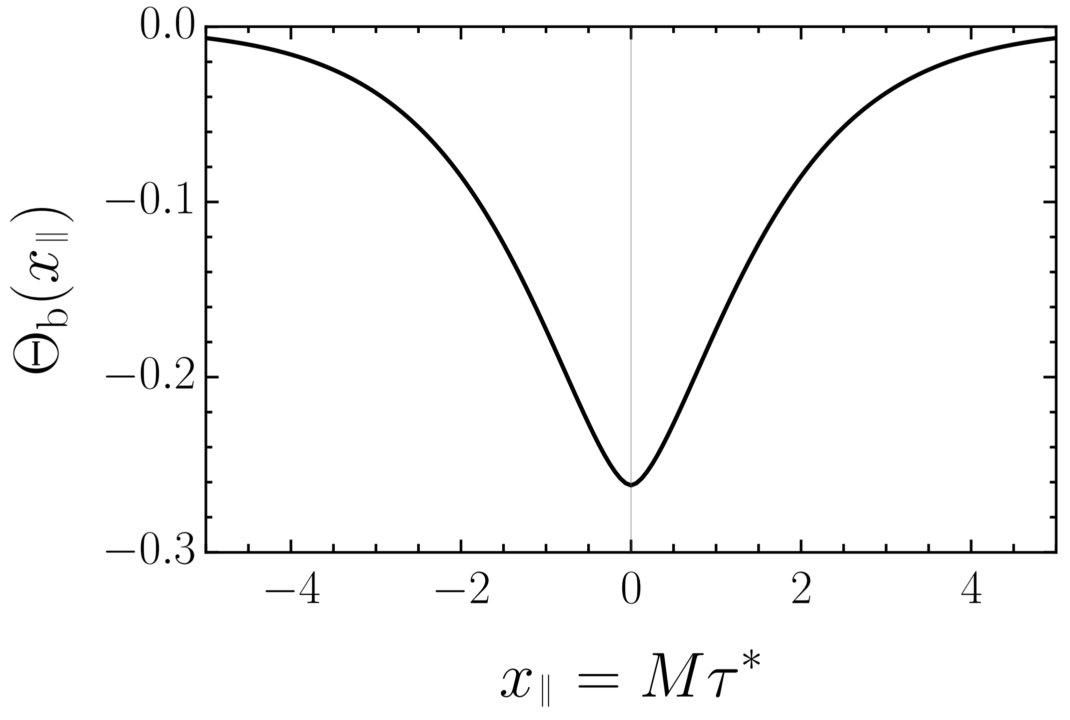


3 Disassembling the Free Energy Scaling Functions

$$\Theta_{\mathrm{b}}(x_\parallel) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\Phi\, \frac{\Phi^2}{\Gamma}\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

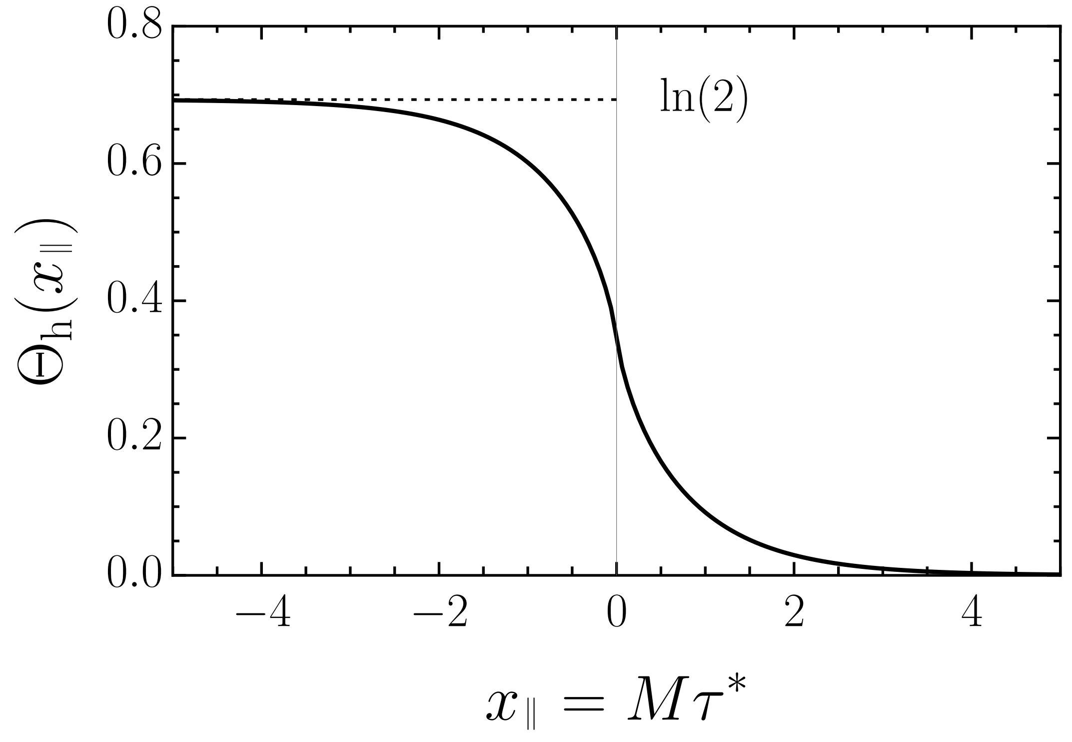
with scaling form of Onsager dispersion

$$\Gamma = \sqrt{x_\parallel^2 + \Phi^2}$$



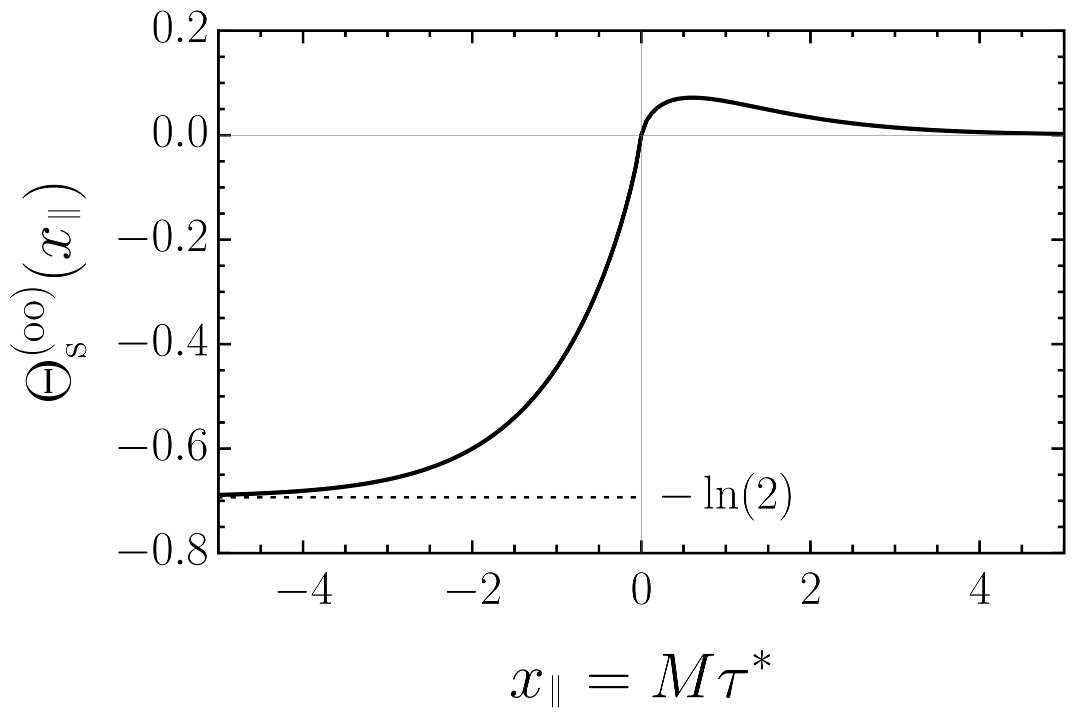
Infinitely strong surface field:

$$\Theta_{\mathrm{h}}(x_\parallel) = \frac{1}{2}\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel)$$



$$\rho\,\Theta_\parallel^{(a/b)}(x_\parallel, \rho) = \rho\,\Theta_{\mathrm{b}}(x_\parallel) + \Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) + n_{\mathrm{h}}\Theta_{\mathrm{h}}(x_\parallel) + \Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel, \rho)$$

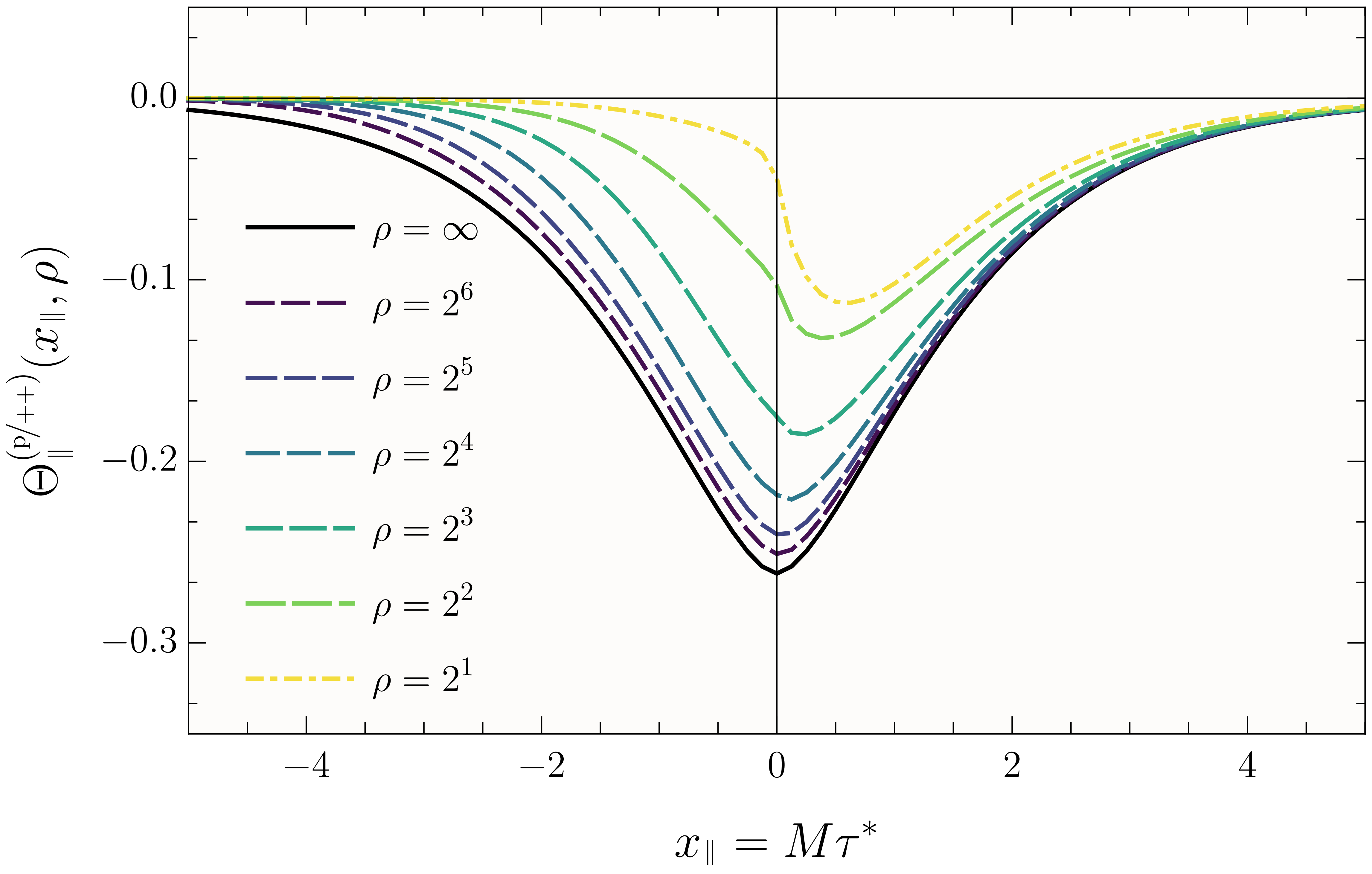
$$\Theta_{\mathrm{s}}^{(\mathrm{oo})}(x_\parallel) = H(-x_\parallel)\ln\left[\frac{2}{1+\mathrm{e}^{-|x_\parallel|}}\right] - \frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\Phi\,\frac{\Phi}{\Gamma}\arctan\left(\frac{x}{\Phi}\right)\left[\tanh\left(\frac{\Gamma}{2}\right) - 1\right]$$

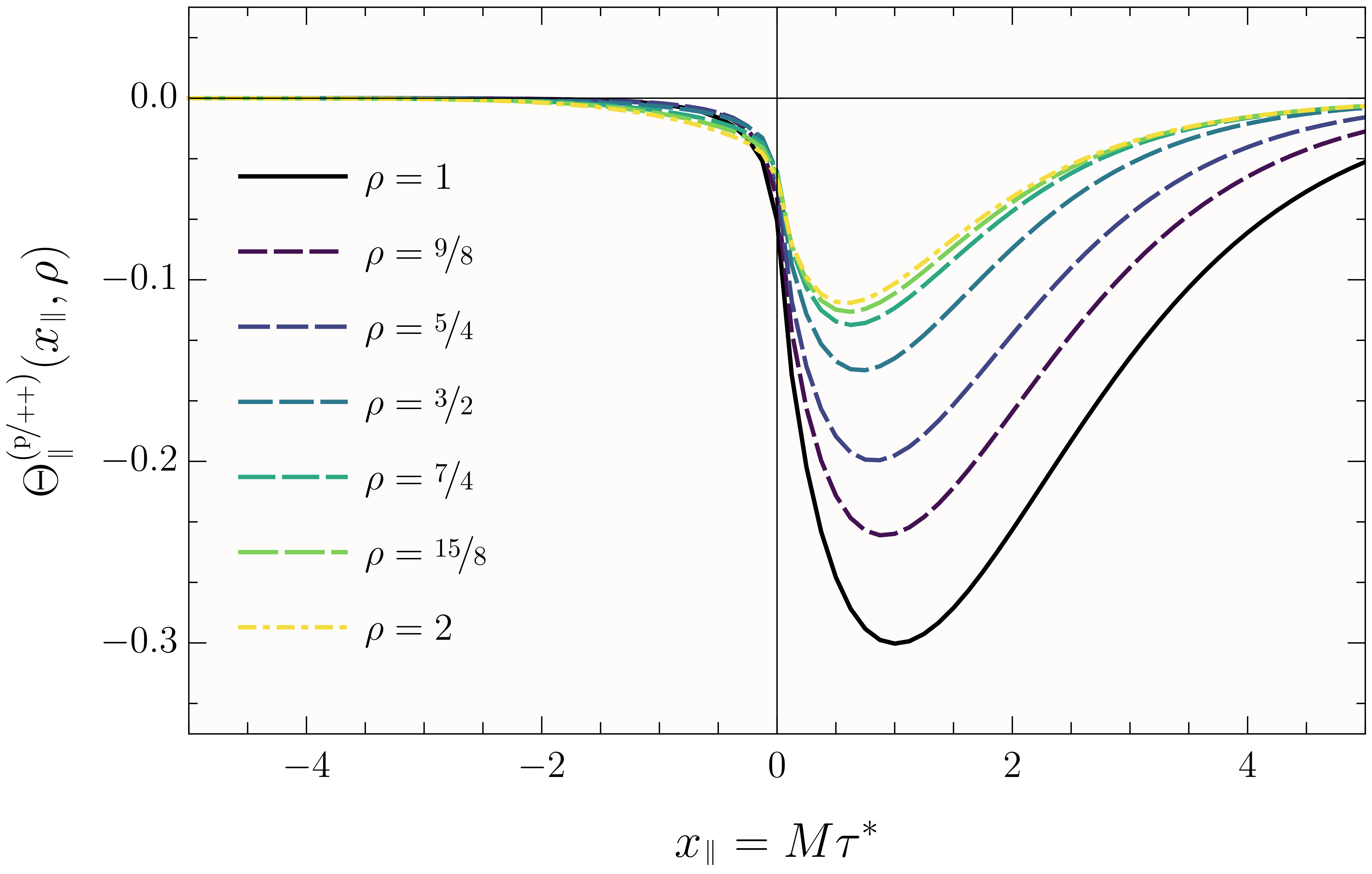


$$\lim_{\rho\to 0}\left[\rho^2\Psi_{\mathrm{strip}}^{(a/b)}(x_\parallel, \rho)\right] = \Theta_\perp^{(a)}(x_\perp)$$

4 Scaling Function for symmetry-breaking Boundaries

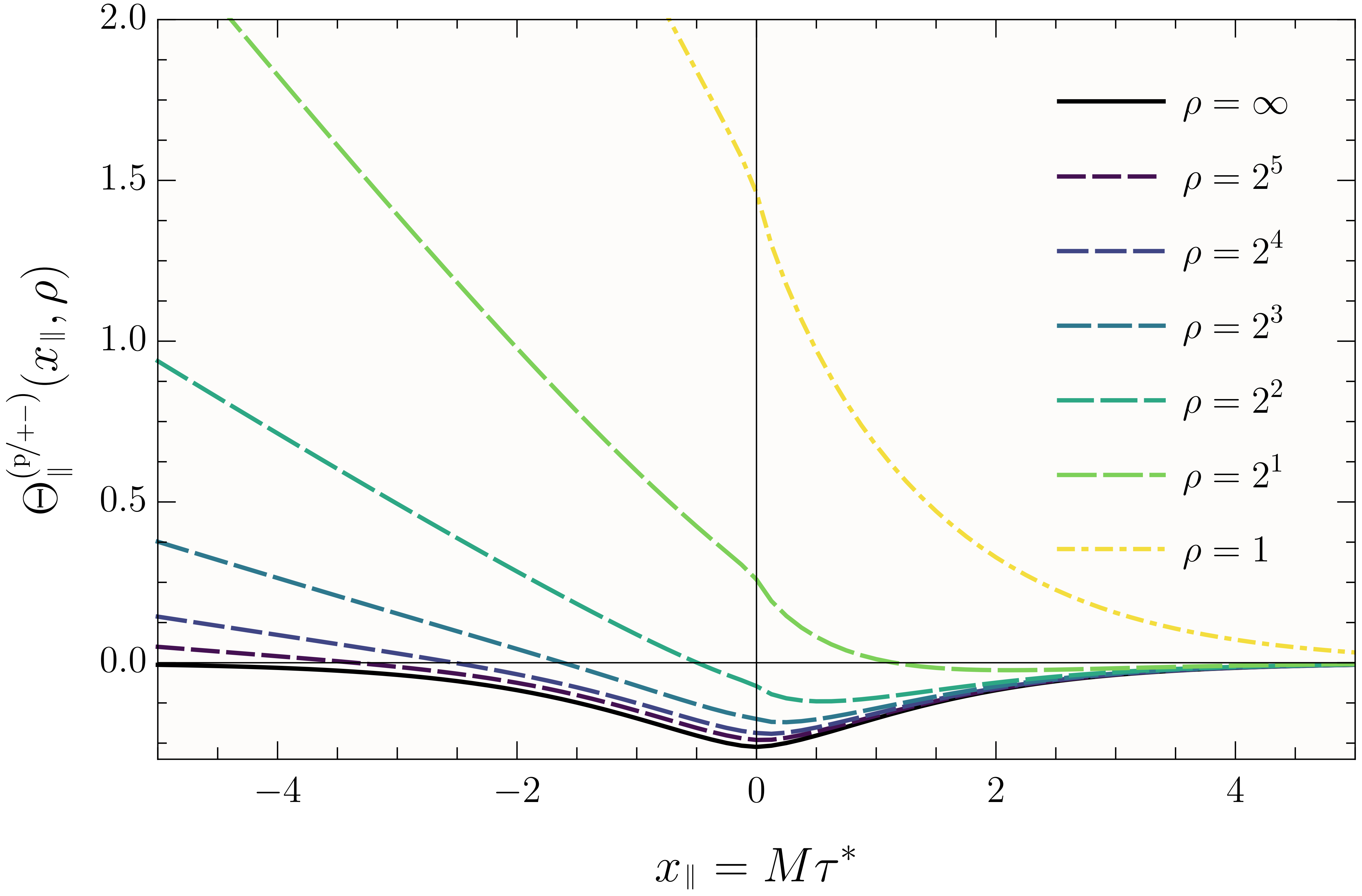
$$\rho\,\Theta_{\parallel}^{(++/\mathrm{p})}(x_\parallel,\rho) = \rho\,\Theta_{\mathrm{b}}(x_\parallel) + \Theta_{\mathrm{s}}^{(\mathrm{oo})} + 2\Theta_{\mathrm{h}}(x_\parallel) + \Psi_{\mathrm{o}}^{(++/\mathrm{p})}(x_\parallel,\rho)$$
$$+ \ln\left[1 + \mathrm{e}^{-\rho[\delta\Theta_{\mathrm{b}}(x_\parallel)+x_\parallel H(x_\parallel)]-\delta\Theta_{\mathrm{s}}^{(\mathrm{oo})}+2\delta\Theta_{\mathrm{h}}(x_\parallel)+\delta\Psi_{\mathrm{o}}^{(++/\mathrm{p})}(x_\parallel,\rho)}\right]$$

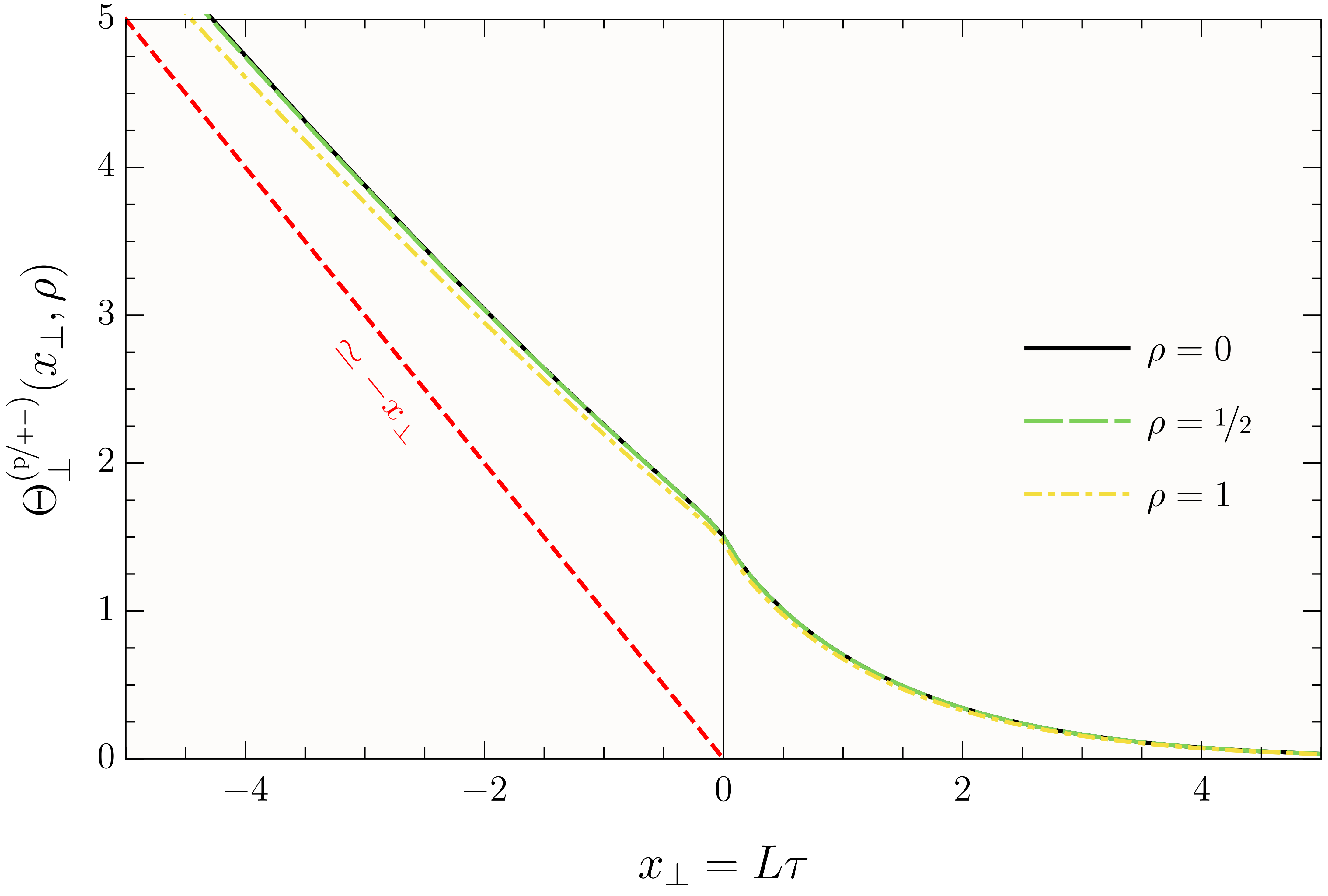




4 Scaling Function for symmetry-breaking Boundaries

$$\rho\,\Theta_{\parallel}^{(+-/\mathrm{p})}(x_\parallel,\rho) = \rho\,\Theta_{\mathrm{b}}(x_\parallel) + \Theta_{\mathrm{s}}^{(\mathrm{oo})} + 2\Theta_{\mathrm{h}}(x_\parallel) + \Psi_{\mathrm{o}}^{(+-/\mathrm{p})}(x_\parallel,\rho)$$
$$+ \ln\left[1 - \mathrm{e}^{-\rho[\delta\Theta_{\mathrm{b}}(x_\parallel)+x_\parallel H(x_\parallel)]-\delta\Theta_{\mathrm{s}}^{(\mathrm{oo})}+2\delta\Theta_{\mathrm{h}}(x_\parallel)+\delta\Psi_{\mathrm{o}}^{(+-/\mathrm{p})}(x_\parallel,\rho)}\right]$$





Conformal Approximation

- Temperatur and geometry scaling variables for two disks

$$x_\perp^{\mathrm{disk}} = \tau \frac{d_0}{\xi_0^+} \qquad \text{and} \qquad \Delta = \frac{d_0}{R}$$

- Aspect ratio ρ and reduced distance Δ connected by

$$\cosh(2\pi\rho) = \left(\frac{\Delta}{2} + 1\right)^2 - 1$$

with $\rho \overset{\Delta \ll 1}{\approx} \Delta^{1/2}/\pi$ for small Δ

- Final formula for conformal approximation:

$$\boxed{\Phi_{\mathrm{C}}^{(ab)}(x_\perp^{\mathrm{disk}}, \Delta) \approx \rho \left[\frac{\pi}{12} + \Theta_\parallel^{(ab/\mathrm{p})}\left(\frac{x_\perp^{\mathrm{disk}}}{\rho}, \rho\right)\right]}$$

vs

Derjaguin Approximation

- Derjaguin approximation for free energy

$$F_{\mathrm{res}}^{(ab)}(\tau; d_0, R) = -\int_{d_0}^{\infty} \mathrm{d}\ell \int_{-\pi/2}^{\pi/2} \mathrm{d}\varphi \frac{R\cos\varphi}{[d(\varphi)]^2}\, \vartheta_\perp^{(ab)}\left(\tau \frac{d(\varphi)}{\xi_0^+}\right)$$

with $d(\varphi) = d_0 + 2R(1 - \cos\varphi)$

- Approximation for (++) BCs and $x_\perp^{\mathrm{disk}} \gg 1$

$$\Phi_{\mathrm{C}}^{(++)}(x_\perp^{\mathrm{disk}}, \Delta) \overset{x_\perp^{\mathrm{disk}} \gg 1}{\approx} -\left[\pi \frac{x_\perp^{\mathrm{disk}}}{\Delta}\right]^{1/2} \mathrm{e}^{-x_\perp^{\mathrm{disk}}}$$

Conformal Approximation

- Temperatur and geometry scaling variables for two disks

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- Final formula for conformal a

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Derjaguin Approximation

- Derjaguin approximation for free energy

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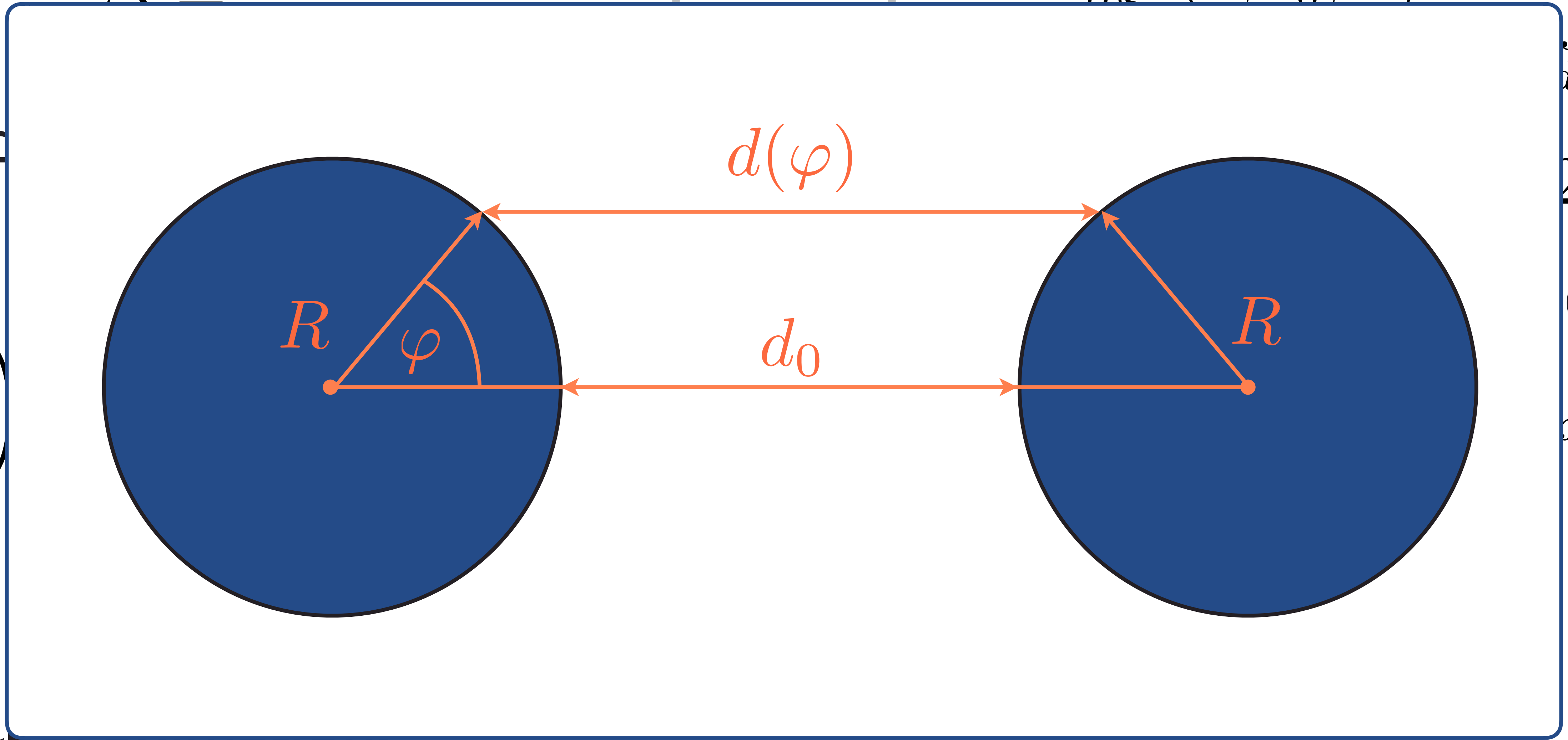
$2R(1 - \cos\varphi)$

(++) BCs and $x_\perp^{\mathrm{disk}} \gg 1$

$$\overset{x_\perp^{\mathrm{disk}} \gg 1}{\approx} \; -\left[\pi \frac{x_\perp^{\mathrm{disk}}}{\Delta}\right]^{1/2} \mathrm{e}^{-x_\perp^{\mathrm{disk}}}$$



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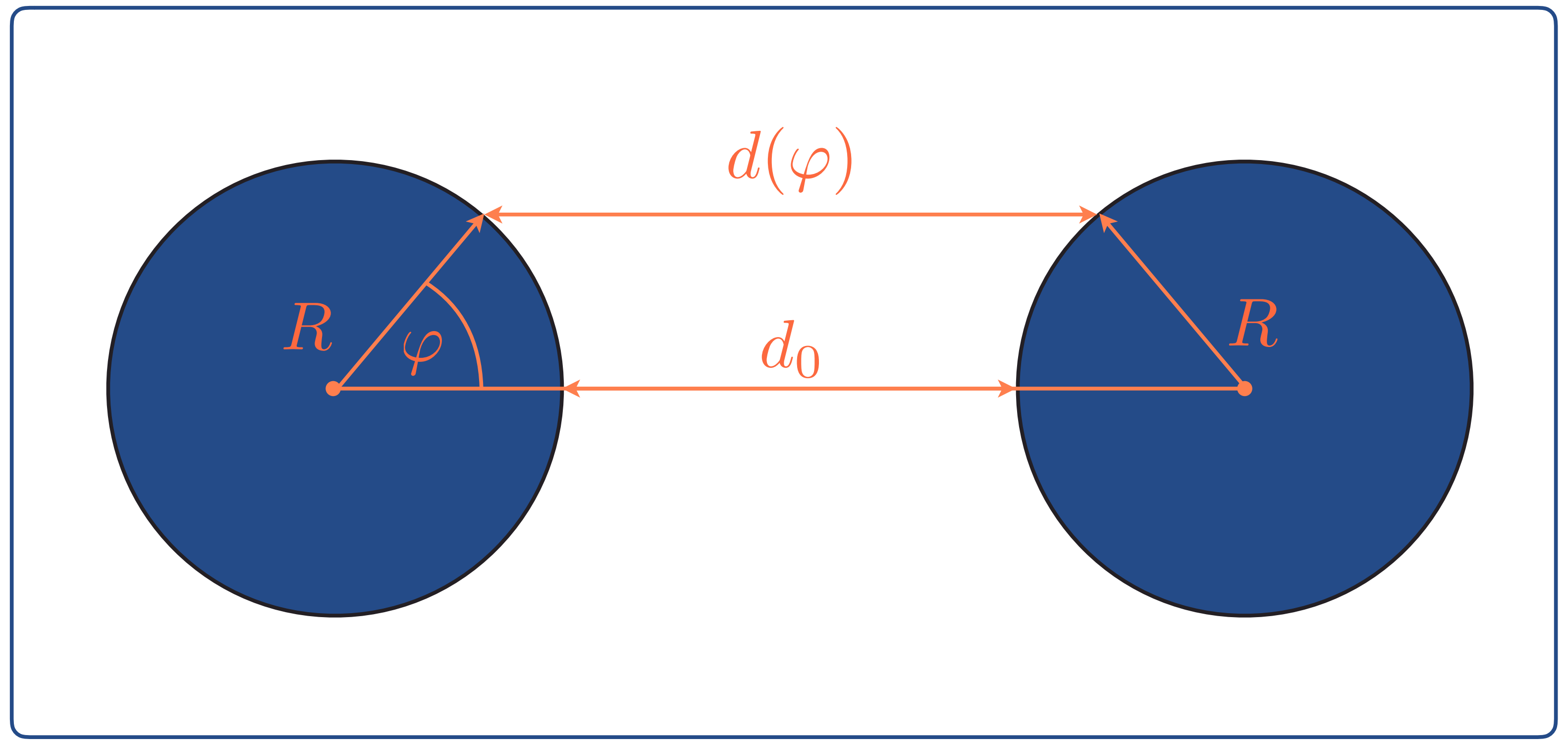

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with $\rho \overset{\Delta \ll 1}{\approx} \Delta^{1/2}/\pi$ for sma

- Final formula for conformal

$$\boxed{\Phi_C^{(ab)}(x_\perp^{\text{disk}}, \Delta) \approx \rho \left[\frac{\pi}{12} + \mathcal{O}_\| \quad \left(\quad \rho \quad , \rho \right)\right]}$$

Derjaguin Approximation

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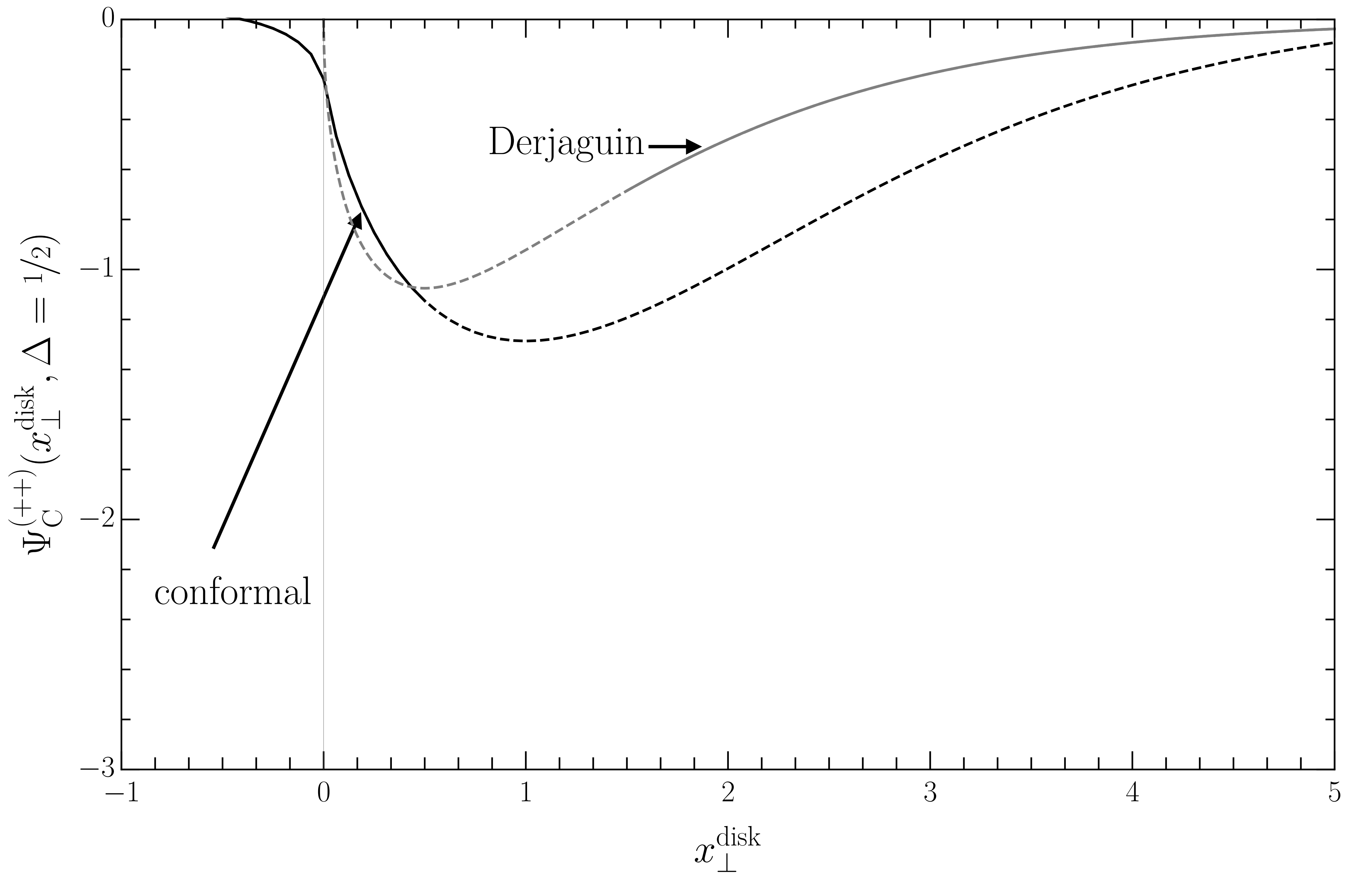
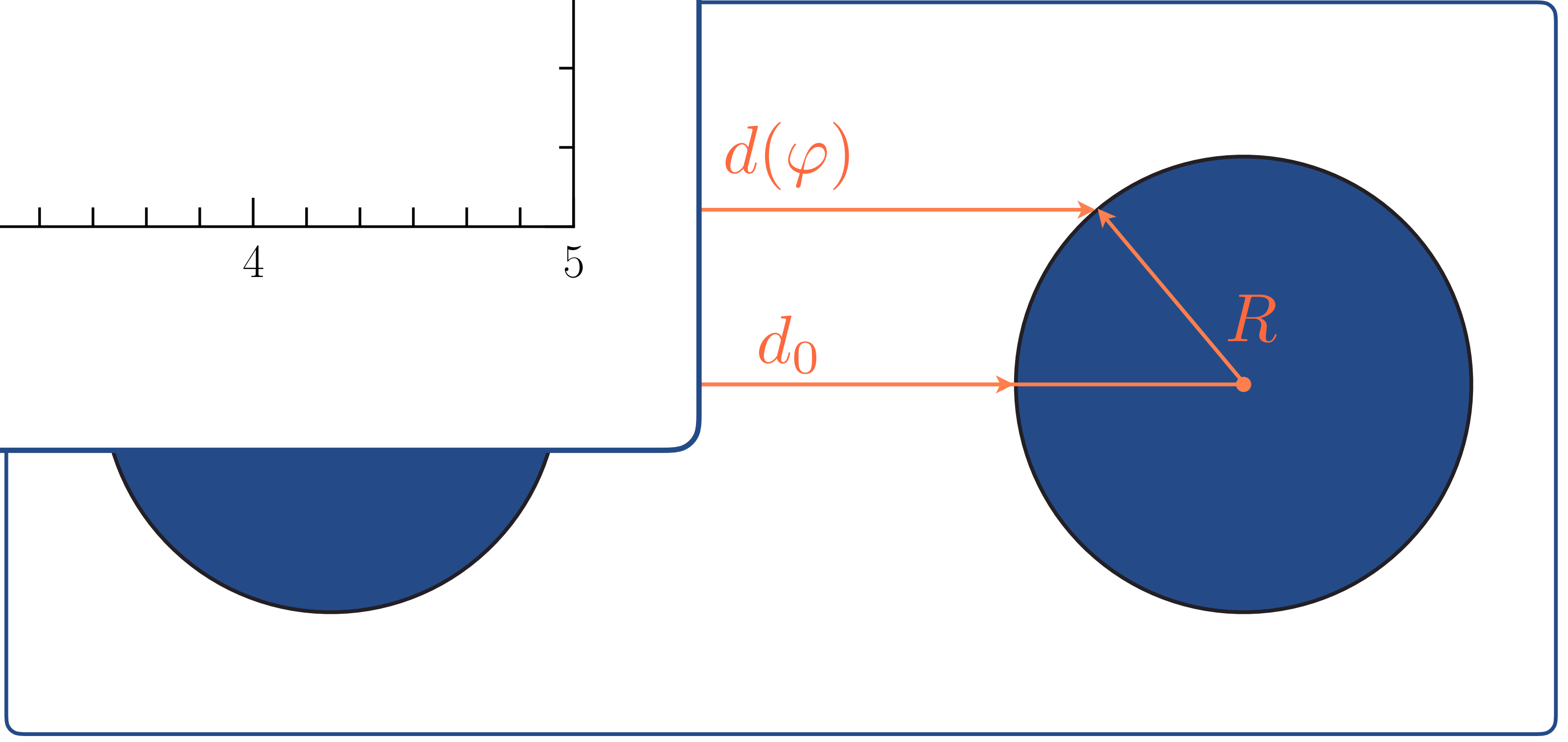
$$\ell \int_{-\pi/2}^{\pi/2} \mathrm{d}\varphi \frac{R\cos\varphi}{[d(\varphi)]^2} \vartheta_\perp^{(ab)} \left(\tau \frac{d(\varphi)}{\xi_0^+}\right)$$

$$(1 - \cos\varphi)$$

) BCs and $x_\perp^{\text{disk}} \gg 1$

$$\overset{\text{k} \gg 1}{\simeq} - \left[\pi \frac{x_\perp^{\text{disk}}}{\Delta}\right]^{1/2} \mathrm{e}^{-x_\perp^{\text{disk}}}$$





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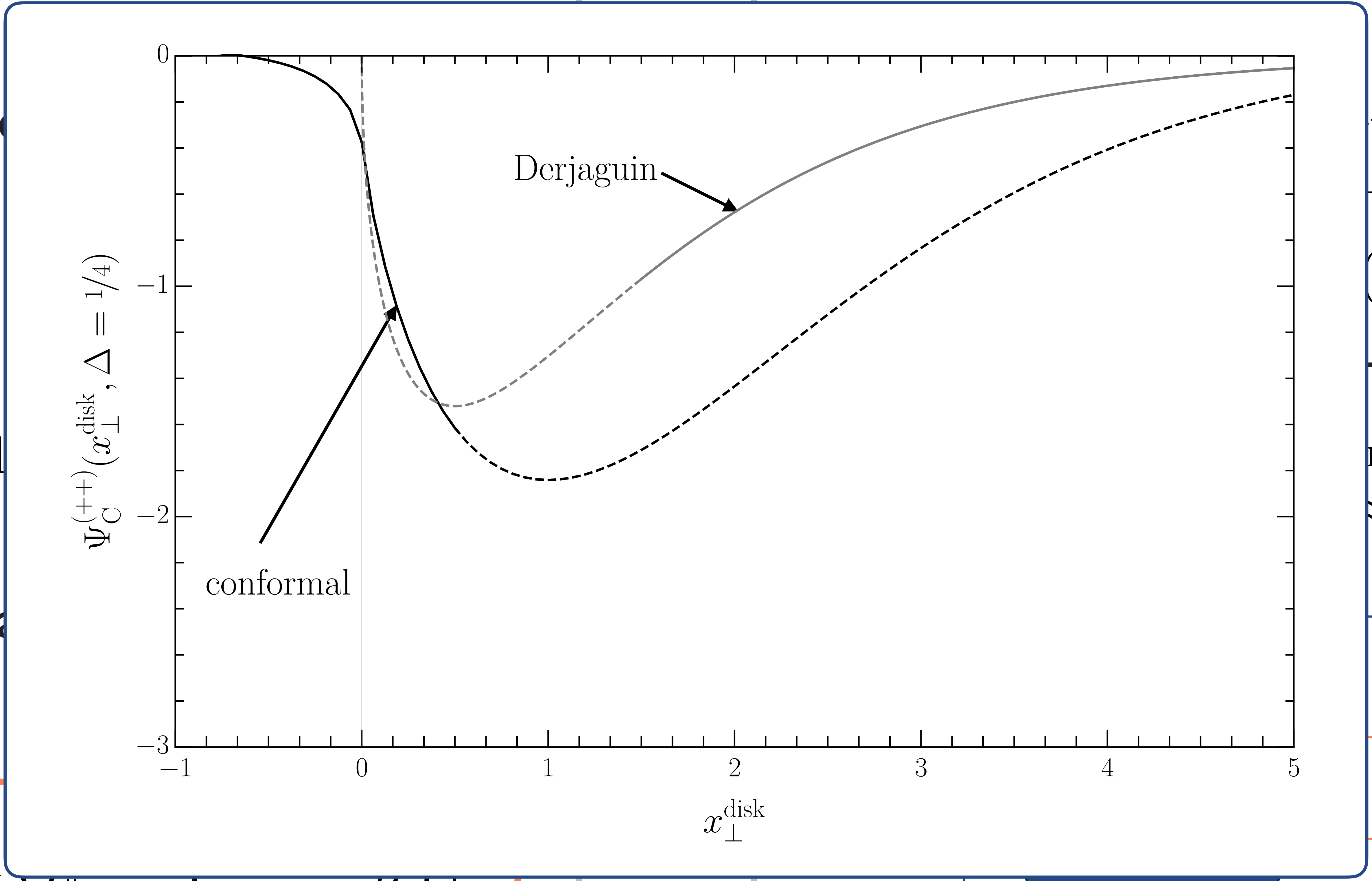

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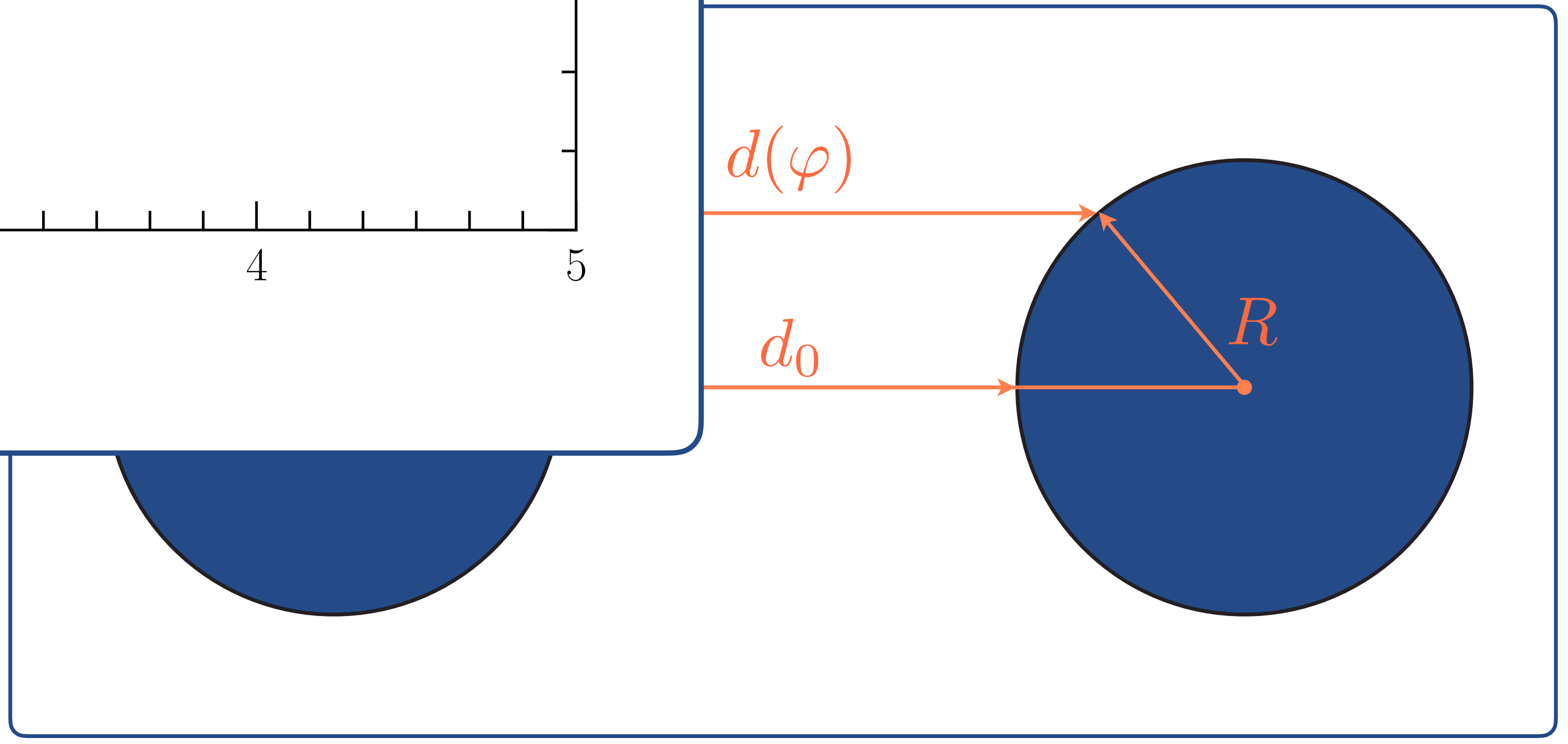

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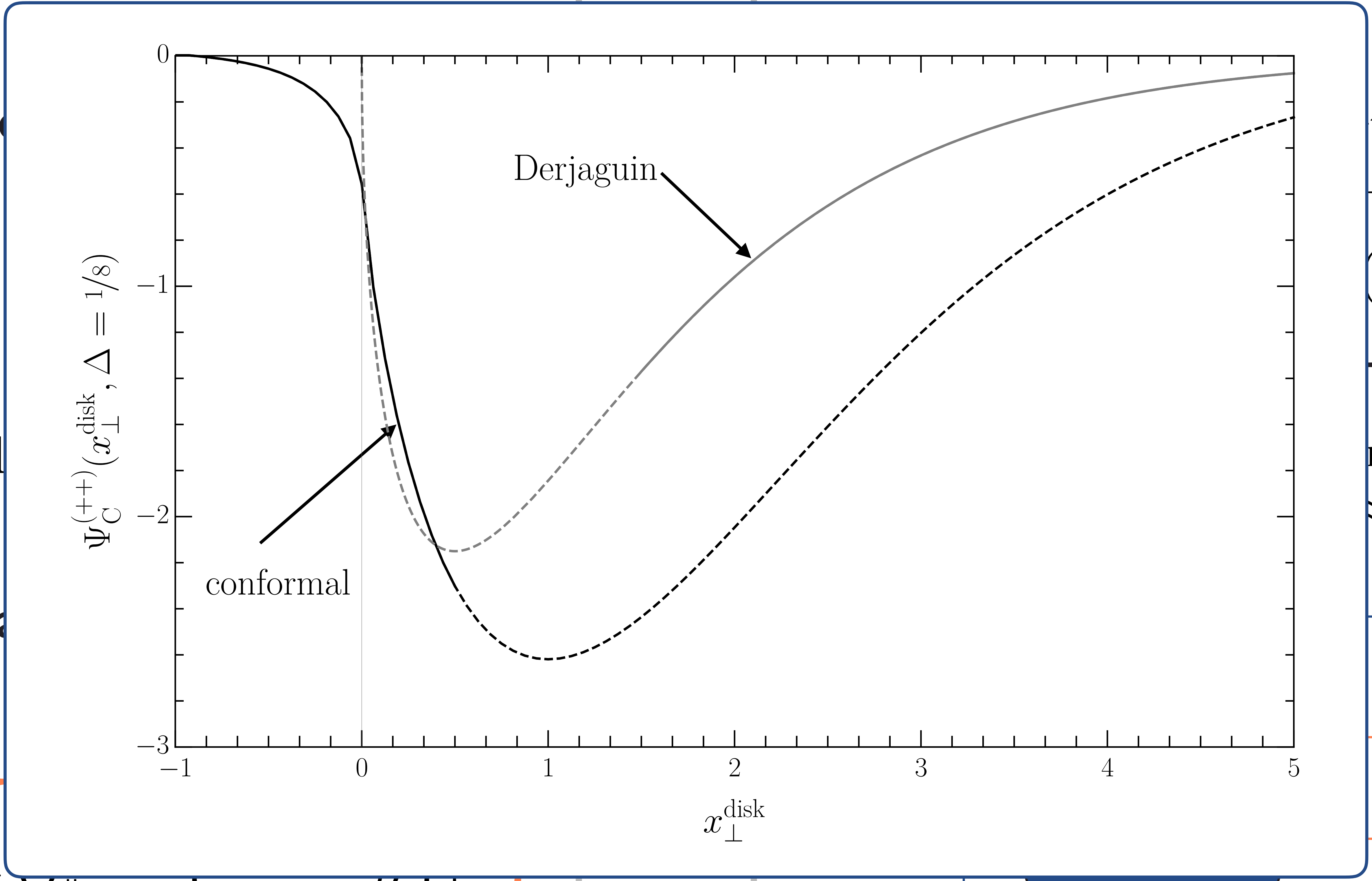

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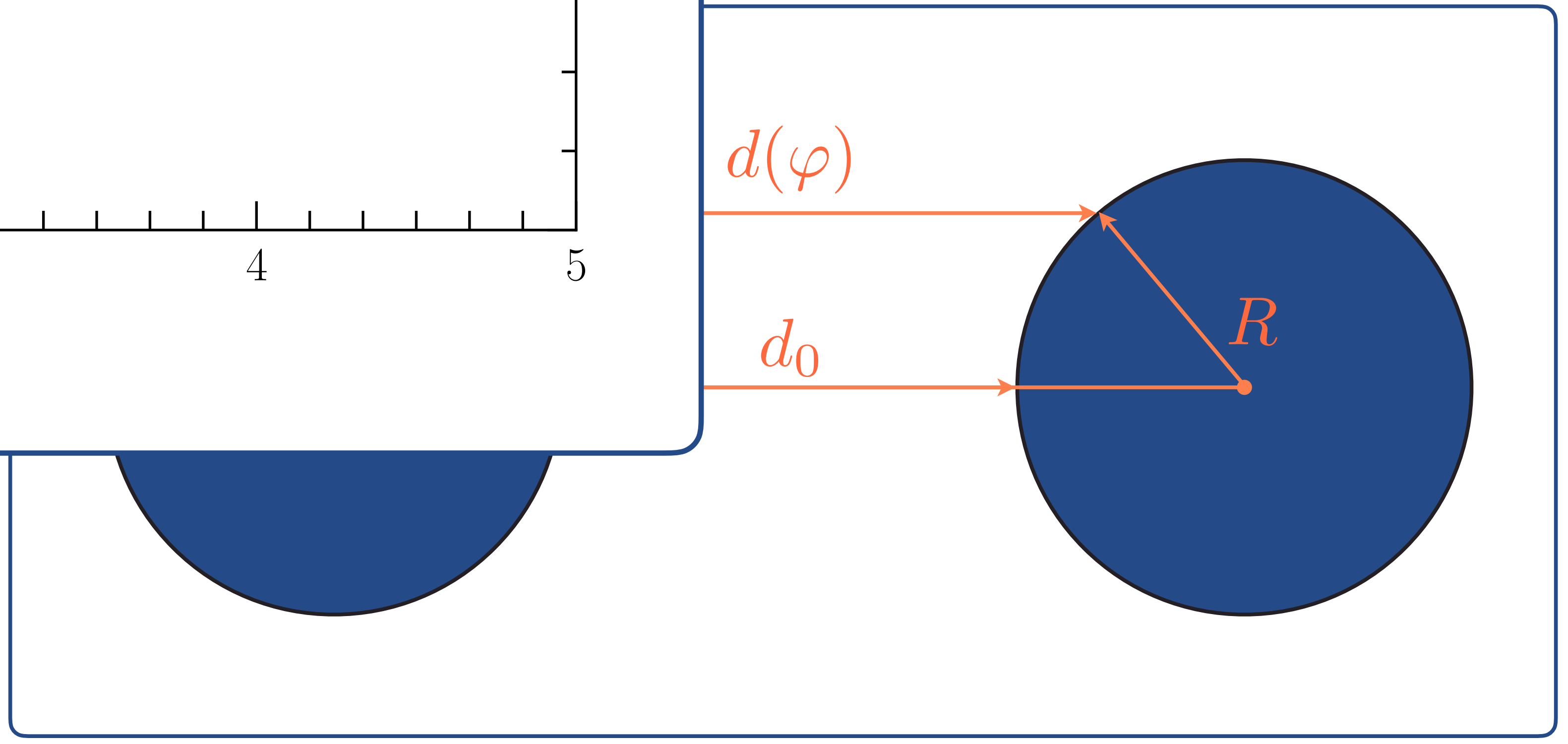

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DFG Deutsche Forschungsgemeinschaft

Thank you for your attention!