Universality from disorder in the random-bond Blume–Capel model

Nikolaos Fytas

Coventry University

December 1st, 2017

Disordered systems

1. Their behavior cannot be inferred from its pure counterpart.
2. Are prototypical examples of complex systems in many aspects.
3. Show incredibly slow dynamic evolution.
4. Give birth to non–perturbative phenomena.

Disordered systems

1. Their behavior cannot be inferred from its pure counterpart.
2. Are prototypical examples of complex systems in many aspects.
3. Show incredibly slow dynamic evolution.
4. Give birth to non–perturbative phenomena.

Non–perturbative methods, like **numerical simulations**, offer a powerful alternative.

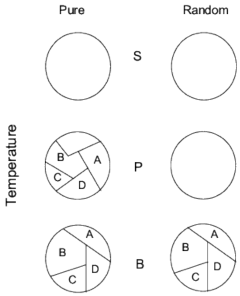
Theoretical arguments on the effect of quenched disorder

Theoretical arguments on the effect of quenched disorder

- Continuous transitions: the Harris criterion (1974) relates the value of the specific-heat exponent α with the expected effects of uncorrelated weak disorder in ferromagnets.

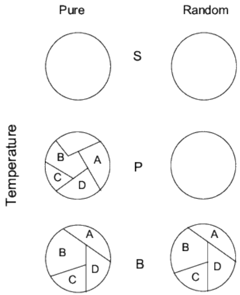
Theoretical arguments on the effect of quenched disorder

- Continuous transitions: the Harris criterion (1974) relates the value of the specific-heat exponent α with the expected effects of uncorrelated weak disorder in ferromagnets.
- Symmetry–breaking first–order transitions are converted to continuous transitions by infinitesimal randomness at $D = 2$ and beyond a threshold amount for $D > 2$ (Aizenmann–Wehr & Berker, 1990).



Theoretical arguments on the effect of quenched disorder

- Continuous transitions: the Harris criterion (1974) relates the value of the specific-heat exponent α with the expected effects of uncorrelated weak disorder in ferromagnets.
- Symmetry–breaking first–order transitions are converted to continuous transitions by infinitesimal randomness at $D = 2$ and beyond a threshold amount for $D > 2$ (Aizenmann–Wehr & Berker, 1990).



What is the fate of a first–order transition that is destroyed by disorder?

Short review on the effect of disorder on 2D first–order transitions

S. Chen, A.M. Ferrenberg, and D.P. Landau, PRL **69**, 1213 (1992)
Ising universality in the random–bond $q = 8$ *Potts model*
(Monte Carlo simulations).

A. Falicov and A.N. Berker, PRL **76**, 4380 (1996)
Disorder–induced continuous transitions are controlled by a distinctive strong–coupling fixed point
(Renormalization group).

J. Cardy and J.L. Jacobsen, PRL **79**, 4063 (1997)
C. Chatelain and B. Berche, PRL **80**, 1670 (1998)
β/ν *varies continuously with* q *and* $\nu \approx 1$ *a weakly varying exponent in the random–bond Potts model*
(Finite-size scaling, conformal invariance, and Monte–Carlo simulations).

A. Malakis, A.N. Berker, I.A. Hadjiagapiou, and N.G. Fytas, PRE **79**, 011125 (2009)
Strong violation of universality in the random–bond Blume–Capel model
(Monte Carlo simulations).

The pure Blume–Capel model

$$\mathcal{H}^{(\mathrm{pure})} = -J\sum_{\langle xy\rangle}\sigma_x\sigma_y + \Delta\sum_x \sigma_x^2 = E_J + \Delta E_\Delta, \quad \sigma_x = \{-1, 0, +1\}, \quad J > 0.$$

The crystal–field coupling Δ controls the density of vacancies ($\sigma_x = 0$). In the limit $\Delta \to -\infty$ the model becomes equivalent to the Ising model.

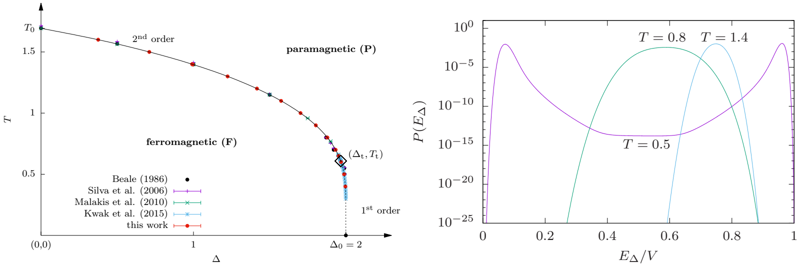


Figure: Left panel: Phase diagram in the $\Delta - T$ plane. Note the tricritical point $(\Delta_t, T_t) = (1.9660(1), 0.6080(1))$. Right panel: $P(E_\Delta)$ for $L = 48$ ($V = 48 \times 48$).

J. Zierenberg, *et al.*, Eur. Phys. J. Special Topics **226**, 789 (2017).

The random–bond Blume–Capel model

$$\mathcal{H}^{(\mathrm{random})} = -\sum_{\langle xy \rangle} J_{xy}\sigma_x\sigma_y + \Delta \sum_x \sigma_x^2 = E_J + \Delta E_\Delta, \quad \sigma_x = \{-1, 0, +1\}, \quad J > 0,$$

where

$$\mathcal{P}(J_{xy}) = \frac{1}{2}\left[\delta(J_{xy} - J_1) + \delta(J_{xy} - J_2)\right]; \qquad \frac{J_1 + J_2}{2} = 1; \qquad J_1 > J_2 > 0; \qquad r = \frac{J_2}{J_1},$$

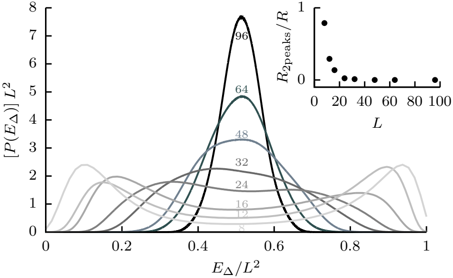
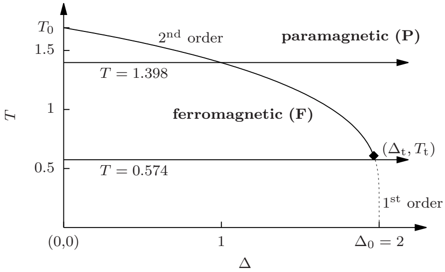


Figure: $P(E_\Delta)$ at $T = 0.574$ with $r = 0.6$ over 256 samples. Up to $L \approx 48$ $R_{2\mathrm{peaks}} \neq 0$.

Simulation details



- Bulk simulations: hybrid scheme combining Metropolis and Wolff cluster moves.
- Dedicated reasons (first-order regime): parallel version of the multicanonical algorithm.
- Pure model ($r = 1$): $L_{\max} = 128$.
- Random model ($r = 0.6$): $L_{\max} = 256$.
- Extensive disorder averaging $R \sim 5 \times 10^3$.

Observables

1. Order parameter: $M = \sum_x \sigma_x$.

2. Specific heat: $C \equiv \frac{\partial \langle E_J \rangle}{\partial \Delta} \frac{1}{V} = -\beta \left(\langle E_J E_\Delta \rangle - \langle E_J \rangle \langle E_\Delta \rangle \right) / V$.

3. Susceptibility: $\chi = \beta \left(\langle M^2 \rangle - \langle |M| \rangle^2 \right) / V$.

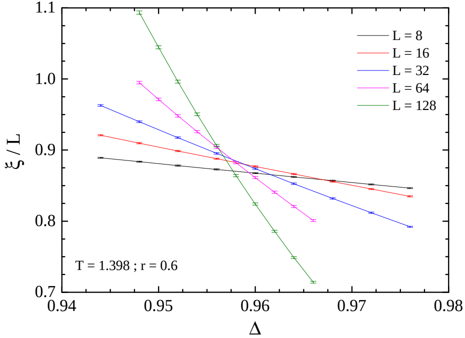
4. Second–moment correlation length. This involves the Fourier transform of the spin field $\hat{\sigma}(\mathbf{k}) = \sum_{\mathbf{x}} \sigma_{\mathbf{x}} e^{i\mathbf{kx}}$. If we set $F = \left\langle |\hat{\sigma}(2\pi/L, 0)|^2 + |\hat{\sigma}(0, 2\pi/L)|^2 \right\rangle / 2$ we obtain

$$\xi \equiv \frac{1}{2\sin(\pi/L)} \sqrt{\frac{\langle M^2 \rangle}{F} - 1}.$$

ξ/L is a dimensionless quantity – for Ising spins on $L \times L$ patches of the square lattice with periodic boundary conditions approaches $(\xi/L)_\infty^{\mathrm{Ising}} = 0.9050488292(4)$.

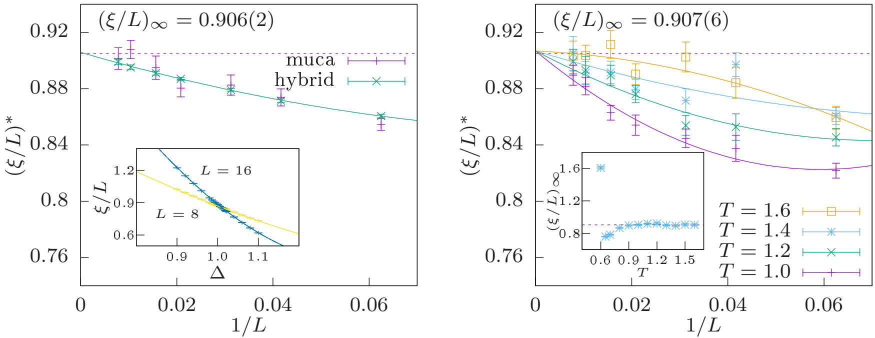
Scaling scheme

We work with pairs $(L, 2L) \rightarrow L_{\rm eff} = (3L/2)$, and define as pseudo–critical points, $\Delta^{(\rm cross)}$, the values of Δ where $\xi_{2L}/\xi_L = 2$.



We denote as $(\xi/L)^*$, C^*, and χ^* the values of ξ/L, C, and χ at $\Delta^{(\rm cross)}$.

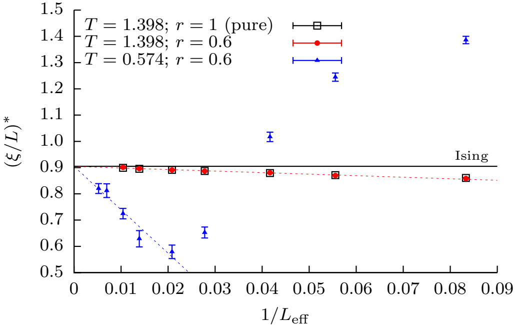
Ising universality for the pure model at $\Delta < \Delta_t$



$$(\xi/L)_\infty^{\mathrm{Ising}} = 0.9050488292(4).$$

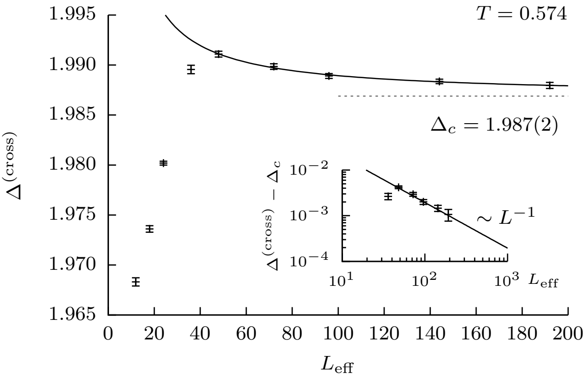
Ising universality for the random model across Δ

Mind the strong scaling corrections for $T = 0.574$!



$$(\xi/L)_\infty = \begin{cases} 0.9050488292(4) & \text{Ising model} \\ 0.906(2) & \text{pure BC model at } T = 1.398 \\ 0.905(2) & \text{random BC model at } T = 1.398 \\ 0.905(35) & \text{random BC model at } T = 0.574 \end{cases}$$

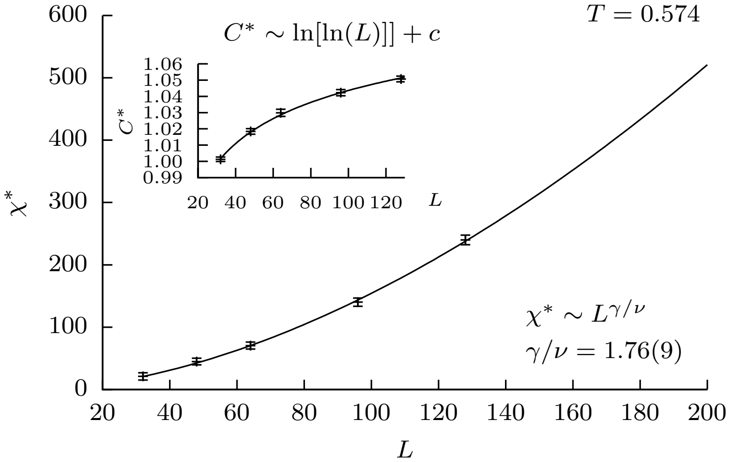
Critical exponent ν at the ex-first-order transition regime



$\Delta^{(\mathrm{cross})} = \Delta_c + bL^{-1/\nu} \to \Delta_c = 1.987(2)$ and $\nu = 1.01(25) \approx 1$.

$(\chi^2/\mathrm{DOF} = 0.48/2)$.

Other instances of Ising universality



N.G. Fytas, *et al.*, work in preparation.

Acknowledgements

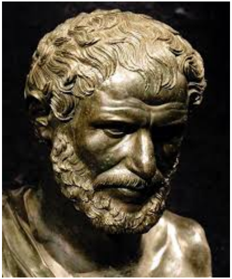
- Work in collaboration with Johannes Zierenberg (Göttingen), Panagiotis Theodorakis (Warsaw), Martin Weigel (Coventry), Wolfhard Janke (Leipzig), and Anastasios Malakis (Athens).

- Funding from the EU FP7 IRSES network DIONICOS under contract No. PIRSES-GA-2013-612707, Coventry University, and the Royal Society.

Acknowledgements

- Work in collaboration with Johannes Zierenberg (Göttingen), Panagiotis Theodorakis (Warsaw), Martin Weigel (Coventry), Wolfhard Janke (Leipzig), and Anastasios Malakis (Athens).

- Funding from the EU FP7 IRSES network DIONICOS under contract No. PIRSES-GA-2013-612707, Coventry University, and the Royal Society.



Heraclitus: *The fairest order in the world is a heap of random sweepings.*

Aristotle: *The whole is more than the sum of its parts.*