ON THE INTERFACE TENSION OF THE ISING MODEL

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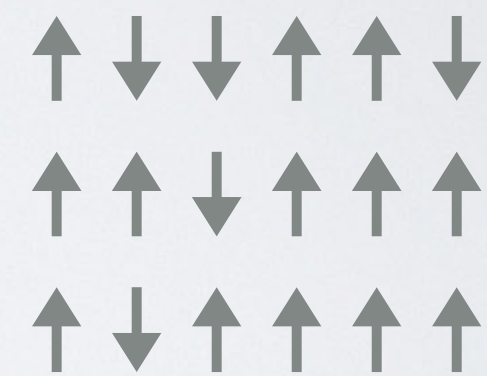
JG|U

Goal

- determine the interface tension for the two and three dimensional Ising model for various temperatures:

$$H = -\sum_{\langle ij \rangle} s_i s_j \quad s_i = \pm 1$$

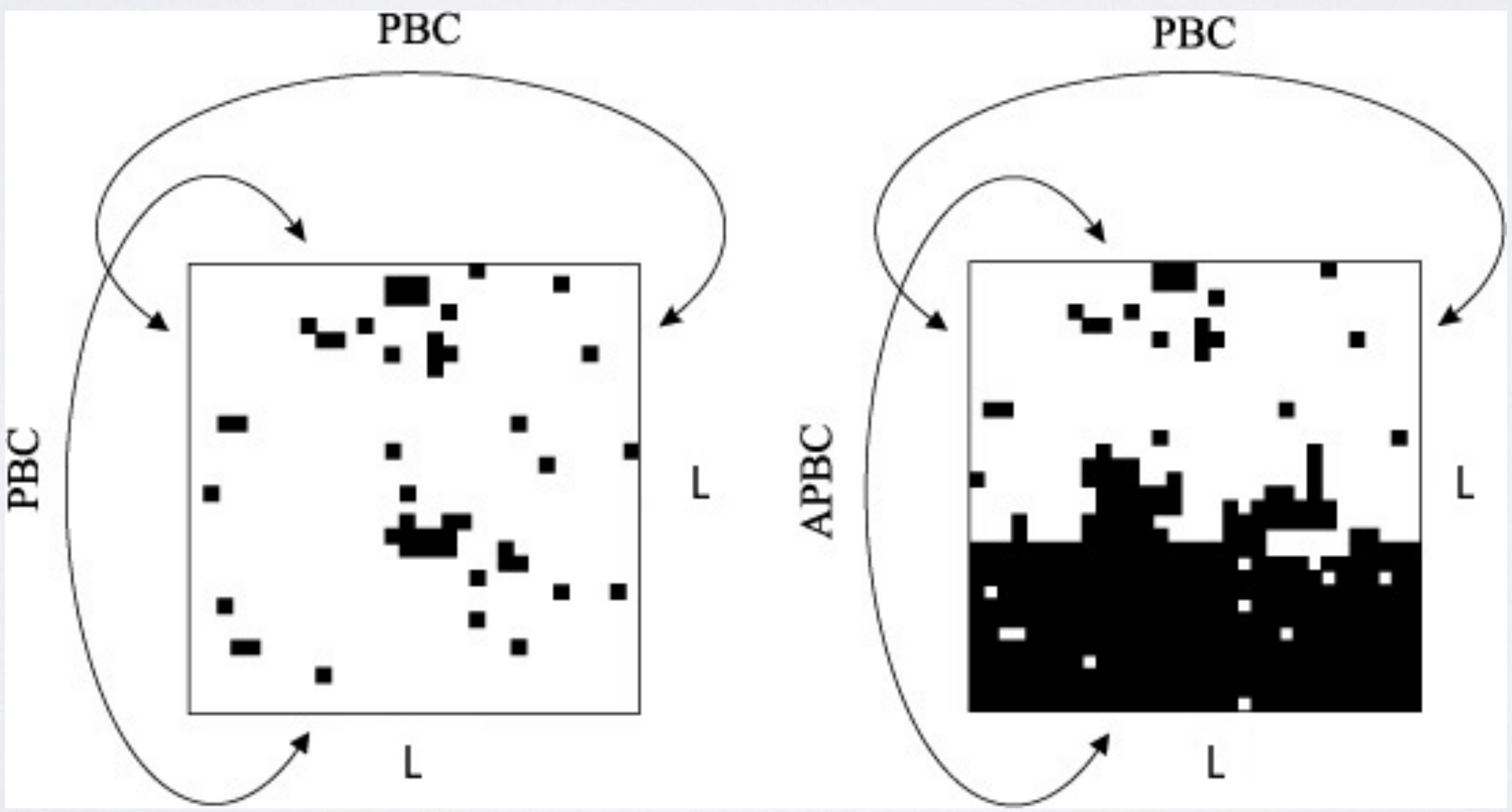


$$Z = \sum_{\{s_i\}} e^{-\beta H} = \sum_E \Omega(E)\ e^{-\beta E}$$

Interface tension

$$\sigma(T, L) = 1/(L^{d-1}T)(F^{\rm ap}(T, L) - F^{\rm pp}(T, L))$$

$$F^{B}(T, L) = -T\ln(Z) = -T\ln(\sum_{E}\Omega(E)e^{-\beta E})$$



two dimensional Ising model

to compare your numerical results we used

Exact finite-size scaling functions for the interfacial tensions of the Ising model on planar lattices

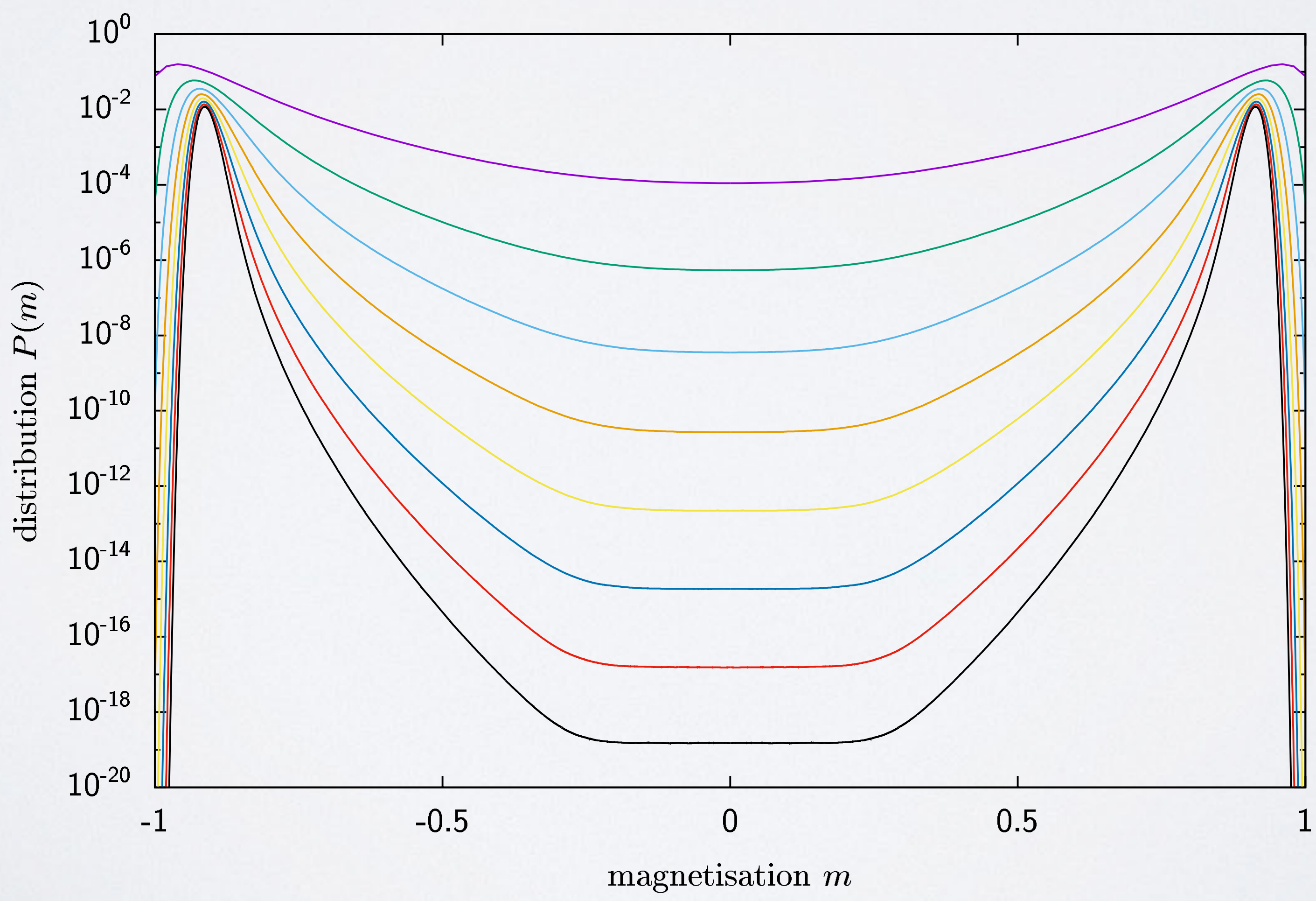
M-C Wu, PRE (2006) 046135

two dimensional Ising model

other definition of the interface tension:

$$\sigma(T,L) = \frac{T}{L^{d-1}} \ln \left(\frac{P_{\max}^{(L)}}{P_{\min}^{(L)}} \right)$$

Binder, Z. Phys. B43 (1981) 119; Phys. Rev. A25 (1982) 1699



L=10 - 80

T=2.0

two dimensional Ising model

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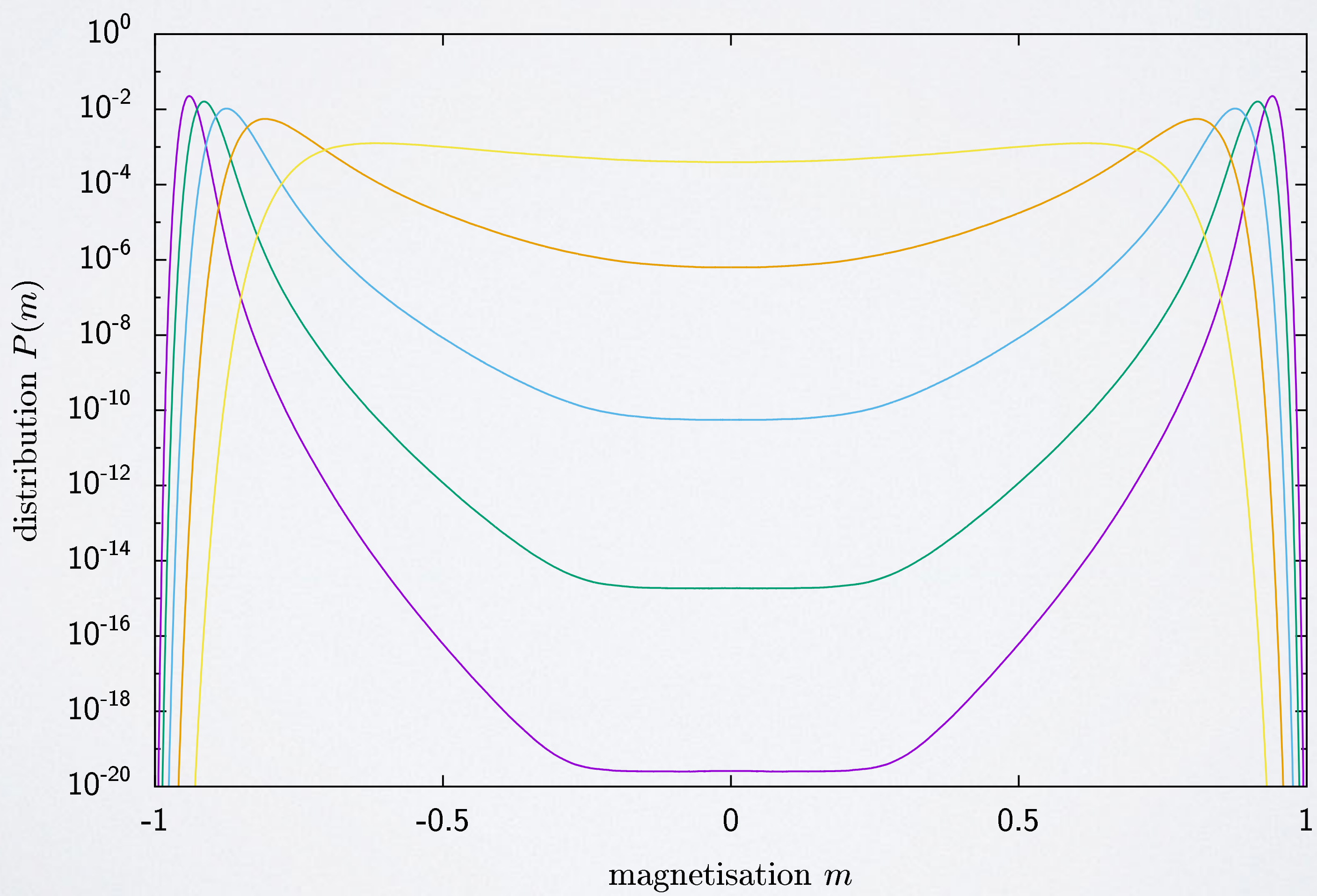

L=60

T=1.9 - 2.3

two dimensional Ising model

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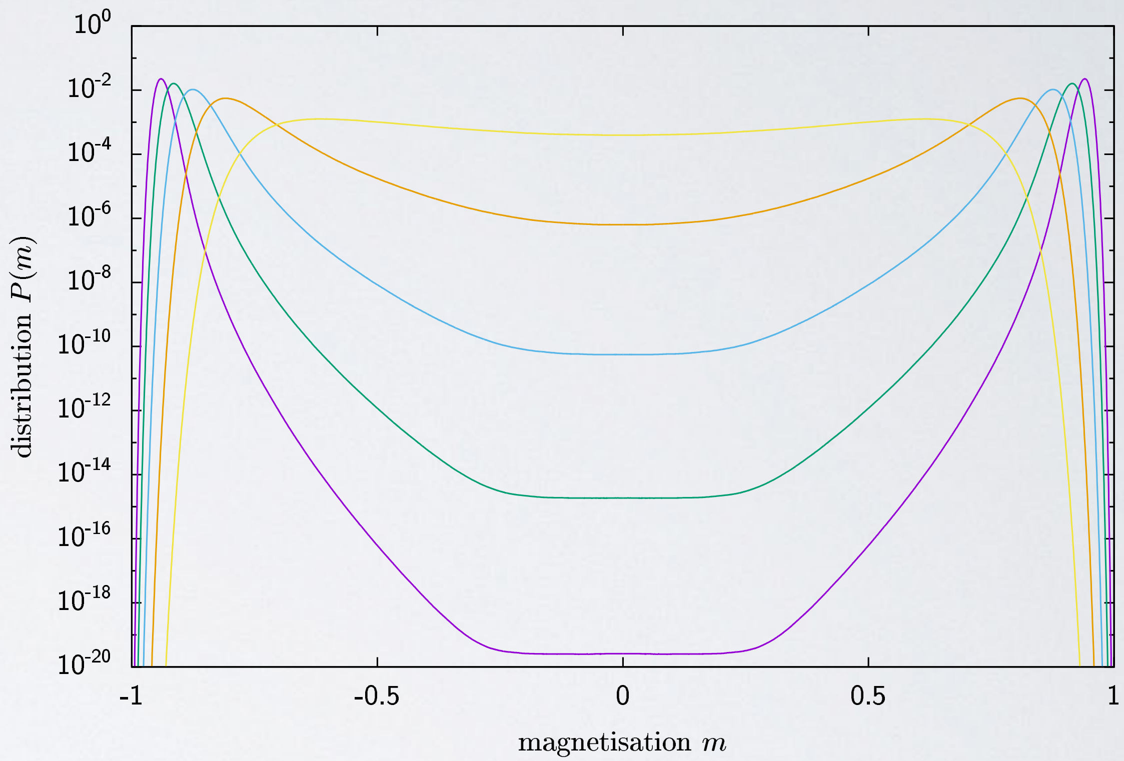


Parallel Tempering Multi-Magnetic Algorithm

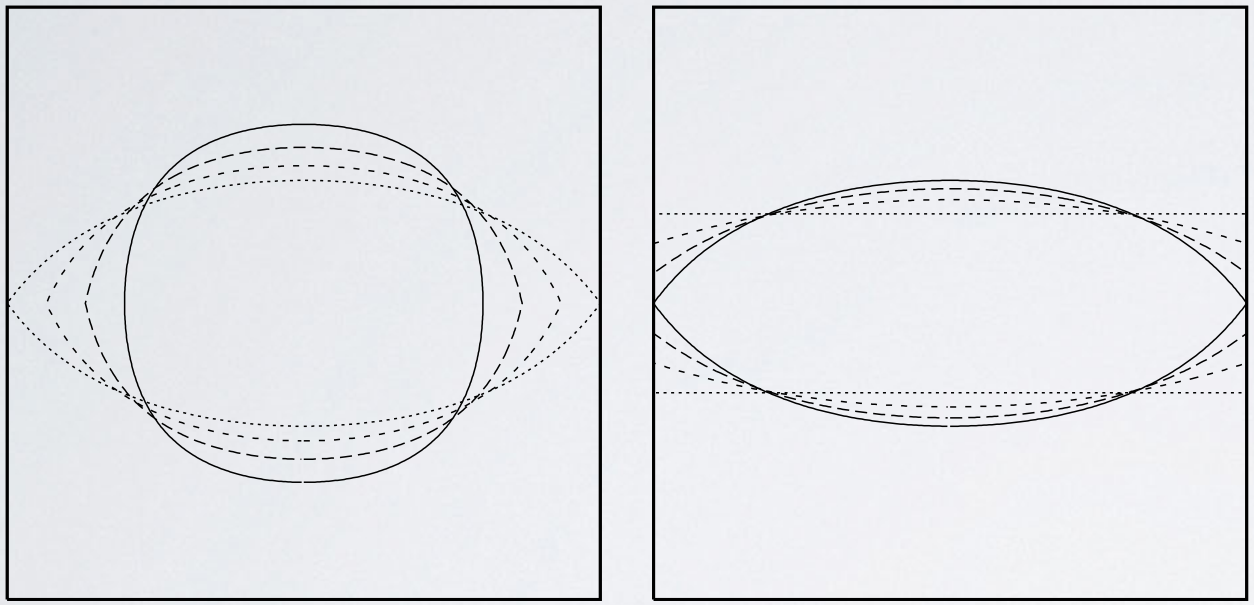
Bittner, Nußbaumer, and Janke, Nuclear Physics B 820 (2009), pp. 694

exponentially large barrier between the strip configuration and the droplet configuration

K. Leung and R. K. P. Zia, J. Phys. A: Math. Gen. 23 (1990) 4593

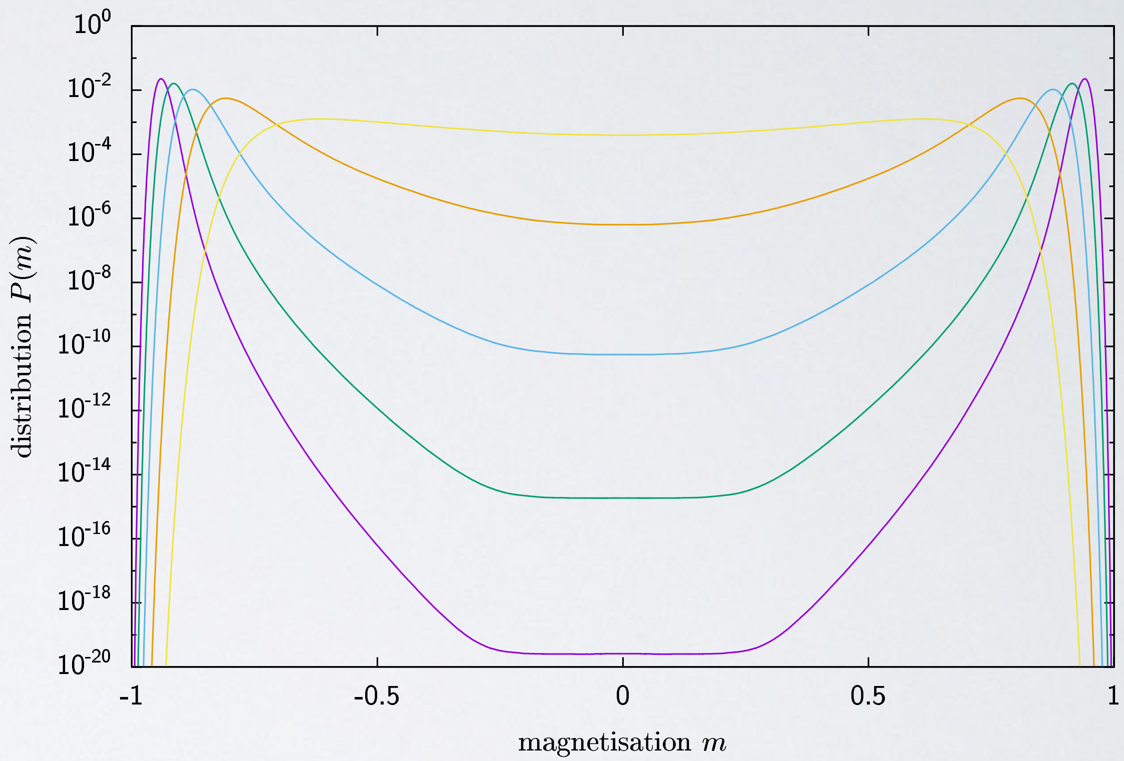
two dimensional Ising model

2D: T=1.0



(a) Droplet

(b) Stripe



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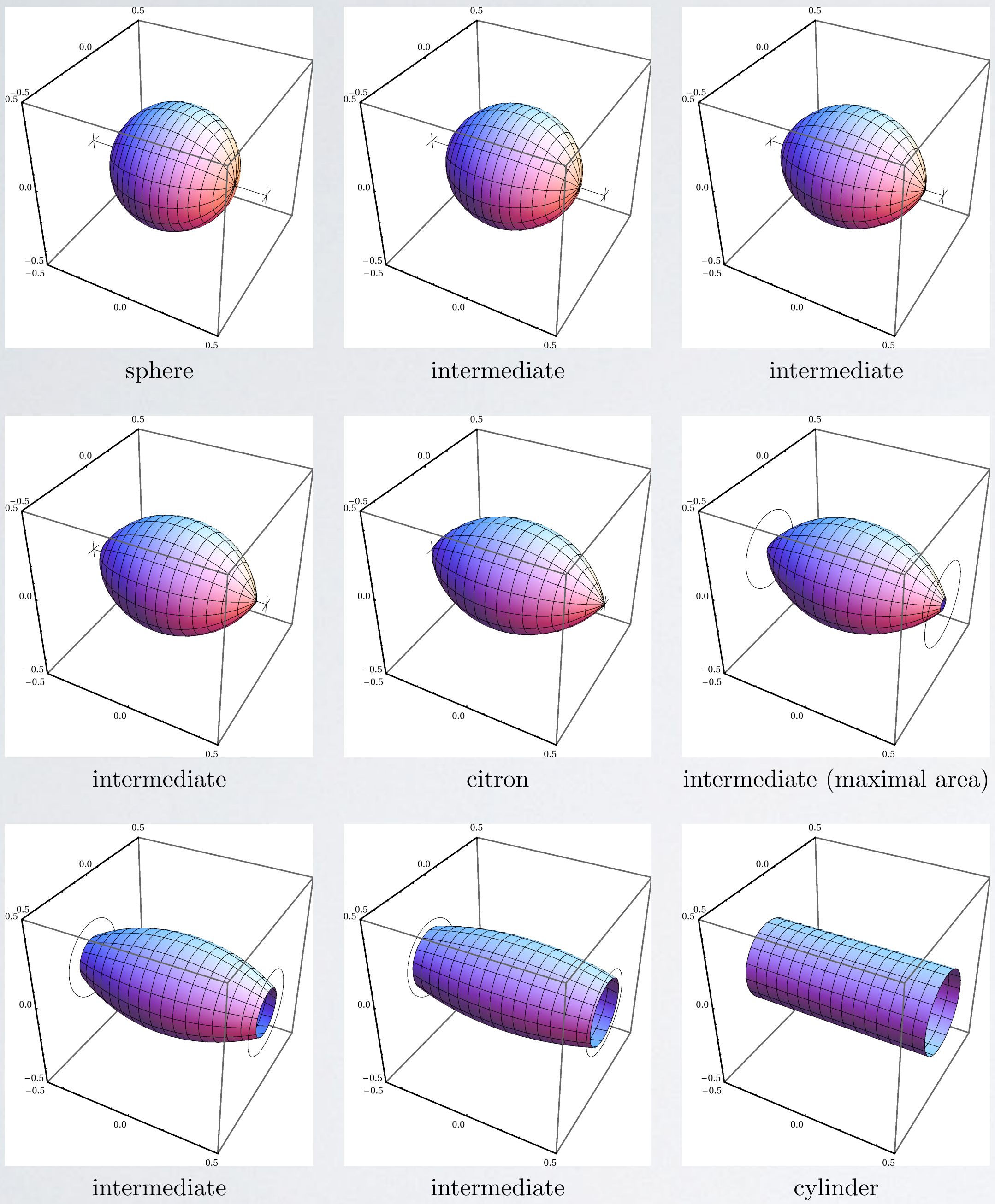
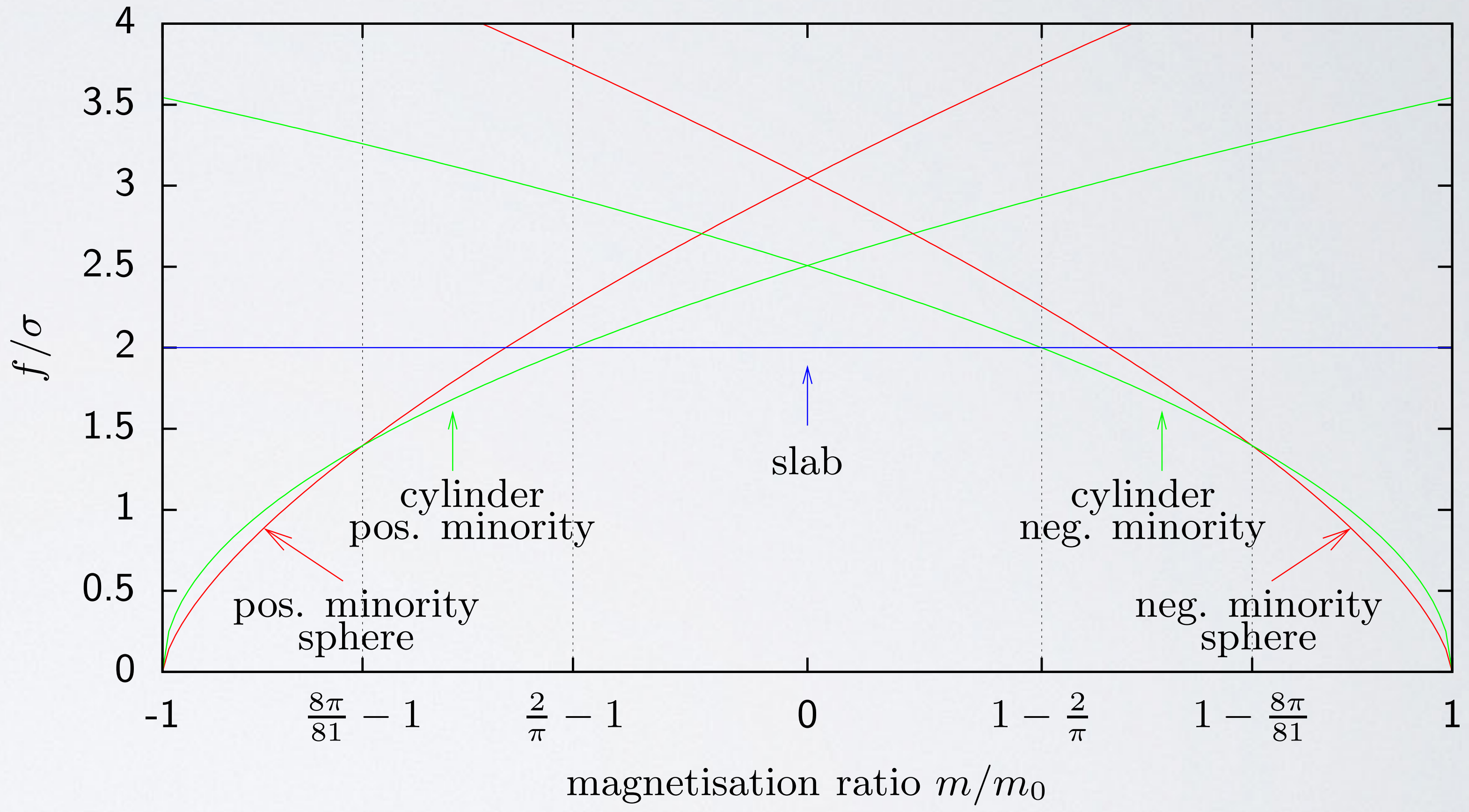
three dimensional Ising model





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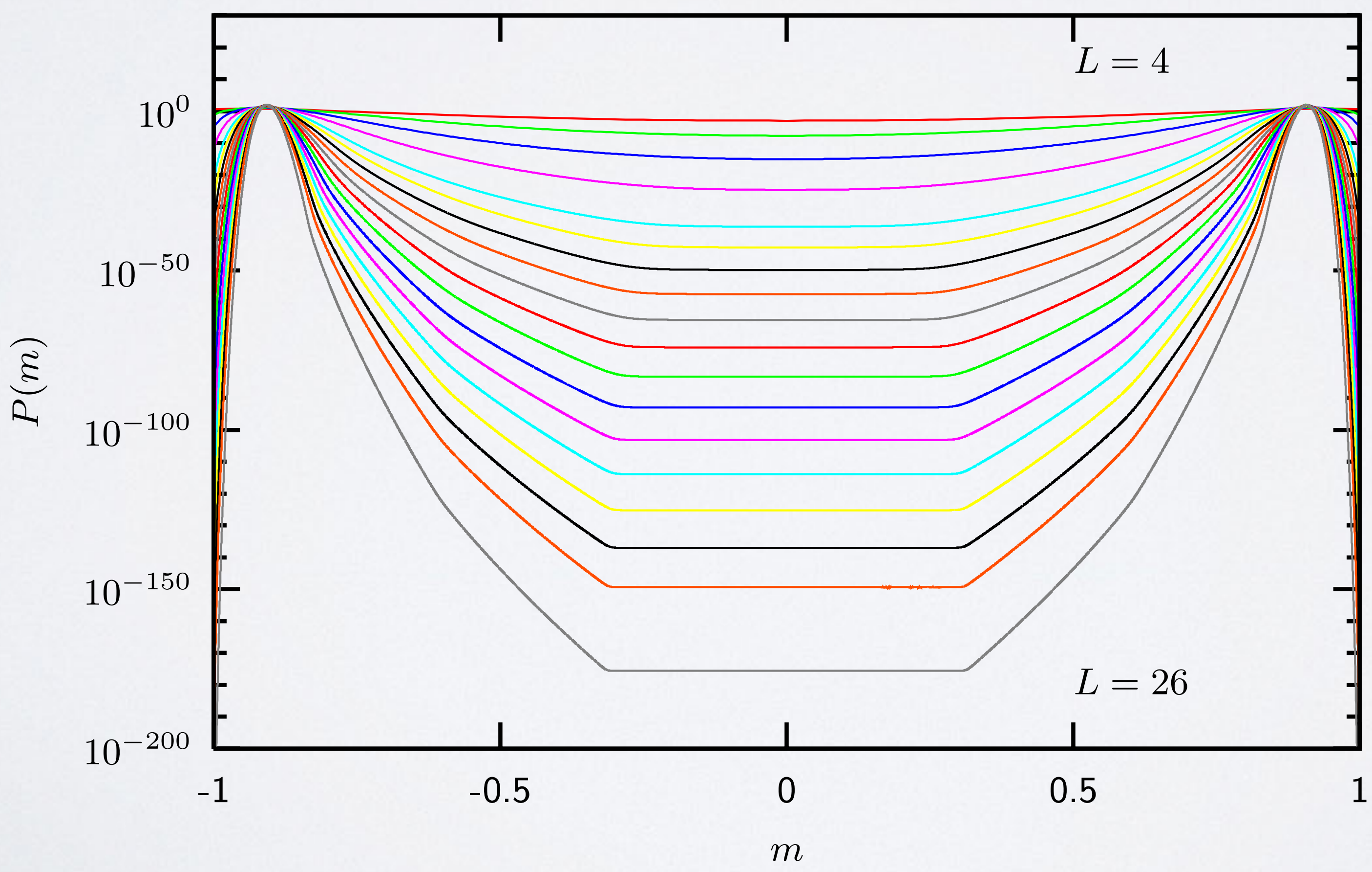
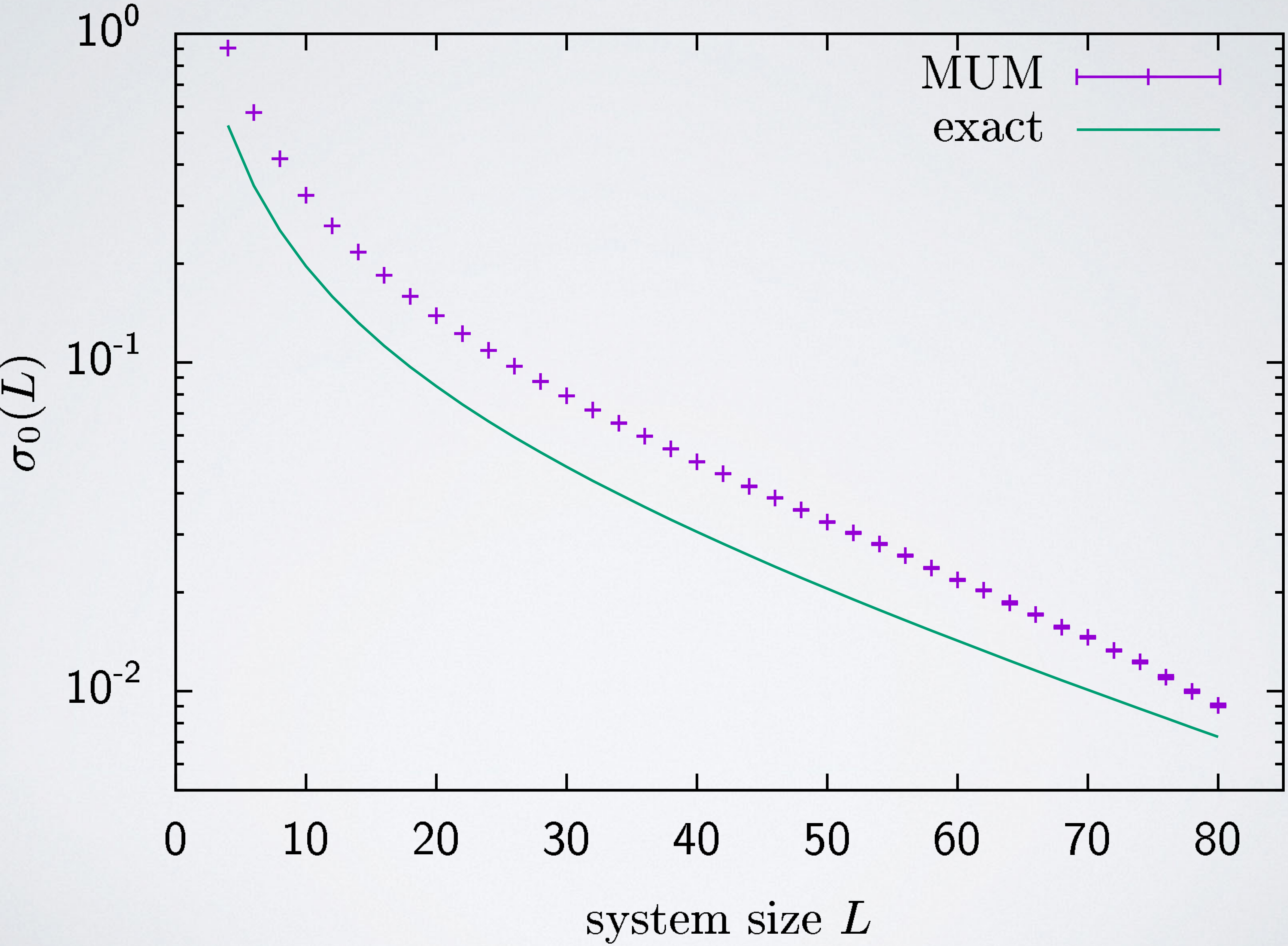


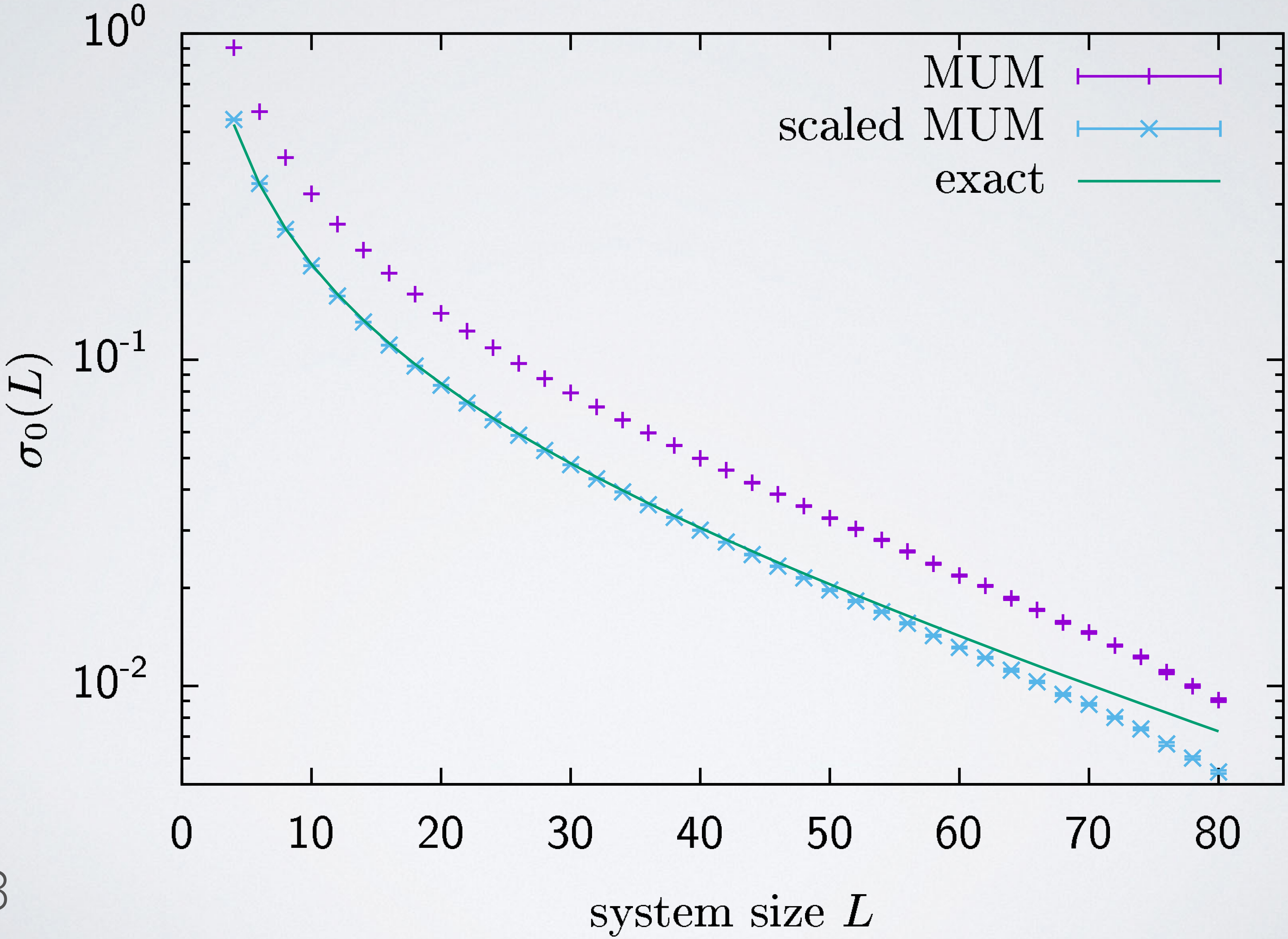
3d: T=3.333

two dimensional Ising model



T=2.3

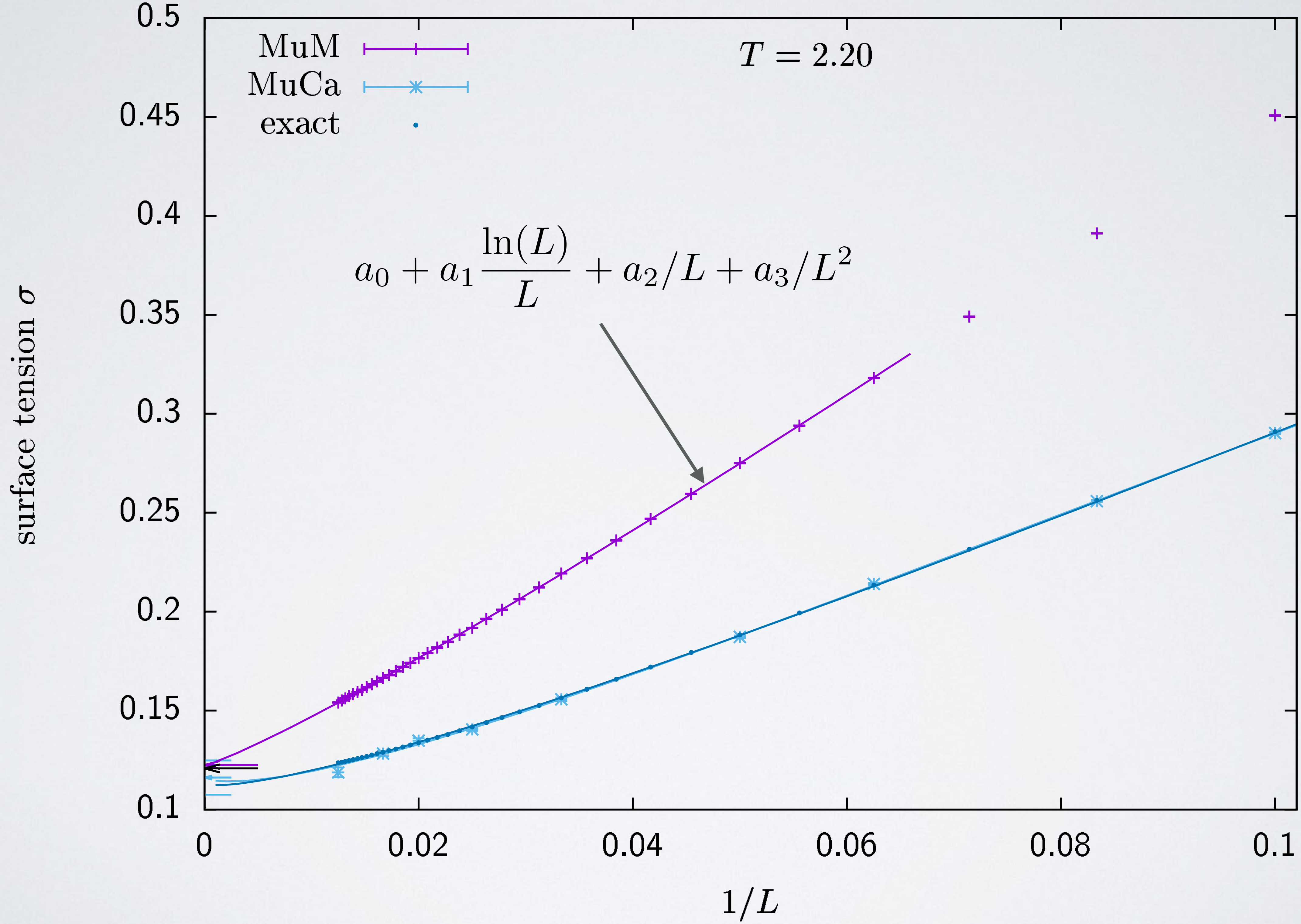
two dimensional Ising model



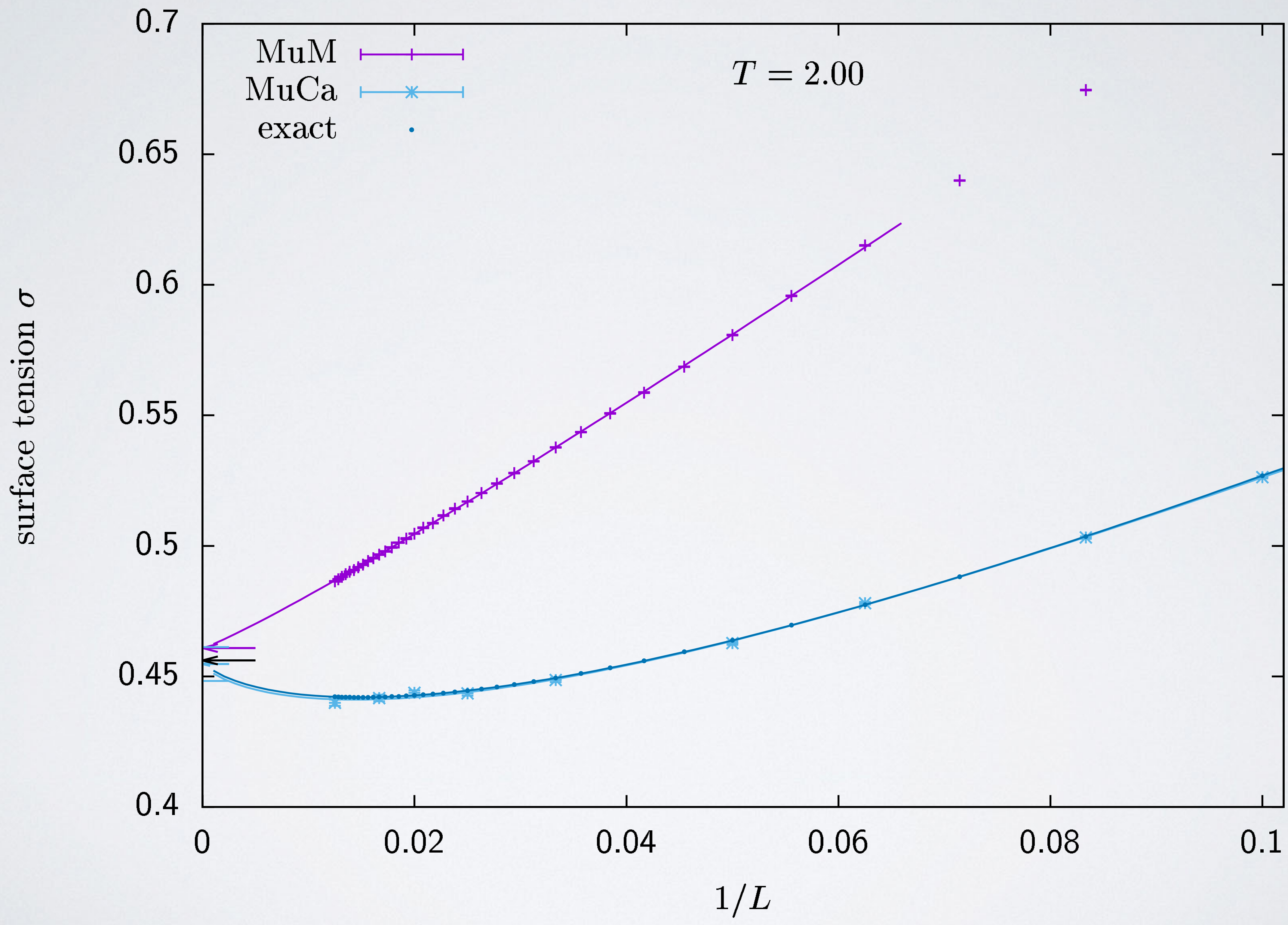
×0.60209

T=2.3

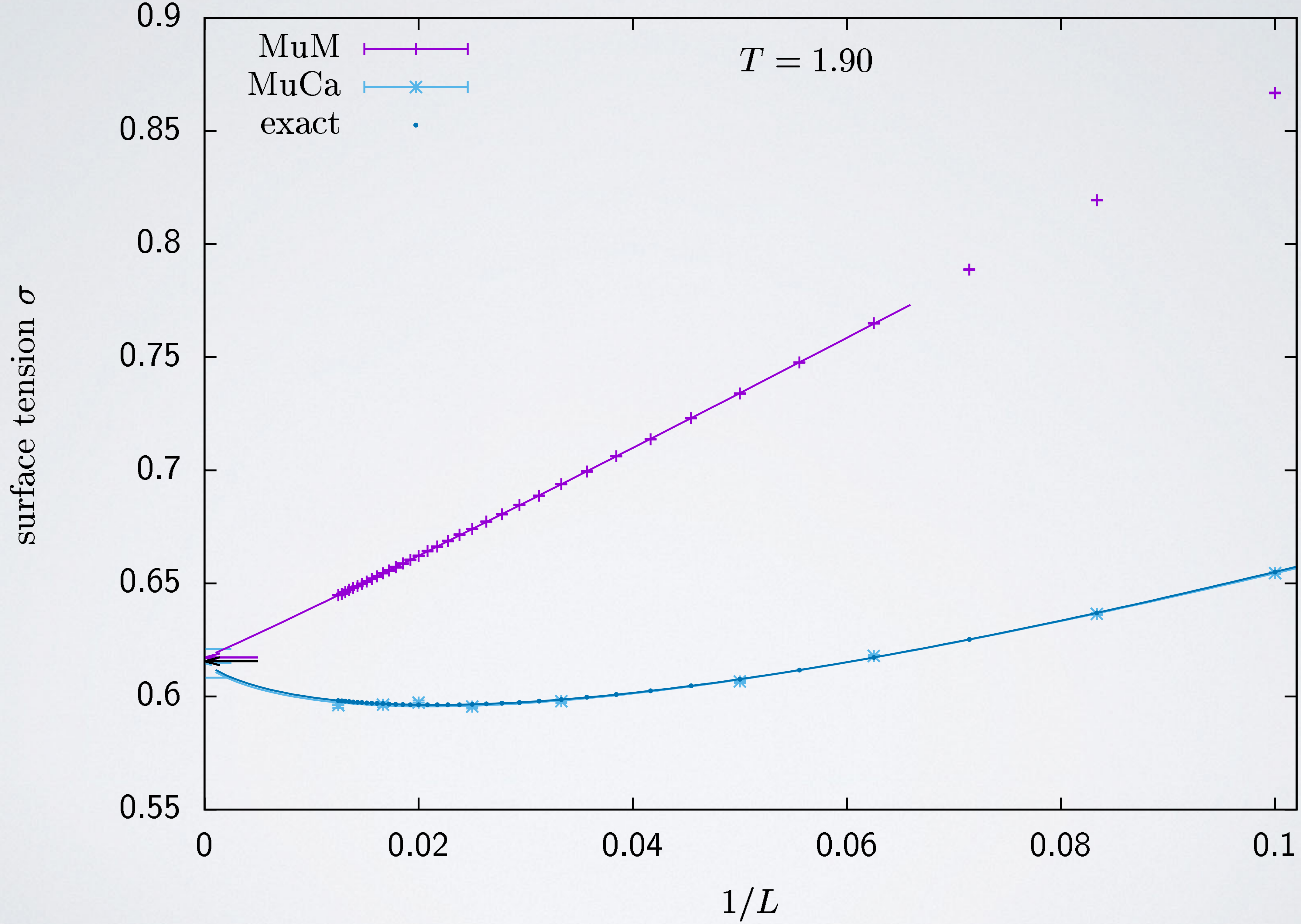
two dimensional Ising model



two dimensional Ising model



two dimensional Ising model



Multicanonical Monte Carlo Algorithm

in the MuCa method one constructs auxiliary weights $W(E)$

$$P_{\mathrm{muca}}(E) = P_{\mathrm{can},\beta}(E) W(E)$$

to construct the weights we use an accumulative recursion

defining the weight ratio $R(E) = \dfrac{W(E + \Delta E)}{W(E)}$

Multicanonical Monte Carlo Algorithm

1. set histogram $H(E)$ to zero, perform m update sweeps with $R(E)$ and measure $H(E)$
2. compute for each bin the statistical weight of the current run $p(E) = H(E)H(E+\Delta E)/[H(E)+H(E+\Delta E)]$
3. Accumulate statistics

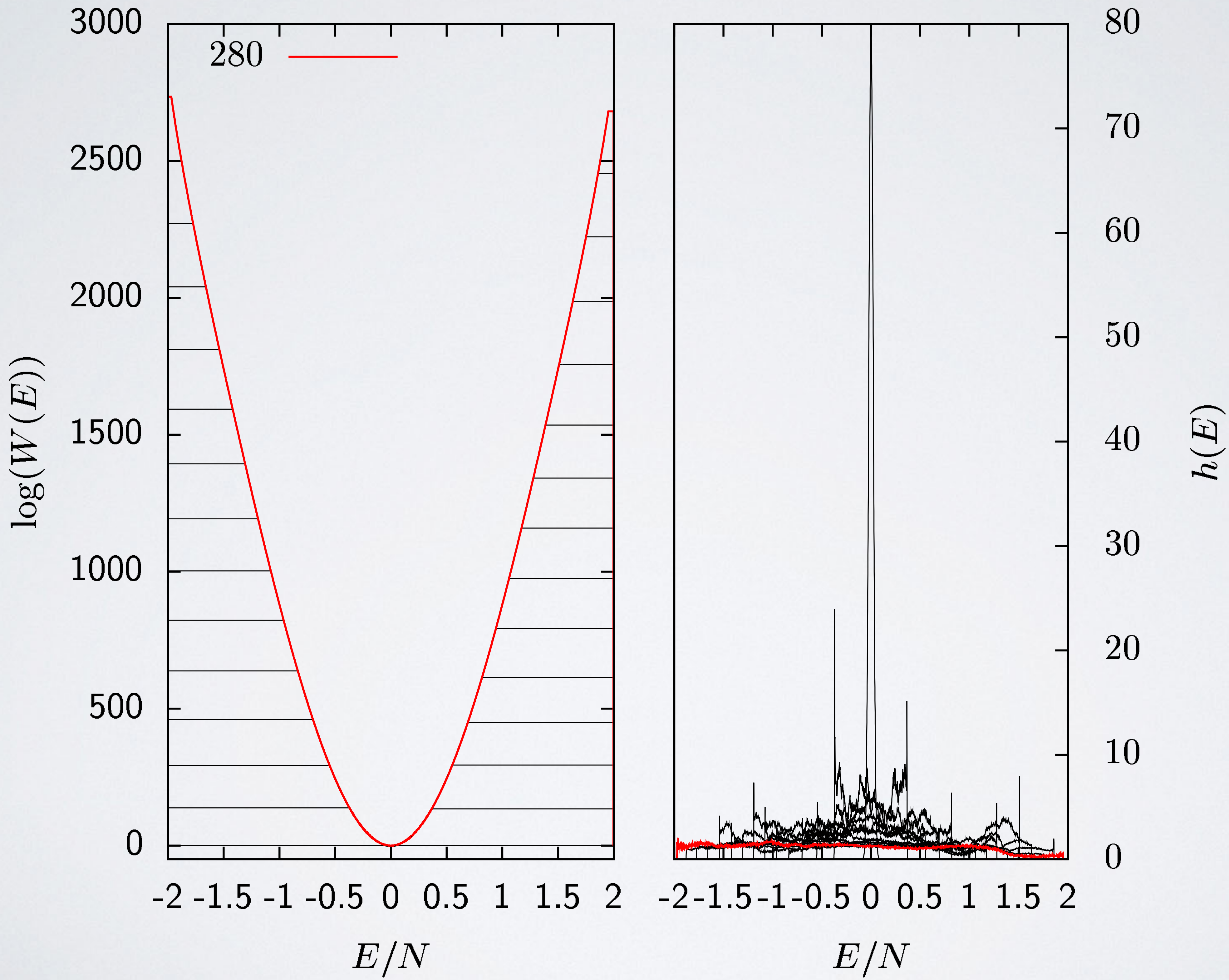
$$p_{n+1}(E) = p_n(E) + p(E)$$
$$\kappa(E) = p(E)/p_{n+1}(E)$$

4. Update weight ratios

$$R_{\mathrm{new}}(E) = R(E)\,[H(E)/H(E+\Delta E)]^{\kappa(E)}$$

set $R(E) = R_{\mathrm{new}}(E)$ and go to 1

Multicanonical Monte Carlo Algorithm



$L = 64$

two dimensional Ising model

- Get exact free energy for PBC:

Ferdinand and Fisher Phys. Rev. 185 (1969) 832

Beale Phys. Rev. Lett. 76 (1996) 78

- Get exact free energy for APBC:

Galluccio, Loebl, and Vondrák Phys. Rev. Lett. 84 (2000) 5924

a algorithm to calculate the density of states for a finite size two-dimensional Edwards-Anderson-Ising model with ±J couplings

```
Input file for ISING
#----------------------
# Lattice width and height:
4 4
```

```
#  Horizontal edge weights:
       1  1  1  1
       1  1  1  1
       1  1  1  1
       1  1  1  1
```

```
# Verticel edge weights:
  -1 -1 -1 -1
   1  1  1  1
   1  1  1  1
   1  1  1  1
```

two dimensional Ising model

Final results for a 4x4 lattice, composed from 1 finite fields
-------------------------------------g_E_exac-------------------------

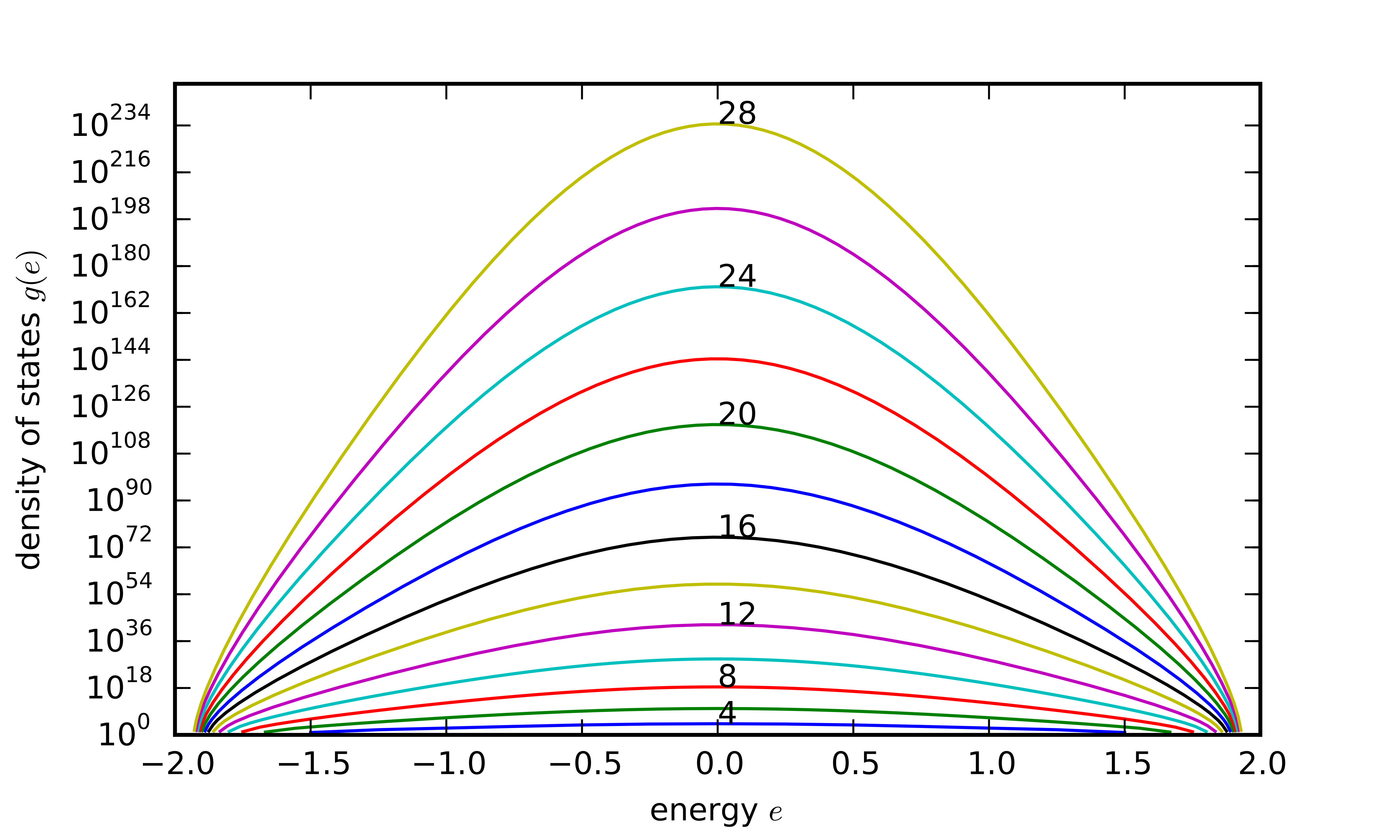
Total modulus = EA71

Energy	-32 ...	0	states
Energy	-28 ...	0	states
Energy	-24 ...	8	states
Energy	-20 ...	60	states
Energy	-16 ...	190	states
Energy	-12 ...	6E0	states
Energy	-8 ...	1978	states
Energy	-4 ...	38C0	states
Energy	0 ...	49E0	states
Energy	4 ...	38C0	states
Energy	8 ...	1978	states
Energy	12 ...	6E0	states
Energy	16 ...	190	states
Energy	20 ...	60	states
Energy	24 ...	8	states
Energy	28 ...	0	states
Energy	32 ...	0	states

In total ... 10000 states

Note the hexadecimal notation for the number of states.

two dimensional Ising model



Interface tension

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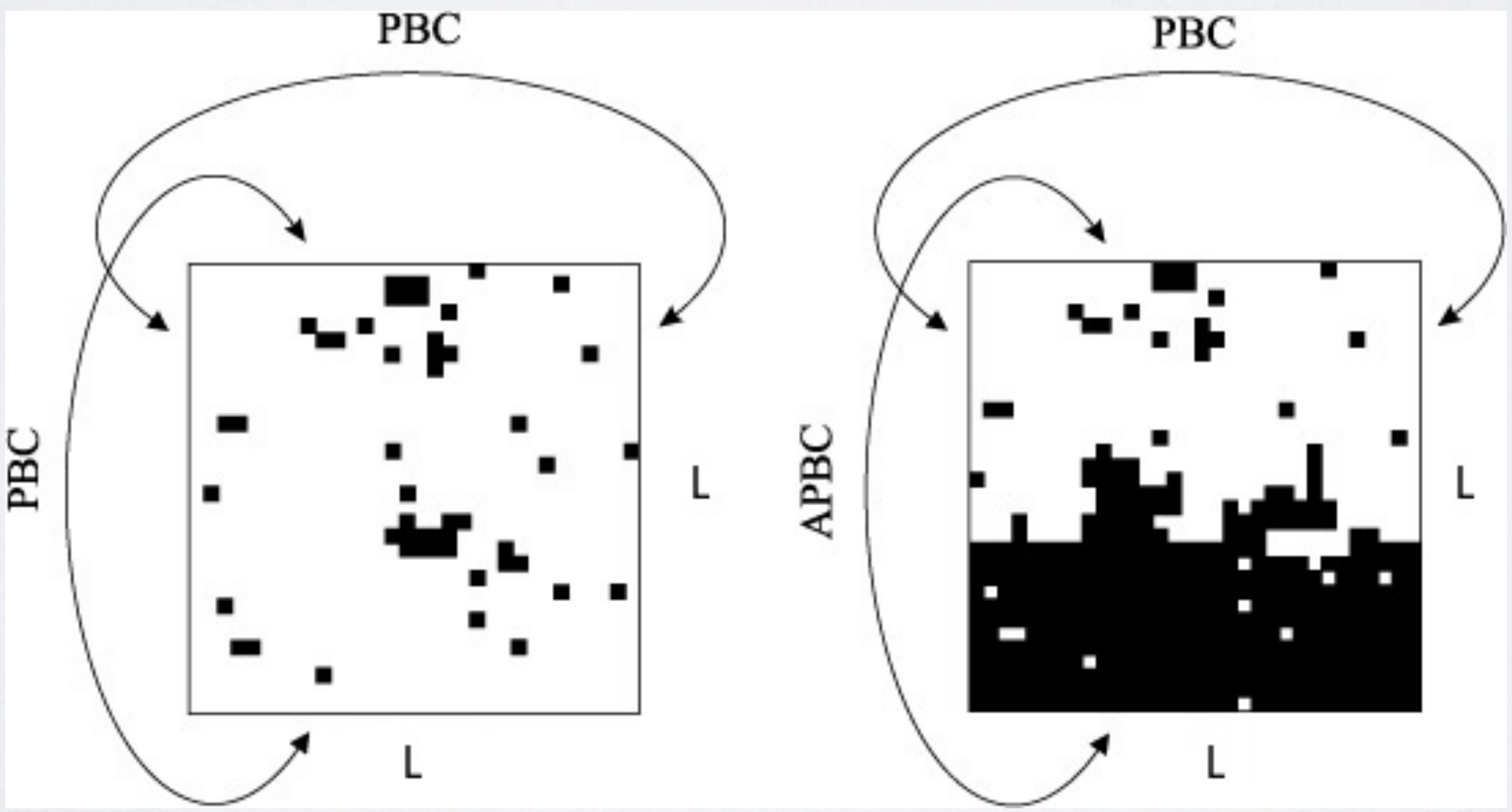

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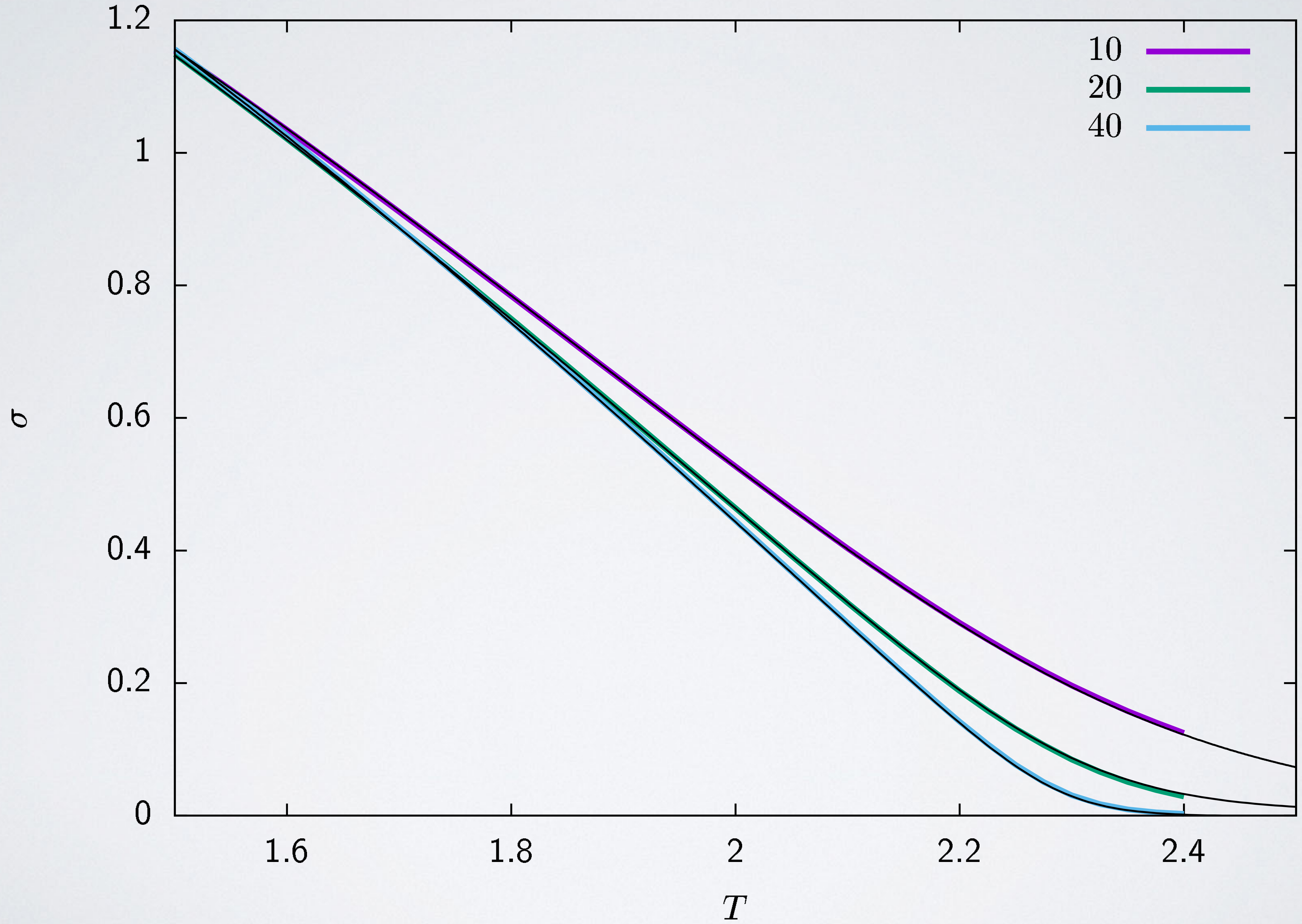
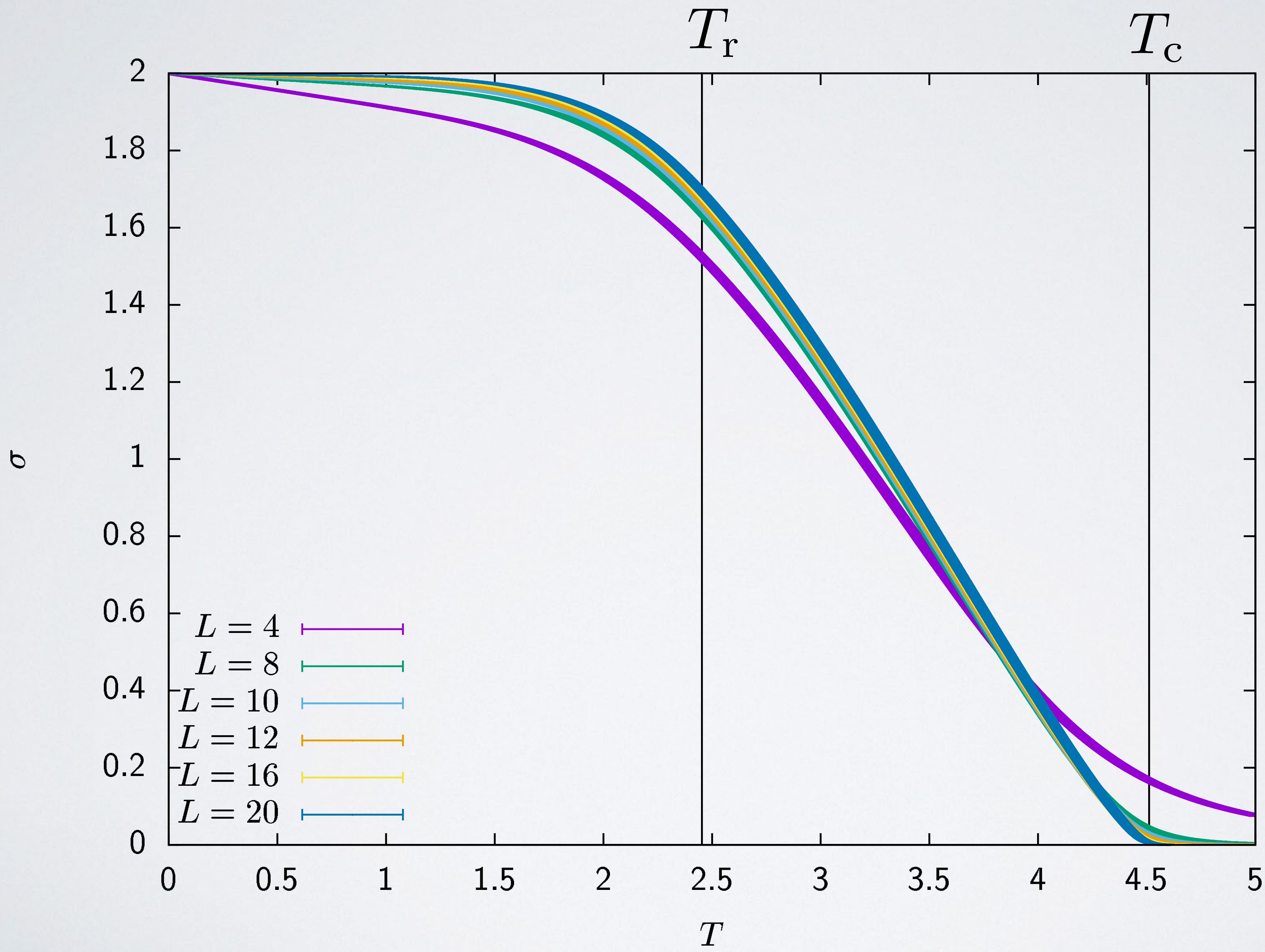


three dimensional Ising model



three dimensional Ising model

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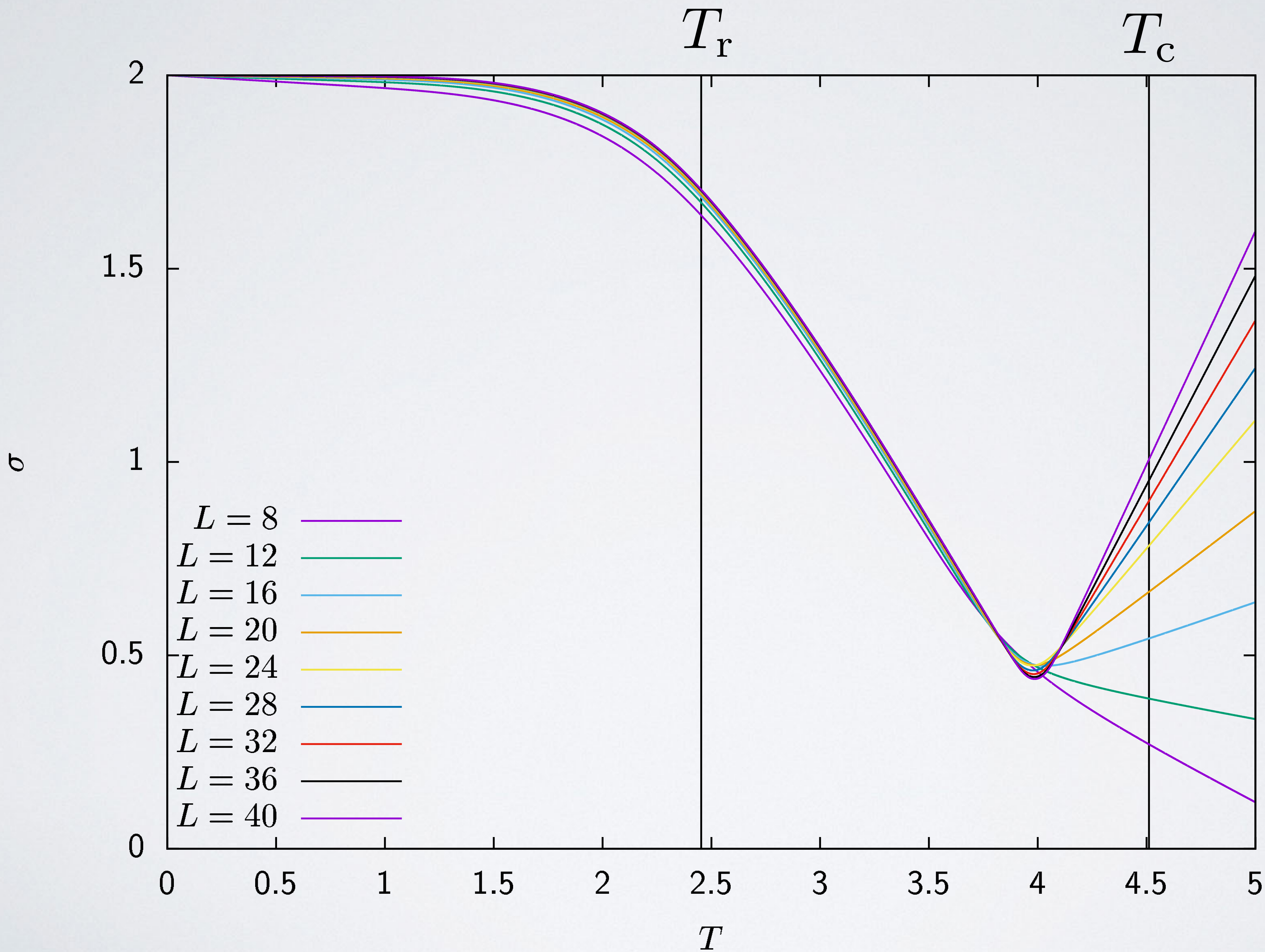

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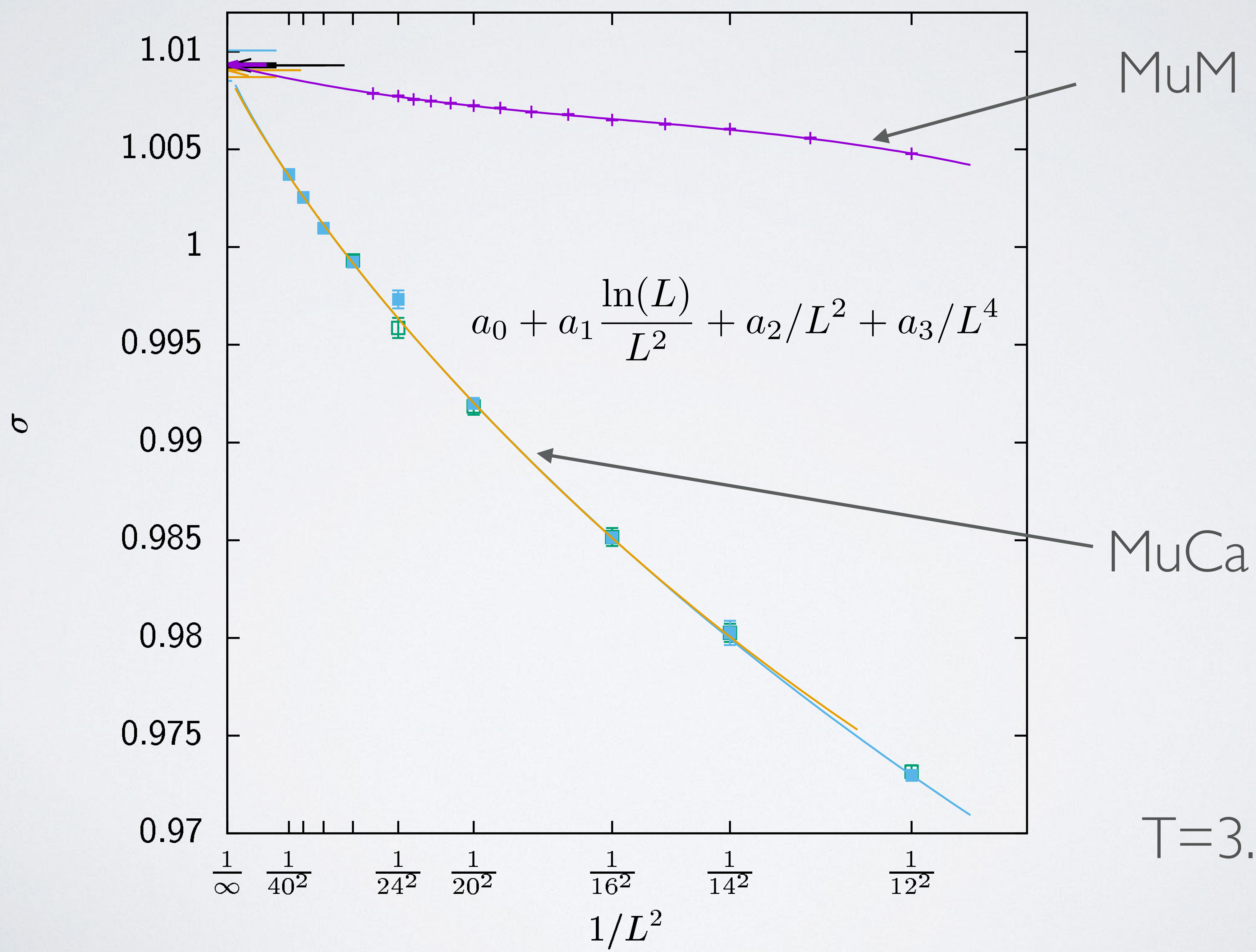
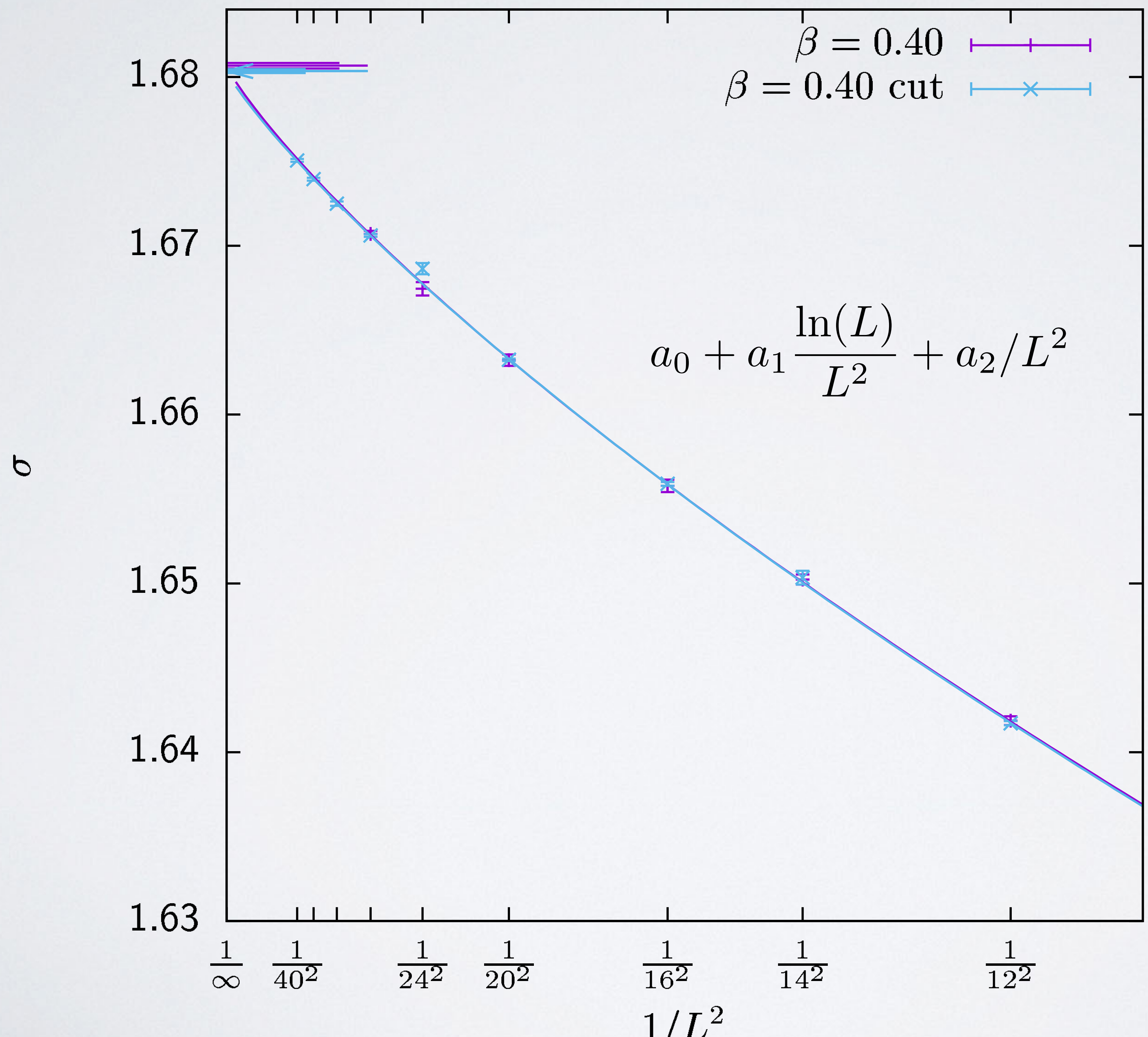


T=3.333

three dimensional Ising model



T=2.5

two and three dimensional Ising model

$$a_0 + a_1 \frac{\ln(L)}{L^2} + a_2/L^2$$

$$\sigma/T$$

Schmitz, Virnau, and Binder, Phys. Rev. Lett. 112 (2014) 125701
and PRE 90 (2014) 012128

$$a_0 + a_1 \frac{\ln(L)}{L} + a_2/L + a_3/L^2$$

two and three dimensional Ising model

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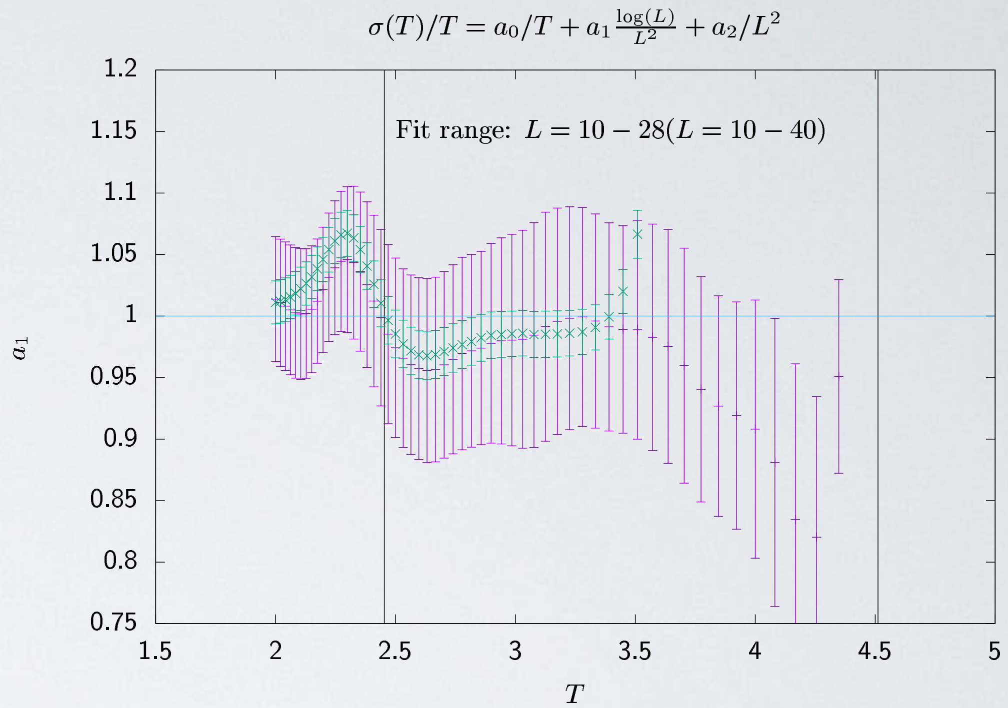

σ/T

Schmitz, Virnau, and Binder, Phys. Rev. Lett. 112 (2014) 125701
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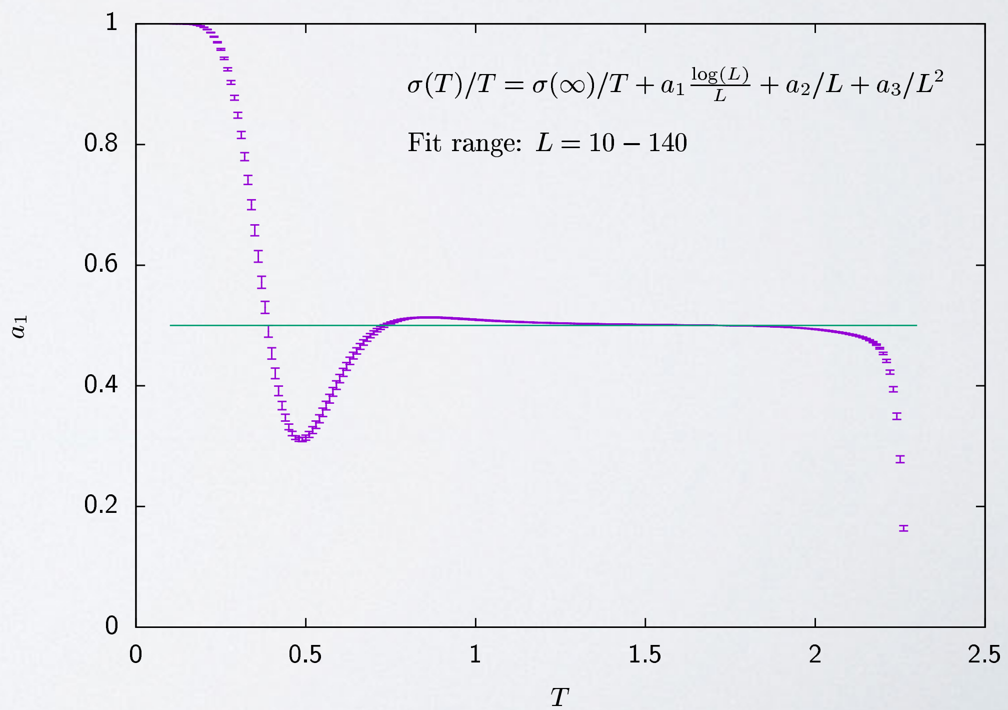
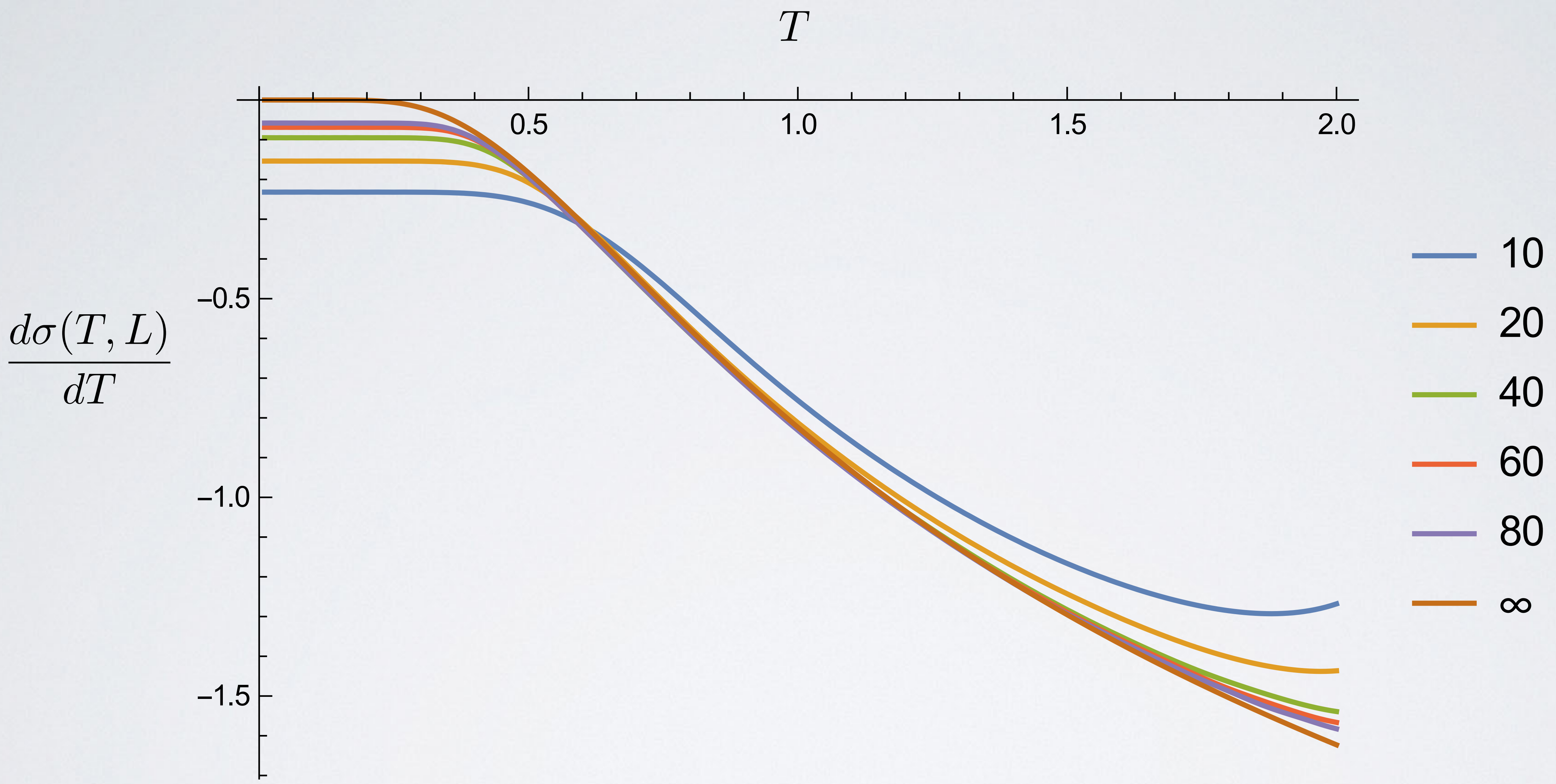


two dimensional Ising model



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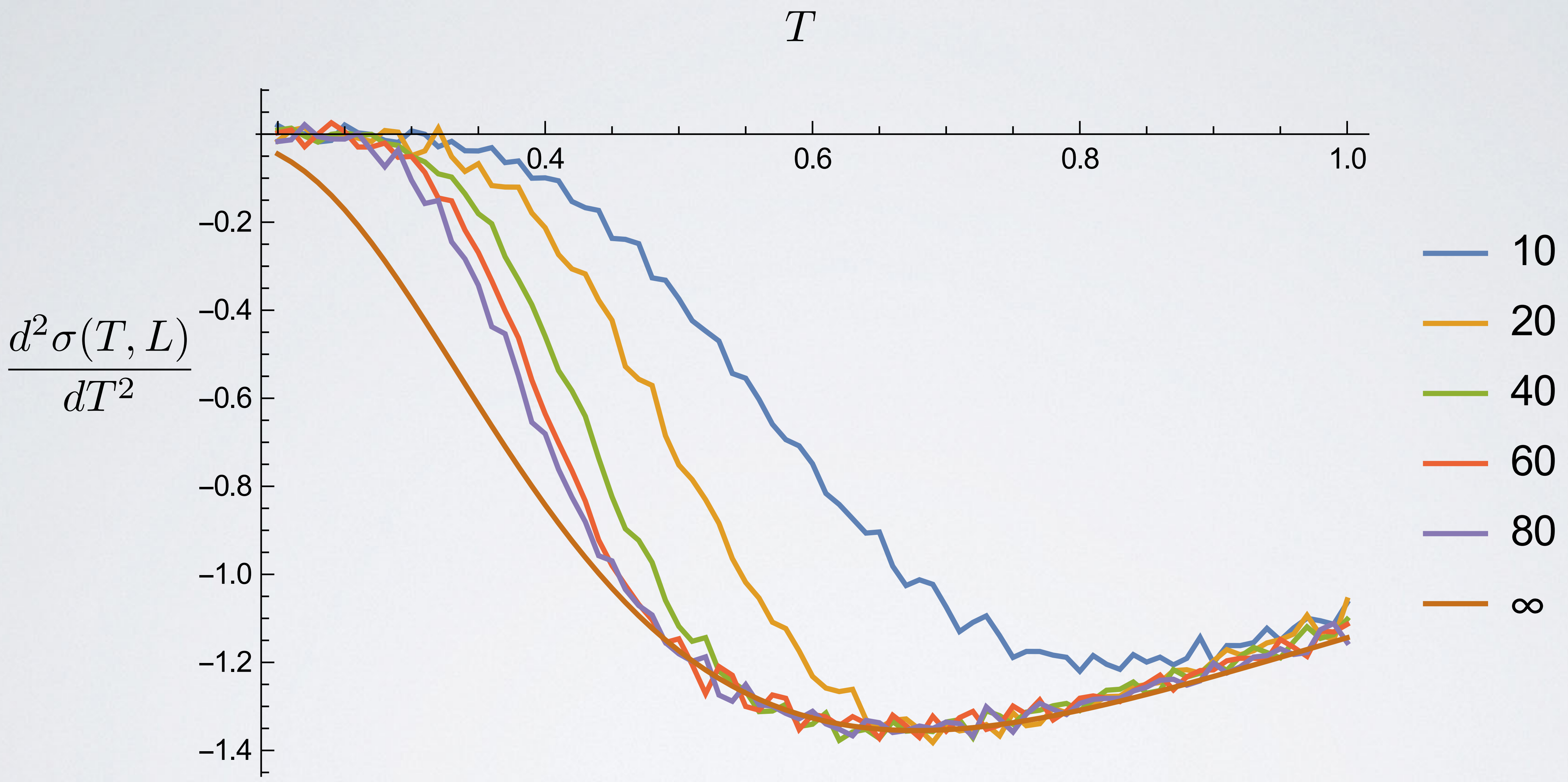

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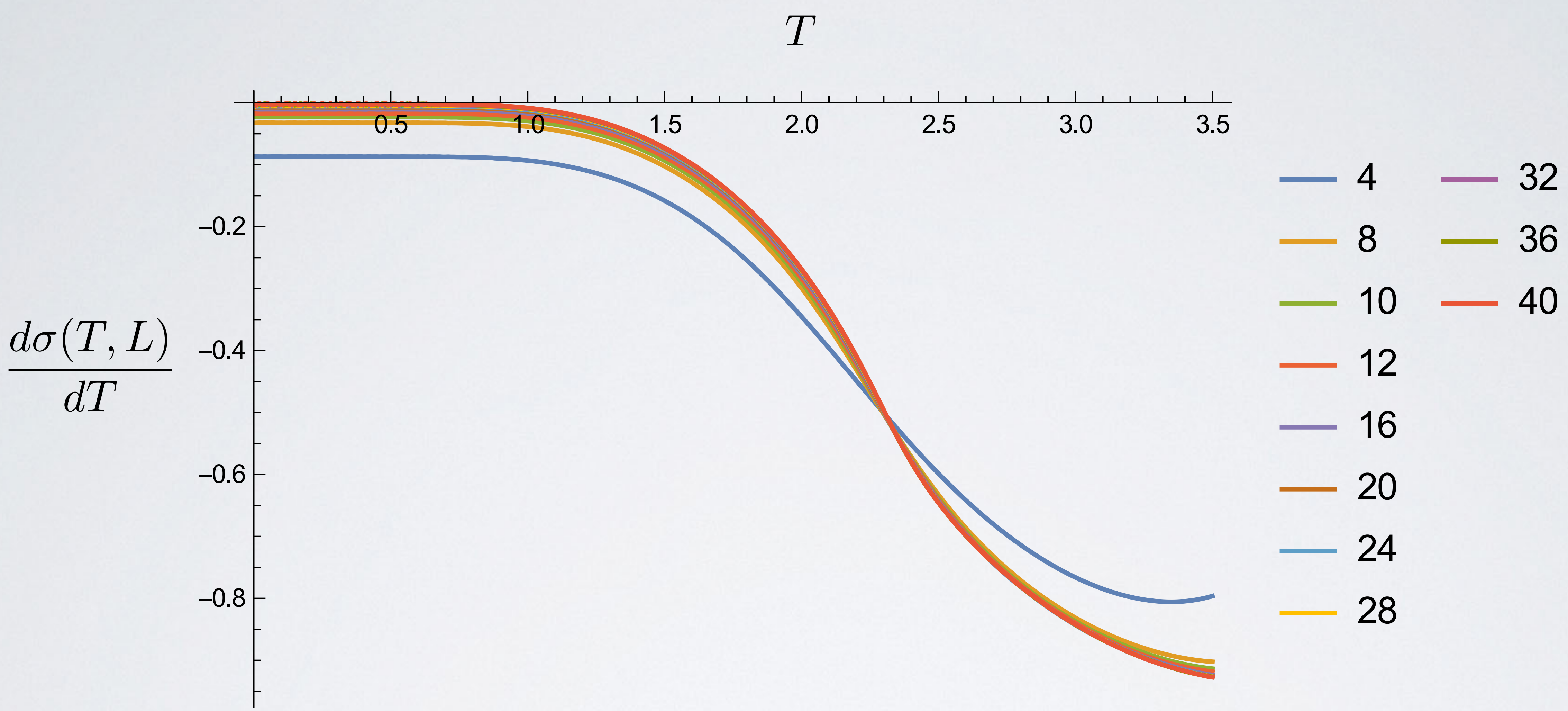
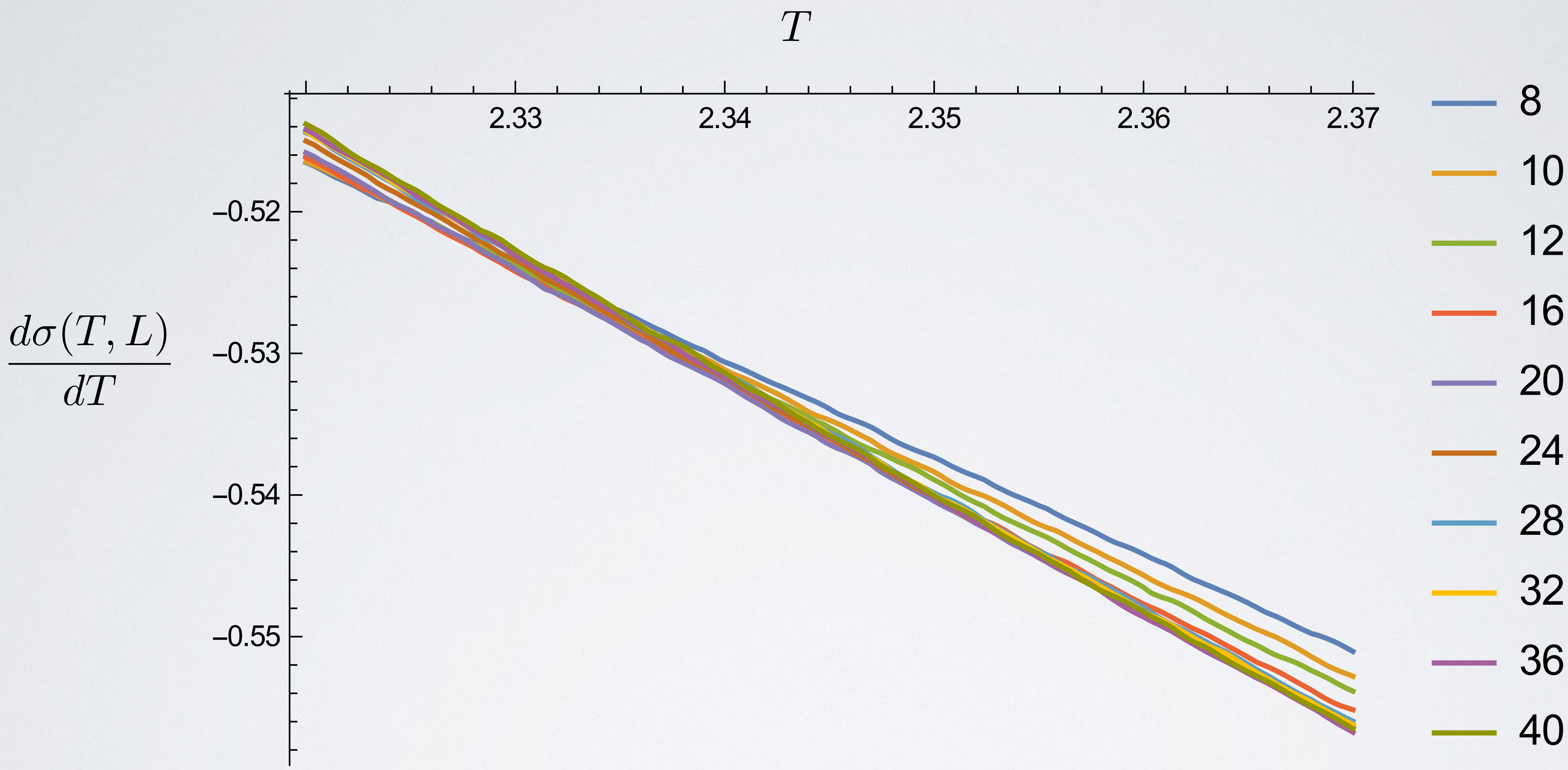


$$T_{\mathrm{R}} \approx 2.4535$$

Hasenbusch, Meyer, and Pütz, Journal of Statistical Physics 85 (1996) 383

Hasenbusch and Pinn, J. Phys. A: Math. Gen. 30 (1997) 63

three dimensional Ising model



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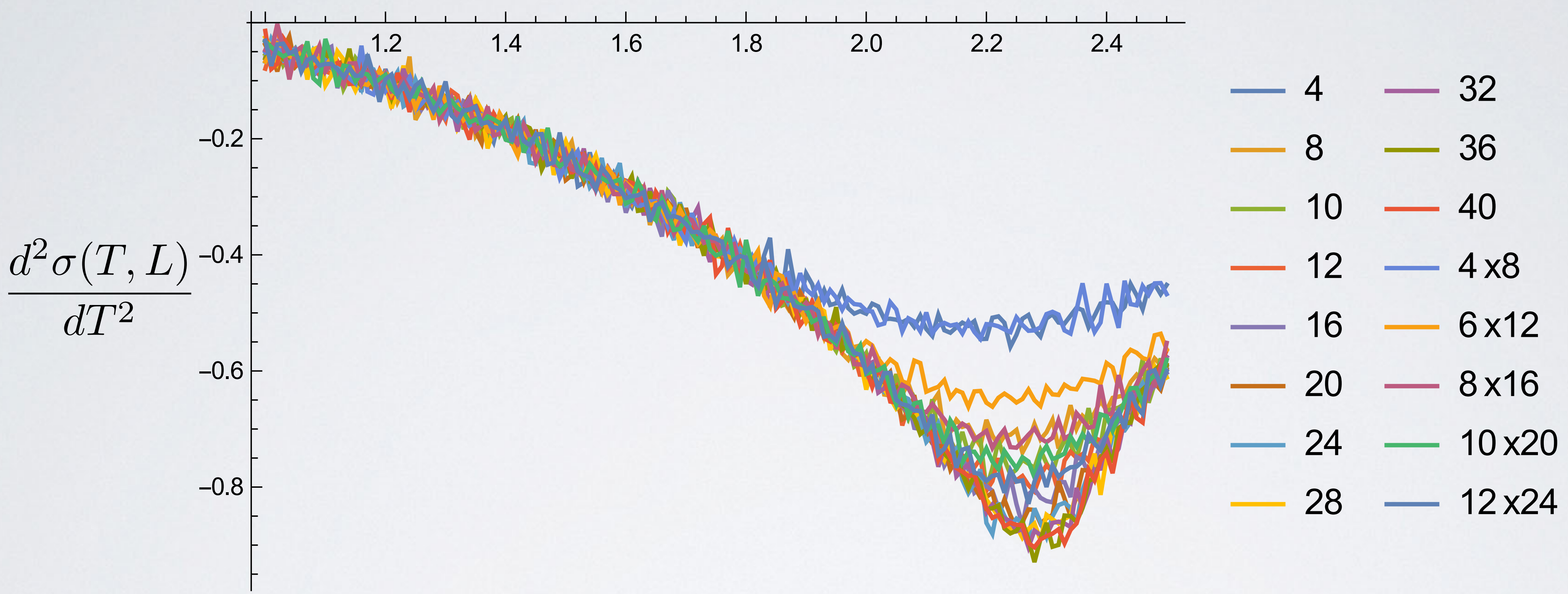

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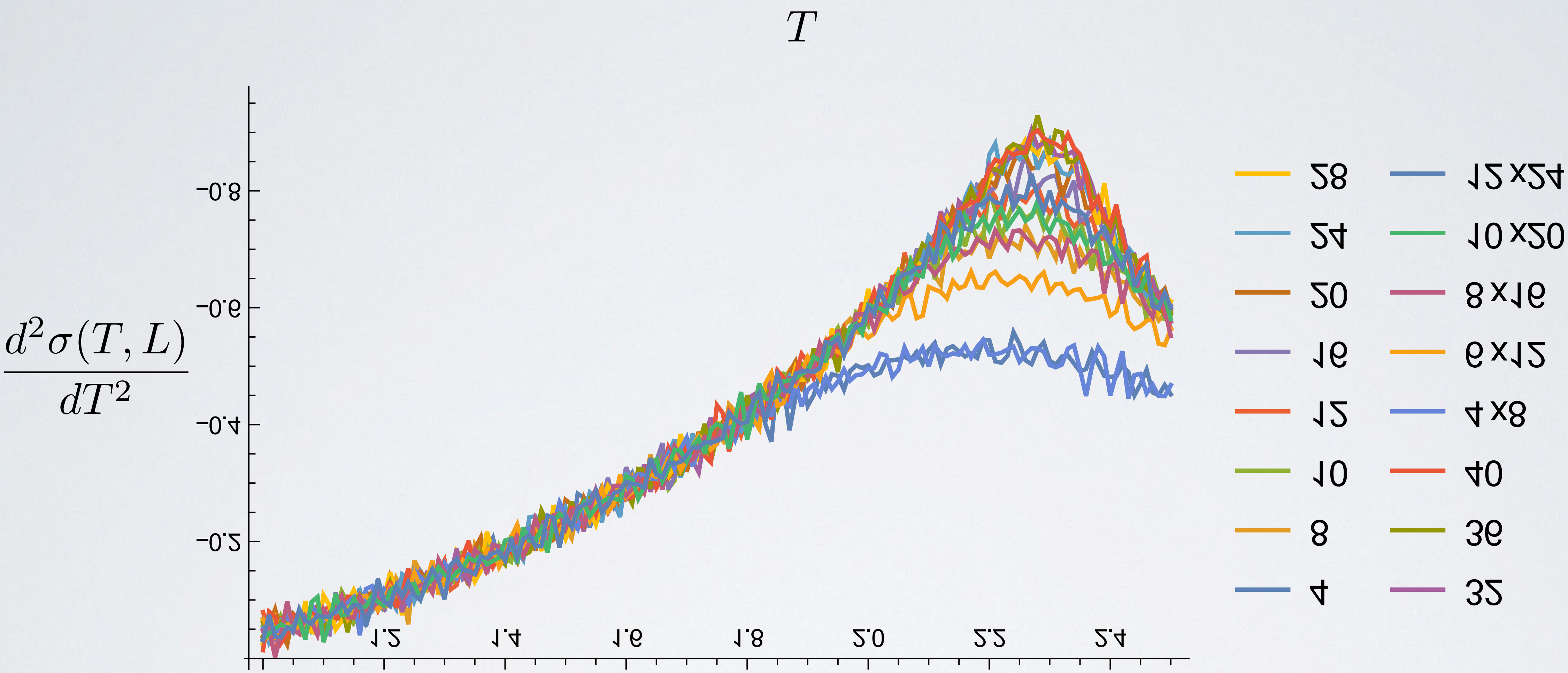

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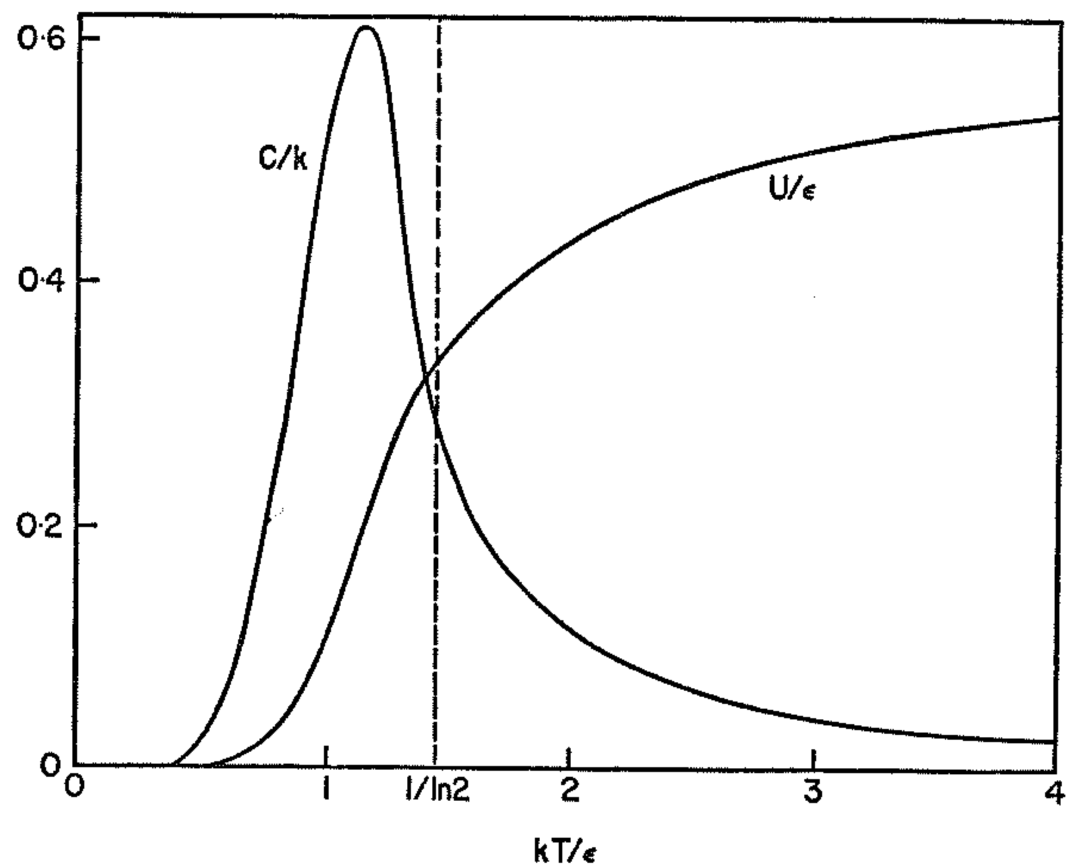


FIG. 16. Energy and specific heat per vertex, U and C, of the F model ($\varepsilon_1 = 0$, $\varepsilon_2 = \varepsilon > 0$) in zero field. The transition temperature is $T_0 = \varepsilon/(k \ln 2)$.

E.H. Lieb, Phys.Rev. Lett. 18 (1967) 1046

Summery

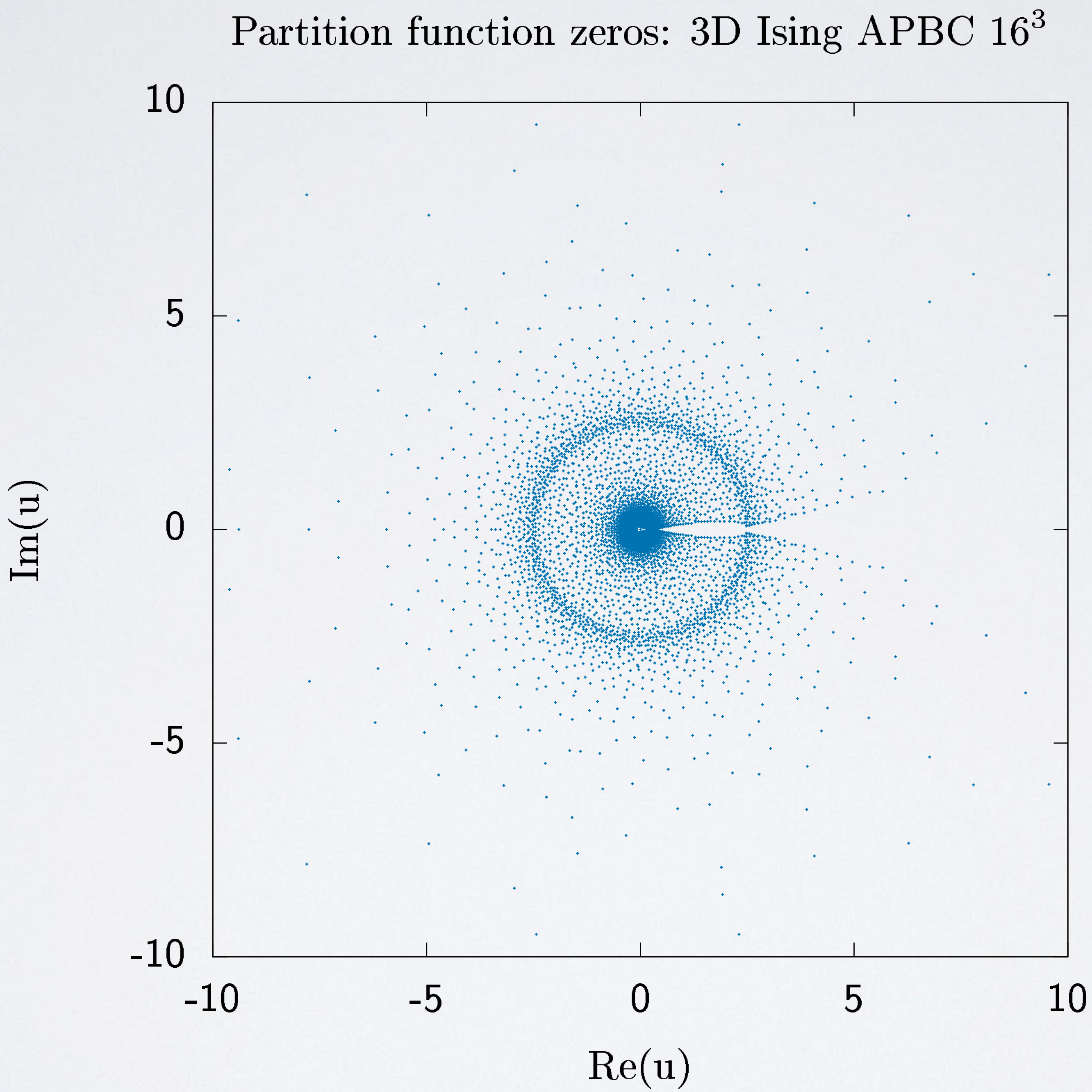
- We determined the interface tension for the two and three dimensional Ising model
- We compared the FSS of the interface tension measured with different methods

Summery & open questions

- We determined the interface tension for the two and three dimensional Ising model
- We compared the FSS of the interface tension measured with different methods

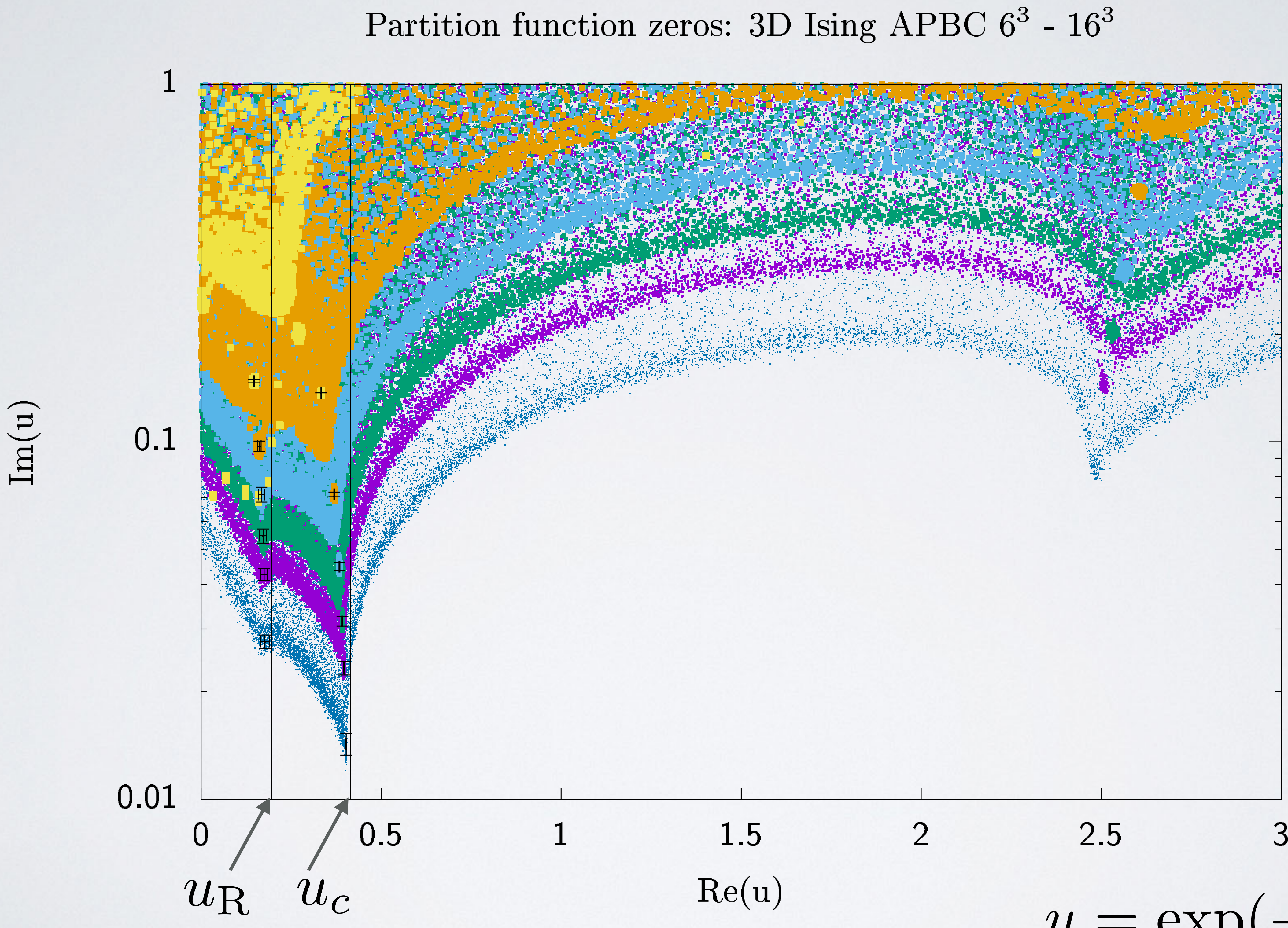
- How can we determine roughening transition?

Summery & open questions & outlook



$$u = \exp(-4\beta J)$$

Summery & open questions & outlook



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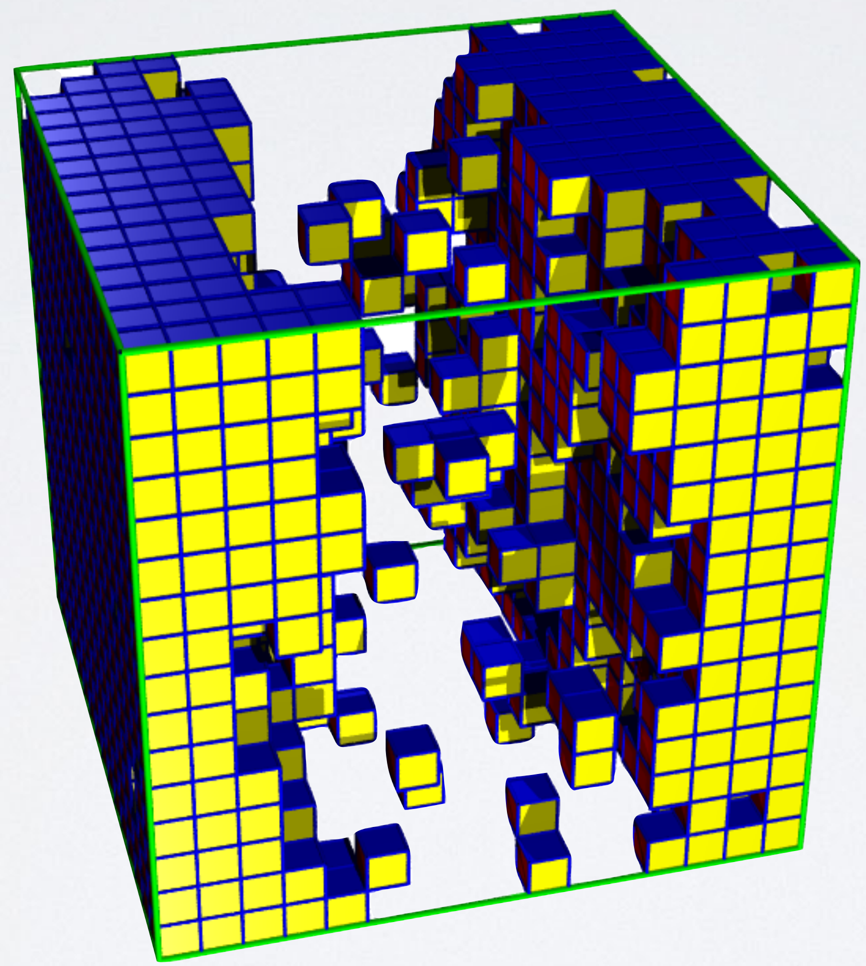
Summery & open questions & outlook

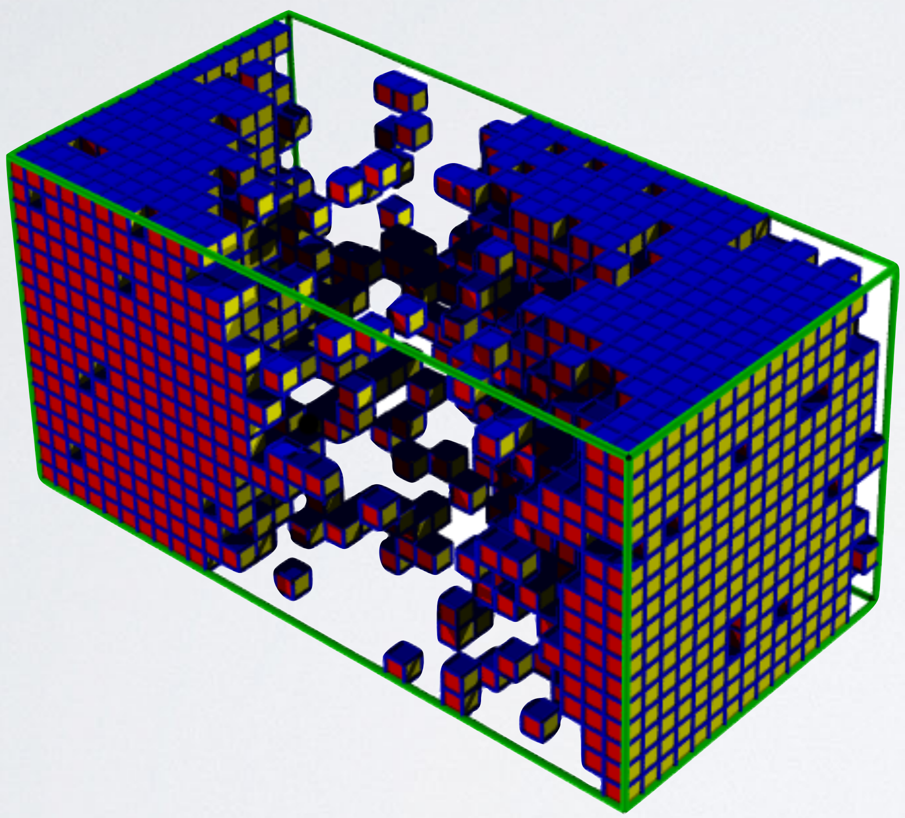
Thanks for your attention!

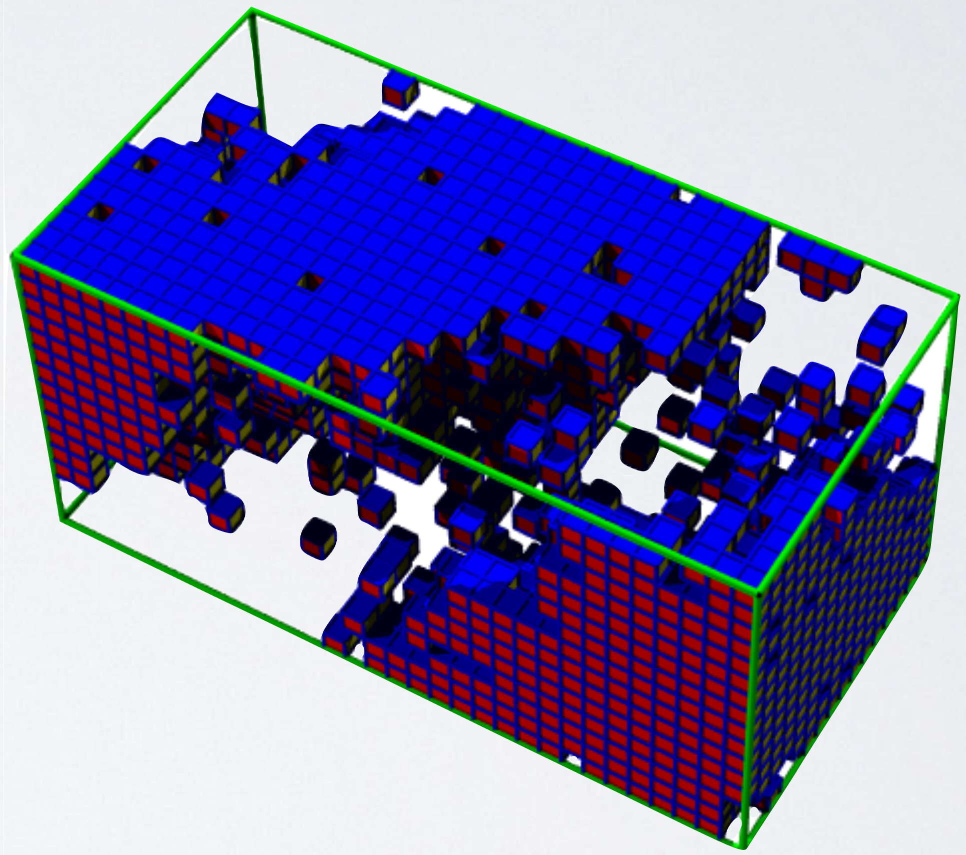
three dimensional Ising model



a typical configuration with two 100 interfaces ($\beta = 0.3$)

three dimensional Ising model





typical configurations with two 110 and 111 interfaces ($\beta = 0.3$)

two dimensional Ising model

- various boundary conditions (free, fixed ferromagnetic, fixed antiferromagnetic, double antiferromagnetic)
  see X. Wu and N. Izmailyan PRE 91, 012102 (2015)

- shape-dependent finite-size effect on a triangular lattice
  see X. Wu, N. Izmailian, and W. Guo PRE 87, 022124 (2013)

- finite-size on a rectangle with free boundaries
  see X. Wu, N. Izmailian, and W. Guo PRE 86, 041149 (2012)

- with Brascamp-Kunz Boundary Conditions
  see W. Janke and R. Kenna PRB 65(2002) 064110

- MxN with periodic boundary conditions
  see N. Izmailian and C-K Hu PRE (2002) 036103