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Critical behavior in the presence of an order-parameter pinning field

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Collaboration

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Motivation

- Pinning-field approach in MC:
  Local ordering field, coupled to the order parameter Assaad, Herbut, 2013

  ⇒symmetry-breaking: order parameter $\neq 0$ in finite V

  Enhanced numerical stability

  Related approaches in DMRG, lattice QCD.

- Extrapolate the order parameter $V \to \infty$

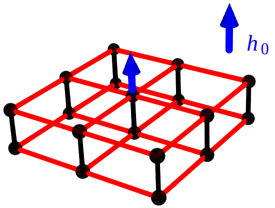
  ⇒detect order, identify phases

Our goals:

- Asses the ability of the method to locate phases
- Study the critical behavior
- Show a relation with a critical adsorption problem

Models

Bilayer quantum Heisenberg model

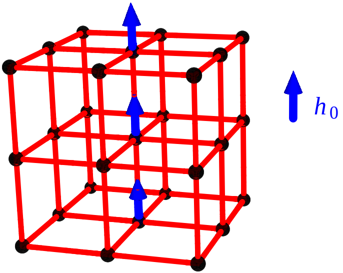


$$H = J \sum_{\alpha=1,2} \sum_{\langle i\ j\rangle} \vec{S}_{i,\alpha}\vec{S}_{j,\alpha} + J' \sum_i \vec{S}_{i,1}\vec{S}_{i,2} - h_0 S_i^z$$

- Quantum phase transition $T = 0$ $(J'/J)_c = 2.5220(1)$ Wang, Beach, Sandvik, 2006
- Critical behavior: $3D$ Heisenberg UC, restoration Lorentz symmetry
- Quantum-to-classical mapping:
  $3D$ model with a constant field h_0 along a line $\parallel$ imaginary time axis

Models

Classical Blume-Capel model



$$\mathcal{H} = -K \sum_{\langle i\ j \rangle} S_i S_j + \Delta \sum_i S_i^2 - h_0 \sum_{i \in line} S_i$$

$$S_i = -1, 0, 1$$

- Continuous phase transition in Ising UC
- Suppressed scaling corrections at $\Delta = 0.655(20)$ Hasenbusch, 2010
  $\Rightarrow$ improved model
- Exp realization: critical adsorption on a rodlike colloid

Scaling behavior: Renormalization Group analysis

- Analogy with surface critical phenomena:
  distinguish between RG flow of "bulk" couplings and "line" couplings

$$\{K_{\text{bulk}}\} \xrightarrow{RG} \{K'_{\text{bulk}}\} = \mathcal{R}(\{K_{\text{bulk}}\})$$
$$\{K_{\text{line}}\} \xrightarrow{RG} \{K'_{\text{line}}\} = \mathcal{R}(\{K_{\text{line}}\}, \{K_{\text{bulk}}\})$$

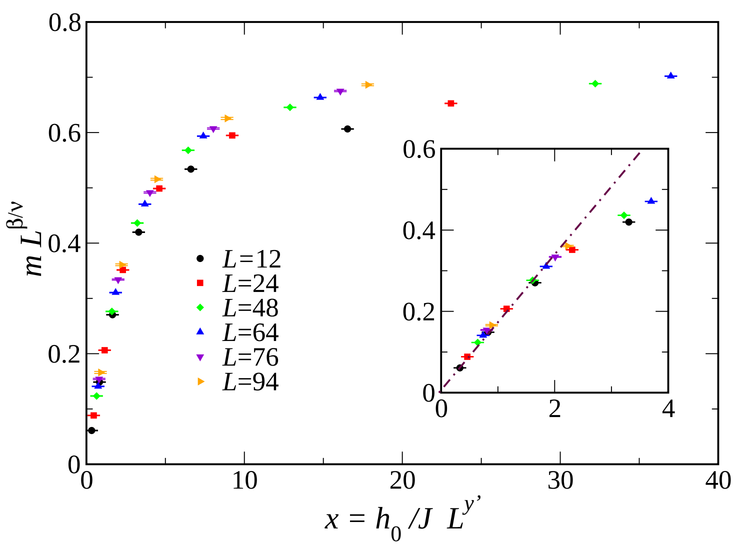
  Bulk fixed point $\Rightarrow$ different line fixed points

- $h_0 = 0$: translational invariance $\Rightarrow$ line fixed point ($\approx$ ordinary UC)
  –no distinction between line and bulk field $[\phi_{\rm line}] = [\phi_{\rm bulk}] = \beta/\nu$
  –scaling dimension of h_0 is $y' = 1 - [\phi_{\rm bulk}] = (1-\eta)/2$

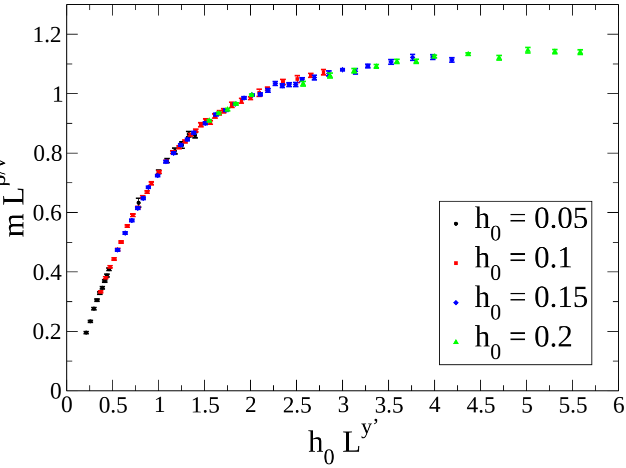
- Crossover behavior for finite h_0

$$m = L^{-\beta/\nu} f(h_0 L^{y'}), \qquad T = T_c$$

Results: crossover behavior in the bilayer Heisenberg model



Results: crossover behavior in the Blume-Capel model



$L = 15 \dots 1000$

Critical adsorption fixed point

- $y' = (1-\eta)/2 > 0 \Rightarrow h_0 = 0$ unstable fixed point for $h_0 \neq 0$
- Analogy with surface critical behavior
  $h_0 \to \infty$ critical adsorption or normal UC
  confirmed by Migdal-Kadanoff calculations Hanke, 2000
- At the critical adsorption fixed point $h_0 = h_0^* = \infty$

$$m = L^{-\beta/\nu} f(h_0^*), \qquad T = T_c$$

- We simulate the models for $h_0 = \infty$
- Fits of m to $AL^{-\beta/\nu}$, $L = 8 \ldots 600$:
  –β/ν deviates from the known value
  –systematic drift of β/ν as function of $L_{\min}$
  –large χ^2/DOF
  $\Rightarrow$ significant scaling corrections

Critical adsorption fixed point

- We simulate the models for $h_0 = \infty$
- Including scaling corrections:

$$m = AL^{-\beta/\nu}(1 + BL^{-\omega})$$

–β/ν agrees with known value
–stable results, good χ^2/DOF
–unusual slowly-decaying scaling corrections

Blume-Capel: $\omega = 0.60(5)$ (expected $\omega = 1$)

Heisenberg: $\omega = 0.32(4)$ (expected $\omega \approx 0.8$)

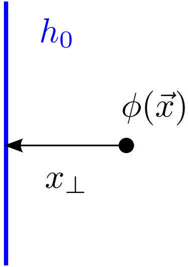
$\Rightarrow$ ω must originate from new irrelevant line scaling field

Short-Distance Expansion of the order-parameter

- Order parameter close to ordered line/surface:

Diehl, Dietrich, 1981

$$\phi(\vec{x}) \underset{x_\perp \to 0}{=} \sum_i C_{\psi_i} x_\perp^{-x_\phi + x_{\psi_i}} \psi_i(\vec{x}_\parallel)$$
$$= A x_\perp^{-\beta/\nu} \left(1 + C_O x_\perp^{x_O} O(\vec{x}_\parallel) + \cdots\right)$$



- Expectation value at criticality in finite size L

$$m(x_\perp \to 0) = A x_\perp^{-\beta/\nu} \left(1 + B \left(\frac{x_\perp}{L}\right)^{x_O} + \cdots\right)$$

- Correlations parallel to the line/surface

$$\langle \phi(\vec{x}_\perp, \vec{x}_\parallel) \phi(\vec{x}_\perp, \vec{x}'_\parallel) \rangle_c \sim |\vec{x}_\parallel - \vec{x}'_\parallel|^{-2x_O}$$

$\Rightarrow$ x_O characterizes the UC

Order-parameter profiles close to an ordered surface

- Order parameter close to ordered surface:

$$\phi(\vec{x}) \underset{x_\perp \to 0}{=} A x_\perp^{-\beta/\nu} \left(1 + C_O x_\perp^{x_O} O(\vec{x}_\parallel) + \cdots \right)$$

- $O = T_{\perp\perp}(\vec{x_\perp} \to 0)$ Stress-energy Tensor
  $x_O = D = 3$ Cardy, 1990; Eisenriegler, Stapper, 1994
- In a finite size $\langle T_{\perp\perp} \rangle$ =critical Casimir force $\propto L^{-3}$

$$m(x_\perp \to 0) = A x_\perp^{-\beta/\nu} \left(1 - C_+(D-1)\Delta_{+a}(x_\perp/L)^3\right)$$

  Distant-wall corrections Fisher, de Gennes, 1978
- Correlations parallel to the surface

$$\langle \phi(\vec{x}_\perp, \vec{x}_\parallel)\phi(\vec{x}_\perp, \vec{x}'_\parallel)\rangle_c \sim |\vec{x}_\parallel - \vec{x}'_\parallel|^{-2D}$$

  In line with early results Bray, Moore, 1977

Order-parameter profiles close to an ordered line

- Order parameter close to ordered line:

$$\phi(\vec{x}) \underset{x_\perp \to 0}{=} A x_\perp^{-\beta/\nu} \left(1 + C_O x_\perp^{x_O} O(\vec{x}_\parallel) + \cdots\right)$$

- Unknown operator $O \neq T_{\perp\perp}$, $x_O = 1.385(25)$ Hanke, 2000
- At $h_0 = \infty$, $T = T_c$ and for $x_\perp \to 0$ fixed

$$m(x_\perp \to 0) = A(x_\perp)\left(1 + B(x_\perp) L^{-x_O}\right)$$

We fit the data for $x_\perp = 2, 3, 4$, $L = 8 \ldots 600$,
with $L \geq L_{\rm min}$ and increasing $L_{\rm min}$

$$\boxed{x_O = 1.52(6)}$$

- Line scaling field with dimension $y = 1 - x_0 = -0.52(6)$
Matches with $\omega = 0.60(5)$ found by fit of m
$\Rightarrow$ O gives rise to the leading scaling corrections

Summary and Outlook

- We studied the scaling behavior of
  –quantum bilayer Heisenberg model with a on-site pinning field
  –3D improved Blume-Capel model with a line field

- for small h_0: crossover behavior
  scaling dimension of h_0: $y' = (1-\eta)/2$

- $y' > 0$: RG flows to a critical adsorption line fixed point

- At the critical adsorption fixed point for the Blume-Capel model
  - finite-size corrections of $m(x_\perp \to 0)$ due to unknown operator O
  - new critical exponent $x_O = 1.52(6)$ governs the finite-size corrections and the two-point function next to the line defect
  - scaling field coupled to O: corrections to scaling $\omega = 0.52(6)$.
  - Exp realization: rodlike colloid immersed in a critical binary mixture

Summary and Outlook

Several open questions and extensions

- What is the operator O?

- Generalization of the analysis to classical 3D $O(N)$ models, $N \neq 1$
  Identification of O as giving the leading corrections to scaling?

- Universal Short-Distance Expansion coefficient C_O

$$\phi(\vec{x}) \underset{x_\perp \to 0}{=} A x_\perp^{-\beta/\nu} \left(1 + C_O x_\perp^{x_O} O(\vec{x}_\parallel) + \cdots\right)$$

  For an ordered surface C_O has been determined by MC FPT, Dietrich, 2010

Ref.: FPT, S. Wessel, F. F. Assaad, `arXiv/1607.04270`, PRB to appear