Local energy minima of the 3d Edwards-Anderson model

Stefan Schnabel and Wolfhard Janke

CompPhys 16

Edwards-Anderson model

- Hamiltonian : $\mathcal{H}(S_1, \cdots, S_N) = -\sum_{\langle ij \rangle} J_{ij} S_i S_j, \quad S \in \{-1, 1\}$
- Gaussian disorder : $P(J) = \frac{1}{\sqrt{2\pi}} e^{-\frac{J^2}{2}}$
- Overlap of two configurations $Q(\mathbf{S}^\alpha, \mathbf{S}^\beta) = \frac{1}{N} \sum_i S_i^\alpha S_i^\beta$
- 3d – simple cubic lattice

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Energy reduction

Greedy algorithm

- Reduces E by **locally** optimal steps.
- Find spin with the highest positive energy e_i, flip it and repeat until all $e_i < 0$.
- "Traditional" application : Randomize many configurations and minimize with greedy algorithm.
- Does not find the global minimum.
- Constitutes basins of attraction:

$$B(\mathbf{S}_{\mathrm{min}}) := \{\mathbf{S}_B : G(\mathbf{S}_B) = \mathbf{S}_{\mathrm{min}}\}$$

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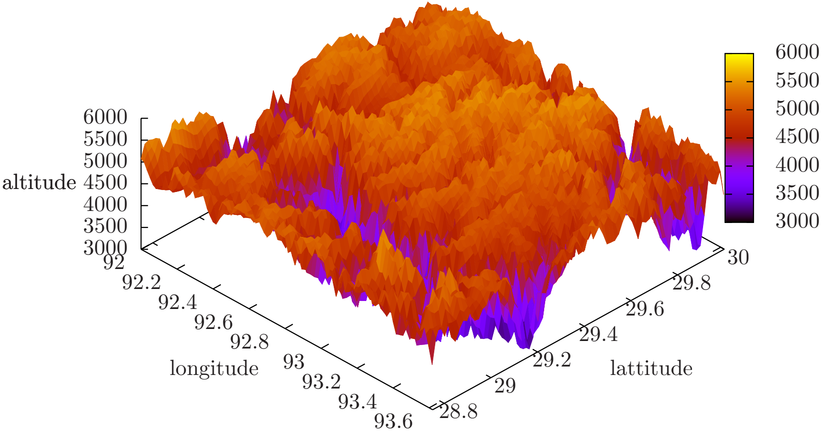
Landscape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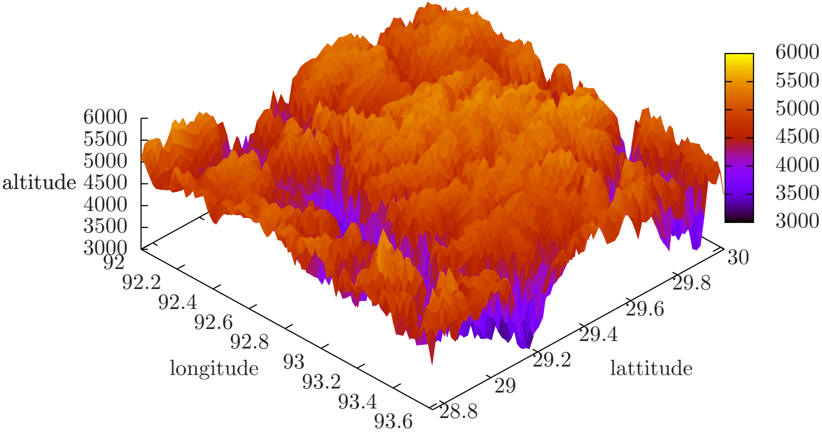
93.6
longitude
28.8
29
29.2
29.4
29.6
29.8
30
lattitude
6000
5500
5000
4500
4000
3500
3000

Reduced Landscape

$$E_{\mathrm{red}}(\mathbf{S}) := E(\mathbf{S}_{\mathrm{min}}) \equiv E\left(G(\mathbf{S})\right).$$

Reduced Landscape



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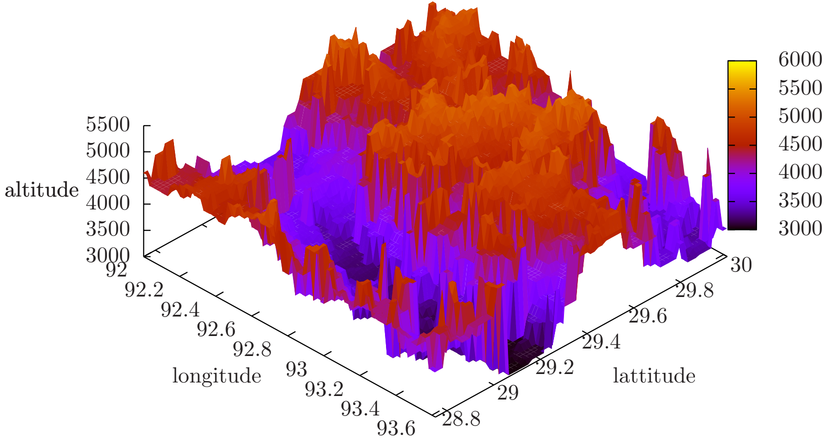

Enable walk in reduced energy landscape

dynamical greedy algorithm

- Stores relevant information of a (standard) greedy optimization $\mathbf{S}_{\mathrm{min}} = G(\mathbf{S})$
- Allows changes of the starting configuration $\mathbf{S} \to \mathbf{S}'$ and efficiently determines $\mathbf{S}'_{\mathrm{min}} = G(\mathbf{S}')$.

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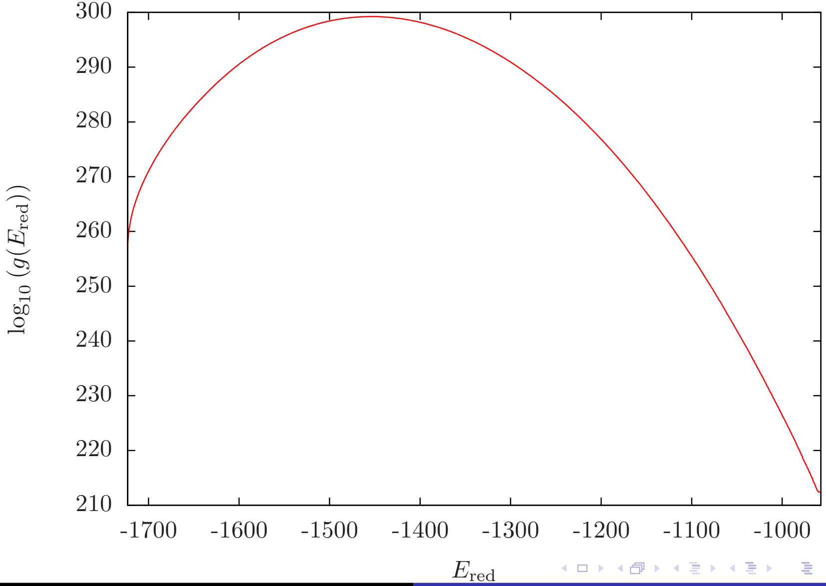
Density of States as function of E_{red} (one sample, $L = 10$)



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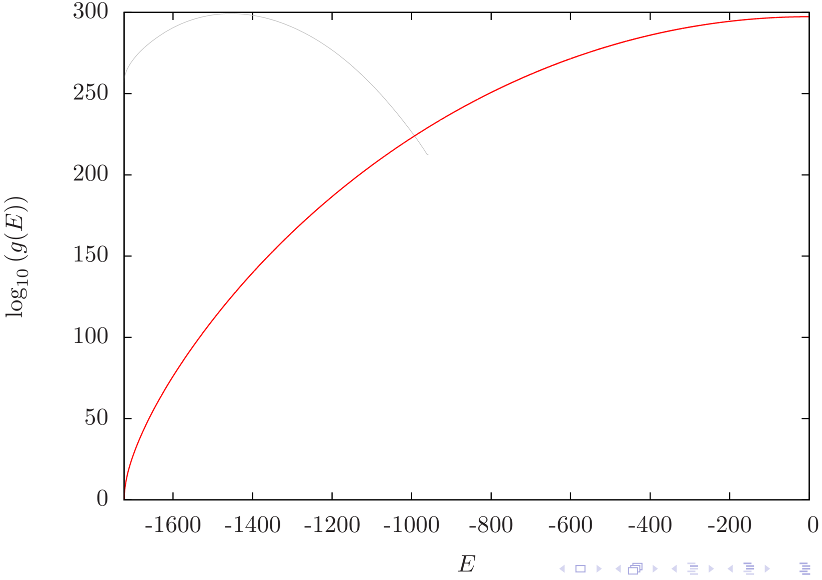

Size of basin of attraction

- Perform a MC simulation within the basin.
  (Reject all MC moves that change the result of the greedy optimization.)
- Relate the weight of the entire basin to the weight of the minimum configuration.
- E.g., flat-histogram over the overlap Q between current configuration $\mathbf{S}$ and local minimum $\mathbf{S}_{\mathrm{red}}$.

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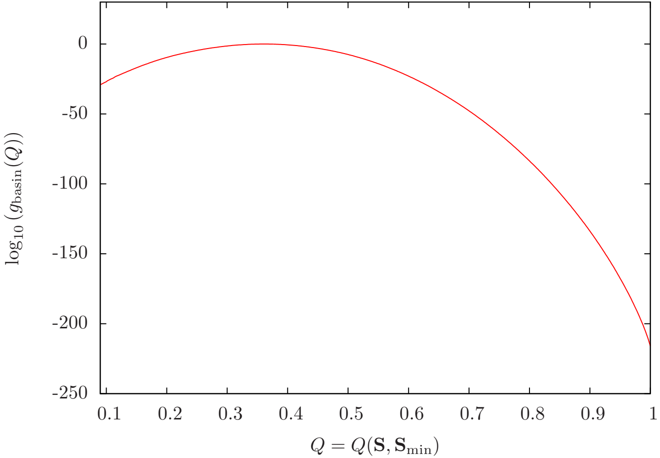
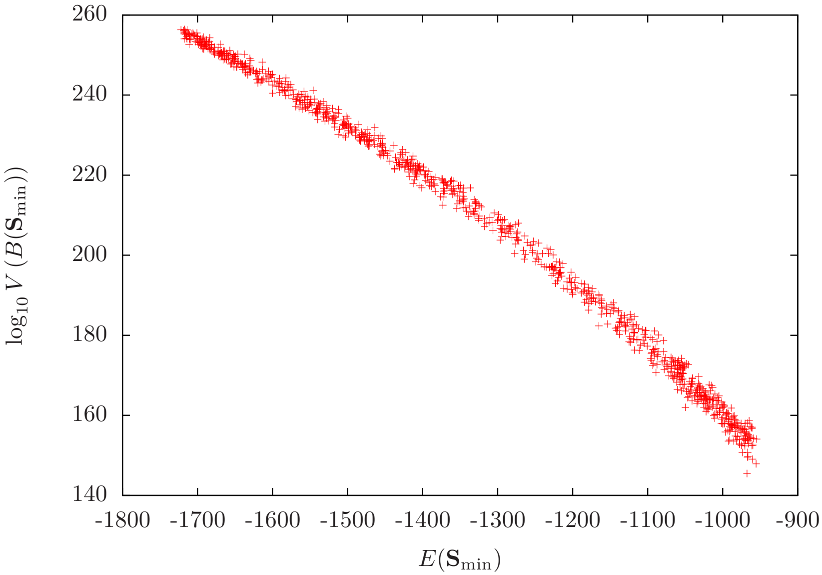


Sizes of various basins



Sizes of various basins

- define logarithmic size : $s = \ln V$
- the sampled distribution $P^s(s)$ is biased towards large basins
- true distribution: $P^s_{\mathrm{true}}(s) \propto \frac{1}{V} P^s(s) = e^{-s} P^s(s)$
- true distribution of sizes: $P^V_{\mathrm{true}}(V) \propto \frac{1}{V^2} P^s(\ln V) = e^{-2s} P^s(s)$
- $\langle V \rangle = \frac{\int_{-\infty}^{\infty} P^s(s) \mathrm{d}s}{\int_{-\infty}^{\infty} e^{-s} P^s(s) \mathrm{d}s}$
- if $P^s(s) \propto e^{-(s-s_0)^2/2\sigma^2}$ then $\langle V \rangle = \frac{e^{S_0}}{\sigma^2}$

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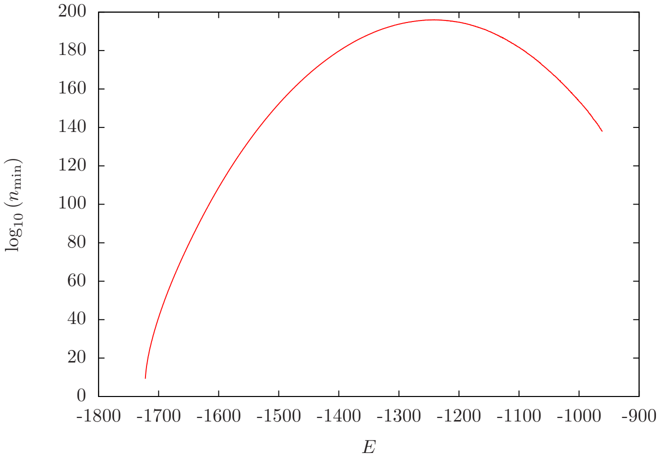
Number of local minima $n_{\rm min} = g(E_{\rm red})/\langle V\rangle(E_{\rm red})$



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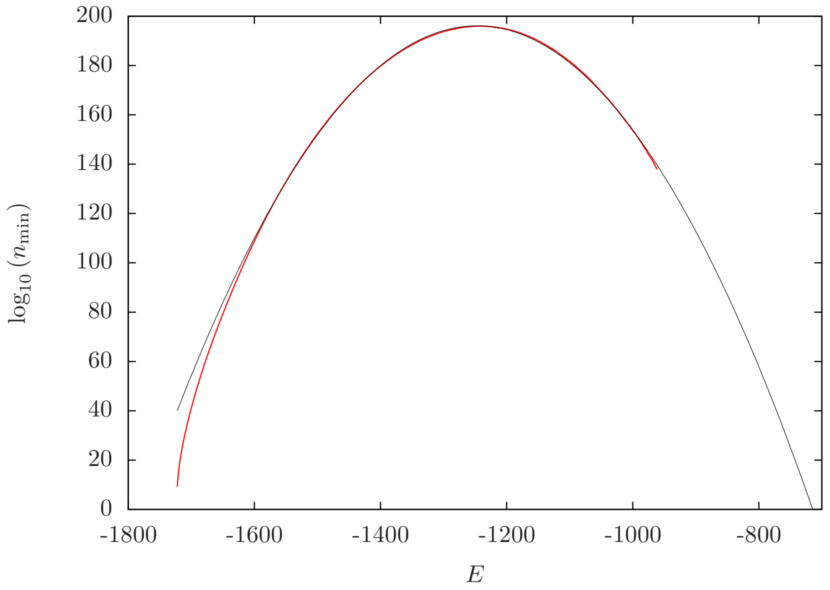

Concluding remarks

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Thanks for your attention

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