Towards the QCD phase diagram using Complex Langevin

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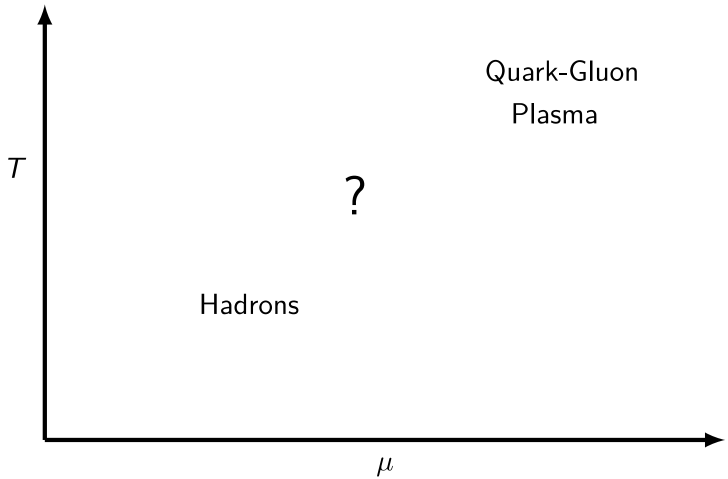
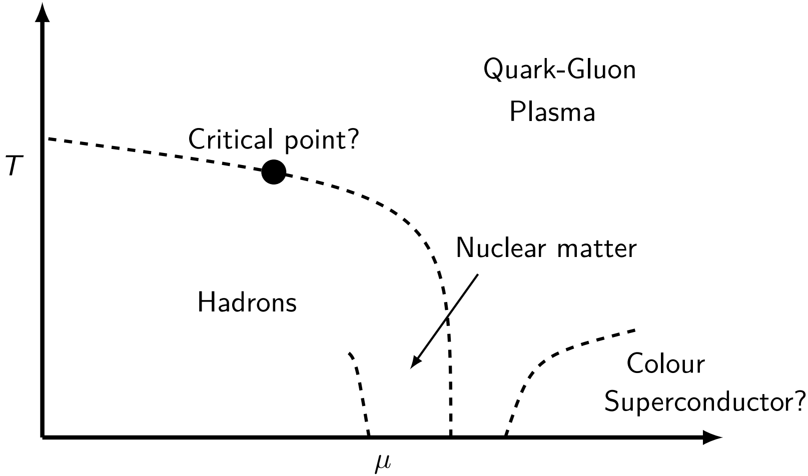
Phase diagram for QCD

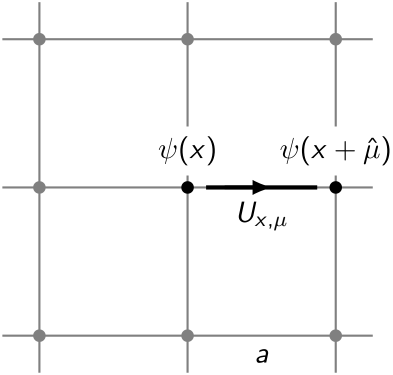


Phase diagram for QCD



- Temperature: $T_c \sim 200\,\text{MeV} \to 2 \cdot 10^{12}\,\text{K}$
- Density: Neutron stars $\mu_c \sim 3 \cdot 10^{17}\,\frac{\text{kg}}{\text{m}^3}$

Overview on Lattice QCD



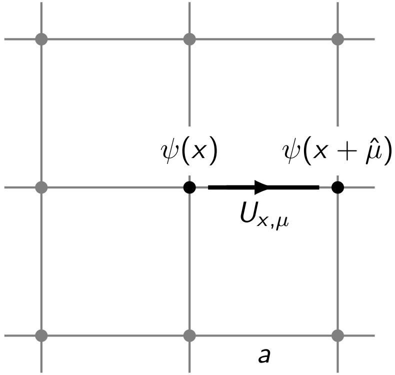
$U_{x,\mu} \in \mathrm{SU}(3)$

- Discretize Euclidean space-time by a hypercubic lattice Λ
- Quantize QCD using Euclidean path integrals
- Calculate expectation values using Monte Carlo techniques:

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, A[U]\, (\det D)^{N_f}\; e^{-S_G[U]}$$

- **Ab initio** method: Only QCD parameters $m_q \to \kappa$, $g_0 \to \beta$.

Uncertainties of the lattice approach



- Monte Carlo $\rightarrow$ Statistical uncertainties
- Finite simulation volume L
- Finite lattice spacing a
- Unphysical heavy quark or pion masses, $m_\pi^2 > m_{\pi,\text{phys}}^2$

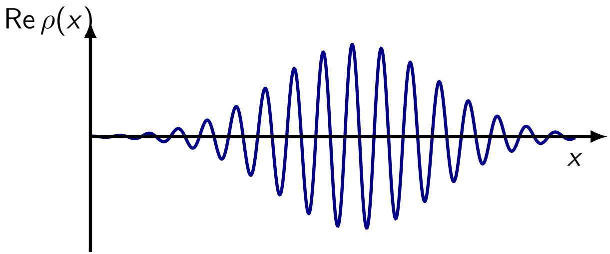
The sign problem

- Finite chemical potential → Sign problem

$$(\det D(\mu))^* = \det D(-\mu^*) \rightarrow \det D(\mu \neq 0) \in \mathbb{C}.$$

- Importance Sampling based Monte Carlo methods fail

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, A(U)\, |\det D|\, e^{i\Theta}\, e^{-S_G(U)}$$



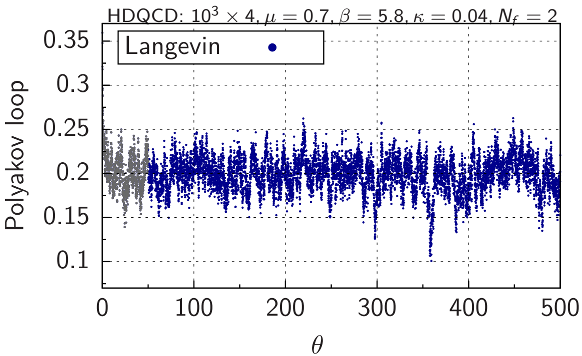
Complex Langevin simulations

- Complexify degrees of freedom $SU(3) \to SL(3,\mathbb{C})$

$$U_{x,\mu} = \exp\left[\mathrm{i}\, a\, \lambda^c \left(A^c_{x,\mu} + \mathrm{i}\, B^c_{x,\mu}\right)\right]$$

- Evolve links according (1st order) Langevin equation

$$U_{x,\mu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\,\lambda^a \left(-\varepsilon\, D^a_{x,\mu} S + \sqrt{\varepsilon}\,\eta^a_{x,\mu}\right)\right] U_{x,\mu}(\theta)$$



Complex Langevin simulations

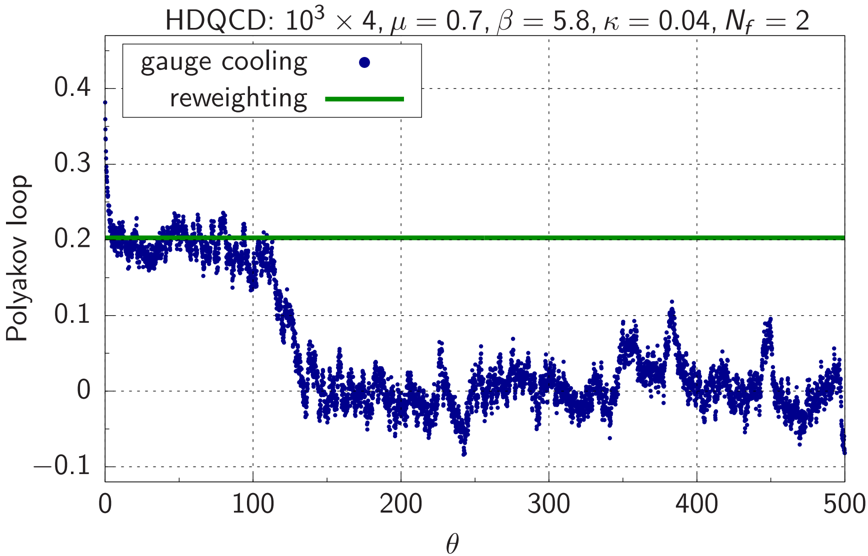
- However, $SL(3,\mathbb{C})$ is not a compact group...
- Convergence $\Leftrightarrow$
  - Action S is holomorphic
  - "Imaginary" direction of $SL(3,\mathbb{C})$ falls off quickly enough
- Measure distance to SU(3) manifold

$$\text{unitnorm} = \text{Tr}\left(U_{x,\mu}\,U^{\dagger}_{x,\mu} - 1\right)^2$$

- Gauge cooling is essential, but sometimes not sufficient...

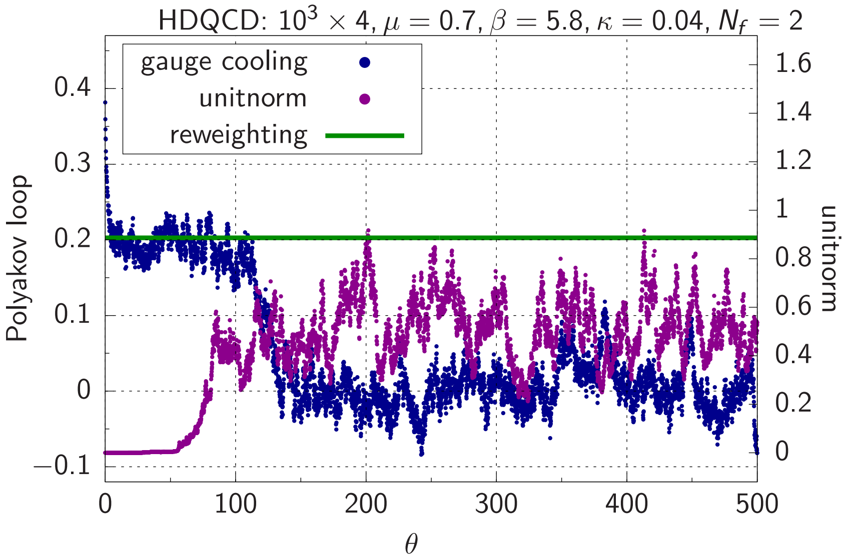
$$U_{x,\mu} \rightarrow \Omega_x\,U_{x,\mu}\,\Omega^{-1}_{x+\mu}$$

Gauge cooling



- Tunneling to wrong results.

Gauge cooling



- Tunneling to wrong results, when unitnorm grows too large.

Dynamic stablization

- Adding a trivial force to the Langevin dynamics

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\lambda^a\left(\varepsilon\, K^a_{x,\nu} + \mathrm{i}\,\varepsilon\,\alpha_{DS}\, M^a_x + \sqrt{\varepsilon}\,\eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

  where

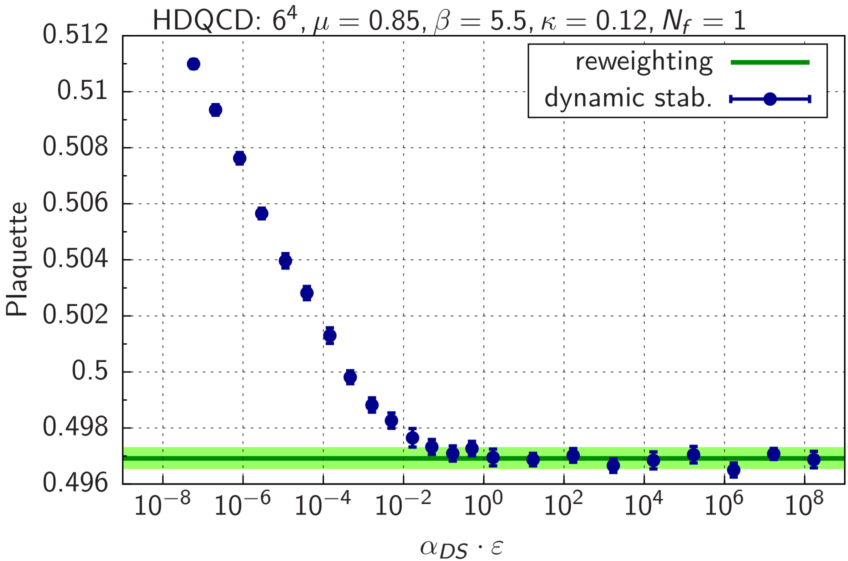
$$M^a_x = \mathrm{i} b^a_x \left(\sum_c b^c_x\, b^c_x\right)^3 \text{ and } b^a_x = \mathrm{Tr}\left[\lambda^a \sum_\nu U_{x,\nu} U^\dagger_{x,\nu}\right].$$

- Expanding the force in terms of gauge fields A and B

$$M^a_x \sim a^7 \left(\overline{B}^c_y\, \overline{B}^c_y\right)^3 \overline{B}^a_x + \mathcal{O}(a^8)\,.$$

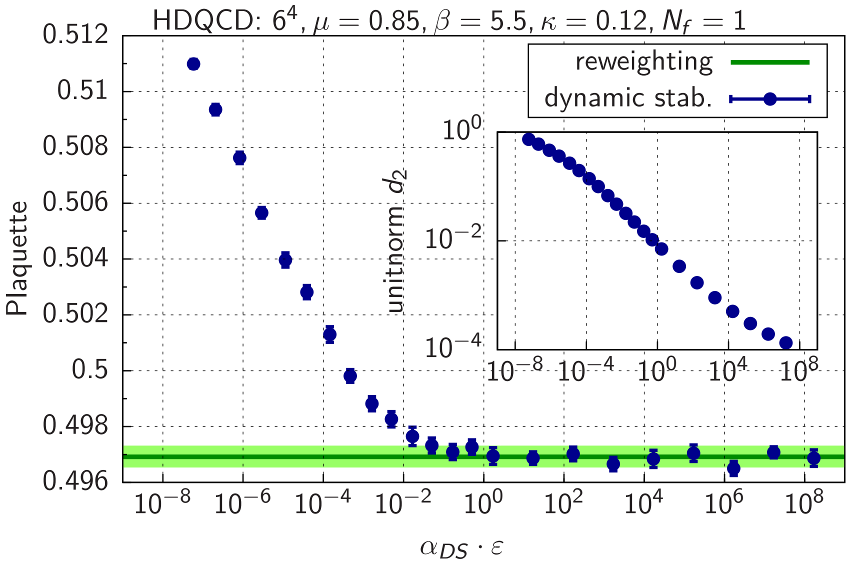
- Dynamic stabilization is numerically cheap and can be combined with gauge cooling (Here: 1 step)

Dynamic stablization



- For sufficient large α_{DS} we find agreement with reweighting.

Dynamic stablization



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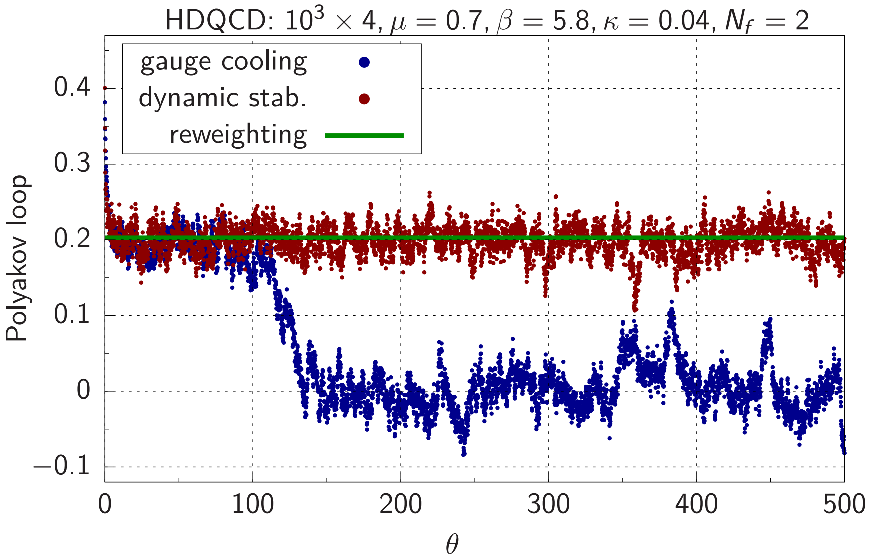
Dynamic stablization

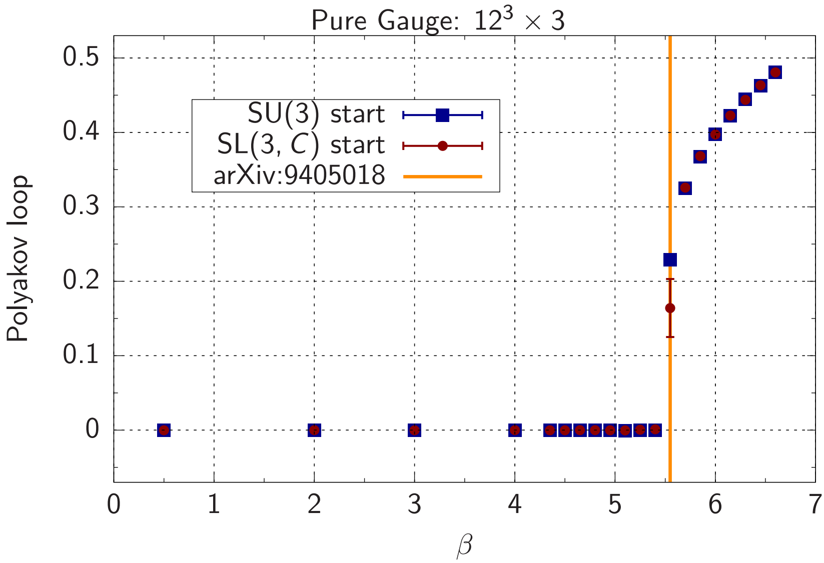


- Improved stability using dynamic stablization

Results

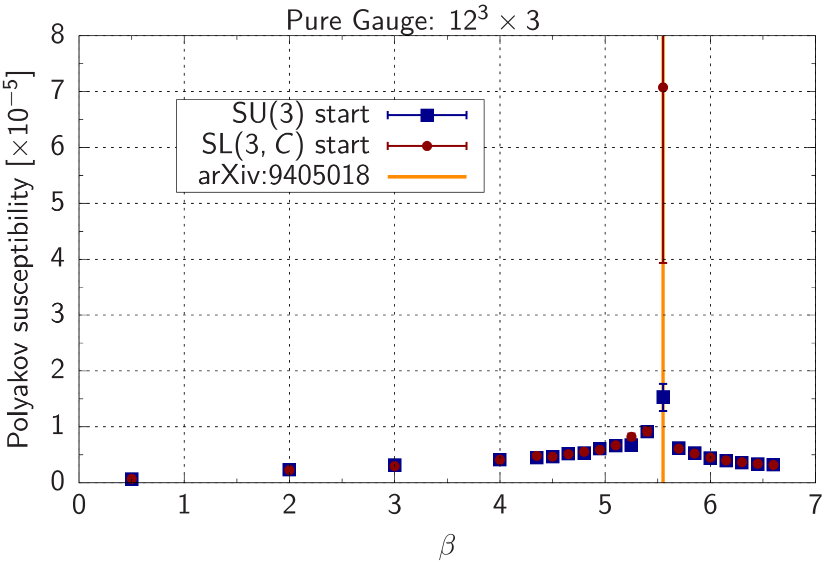
- **Pure Yang-Mills** ⇔ No sign problem
  - Checking Langevin simulations
  - Compare to standard Hybrid Monte Carlo methods
- **Heavy dense QCD** (HDQCD)
  - Heavy and dense approximation of QCD
  - Has a sign problem and phase transitions
  - Numerical cheap :)
- **Full (Staggered) QCD**
  - Including light dynamical fermions
  - Very preliminary results

Pure Yang-Mills



- The correct transition is obtained, even for $SL(3,\mathbb{C})$ start.

Pure Yang-Mills



- The correct transition is obtained, even for $SL(3, \mathbb{C})$ start.

Heavy Dense QCD

- Here: QCD in the limit of heavy quarks (HDQCD).
- Fermion determinant simplifies with the Polyakov loops $P_{\vec{x}}$ and $P_{\vec{x}}^{-1}$ as

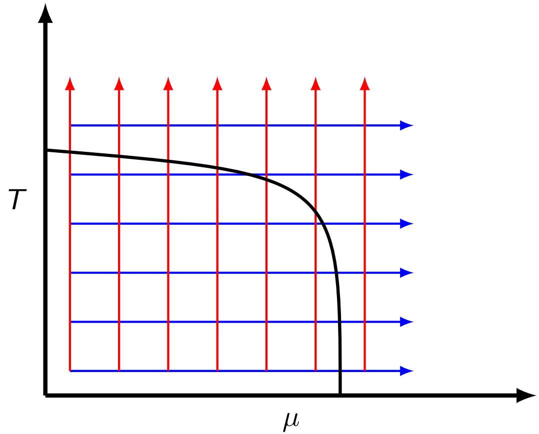
$$\det D(\mu) = \prod_{\vec{x}} \det\left(1 + C\, P_{\vec{x}}\right)^2 \det\left(1 + C'\, P_{\vec{x}}^{-1}\right)^2,$$

where the Polyakov loop $P_{\vec{x}}$ is defined as

$$P_{\vec{x}} = \frac{1}{V} \sum_t U_0(\vec{x}, t)$$

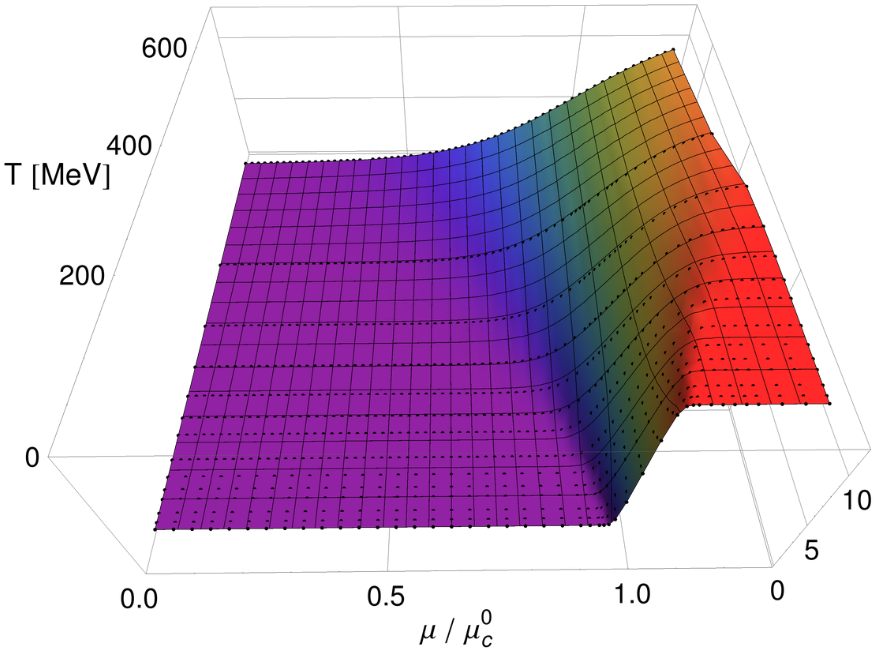
- For the gluonic part, we use the full Wilson gauge action.
- Map out the phase diagram for HDQCD.
- Expected transition: $\mu_c^0 = -\ln(2\kappa)$

Heavy Dense QCD



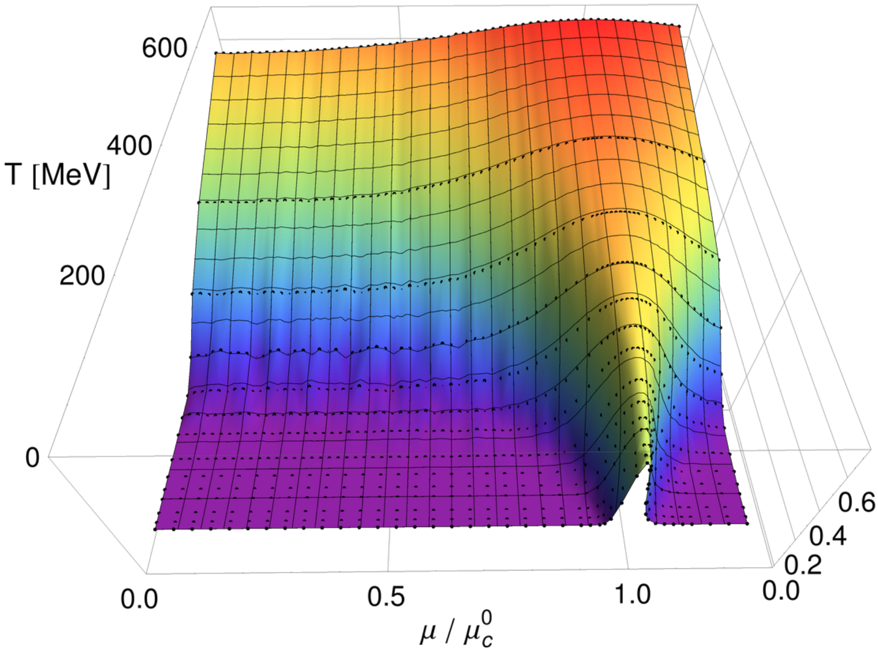
- Strategy:
  - Determine μ-transition in Fermion density
  - Determine T-transition in Polyakov loop

Heavy Dense QCD



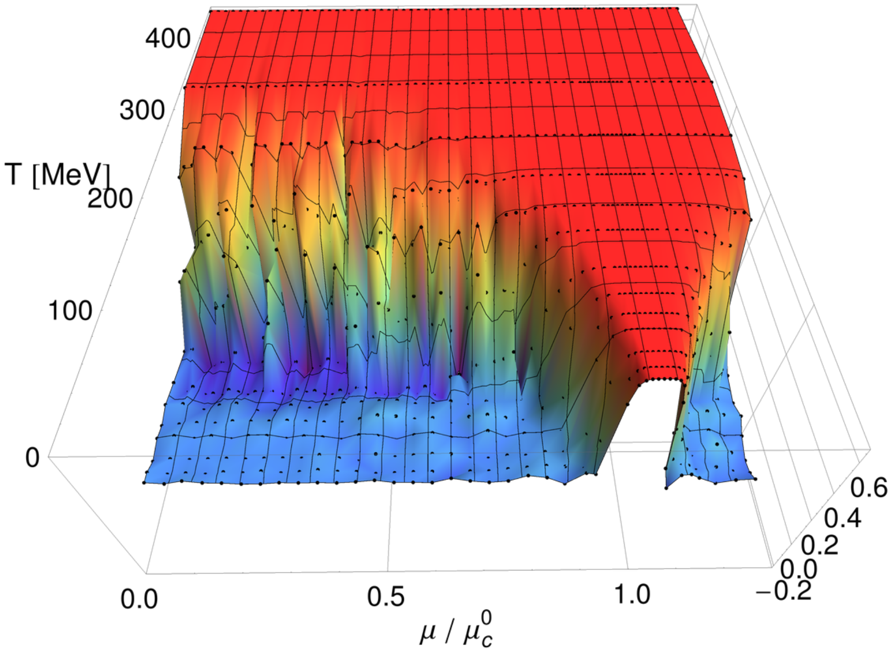
- Fermion density: $n = \frac{1}{N_t}\frac{\partial \ln Z}{\partial \mu}$

Heavy Dense QCD



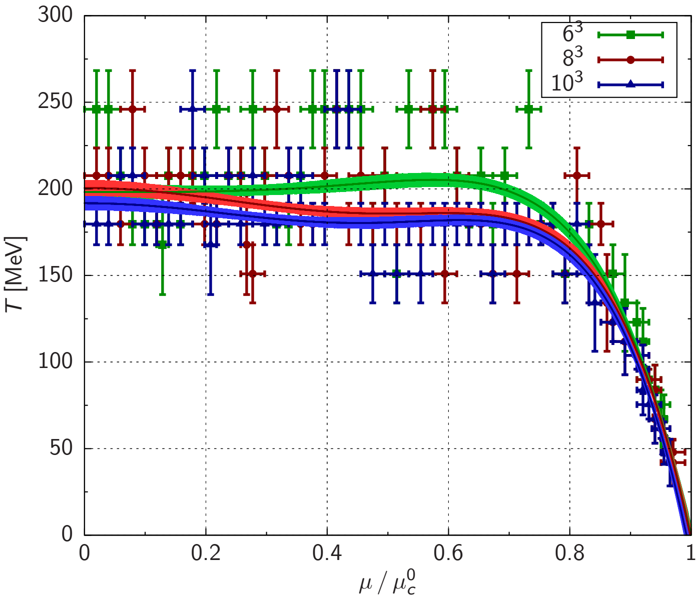
- Polyakov loop

Heavy Dense QCD



- Binder cumulant of the Polyakov loop $B = 1 - \frac{\langle P^4 \rangle}{3 \langle P^2 \rangle^2}$

Heavy Dense QCD



- Fit the phase boundary using $T_c(\mu) = \sum_k b_k \left(1 - \mu/\mu_c\right)^k$

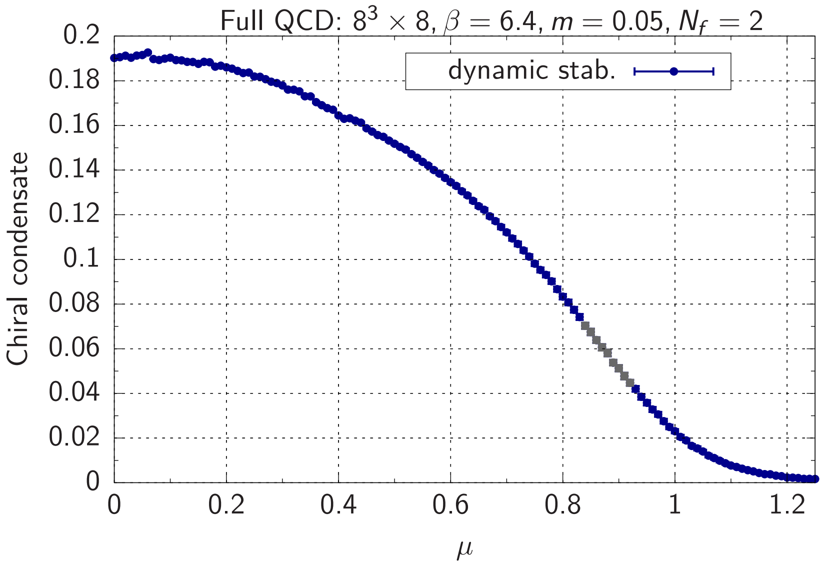
Full (Staggered) QCD

- Here: QCD using unimproved Staggered quarks
- Large gauge coupling $\beta = 6.4$
- Small volume $V = 8^3$
- (Small) pion masses $\sim \mathcal{O}(700)\,\mathrm{MeV}$
- Rough inversion of Dirac operator $M \sim 10^{-3}$

$$D^a_{x,\mu} S = D^a_{x,\mu} S_{YM} - \frac{N_f}{4} \mathrm{Tr}\left[M^{-1} D^a_{x,\mu} M\right]$$

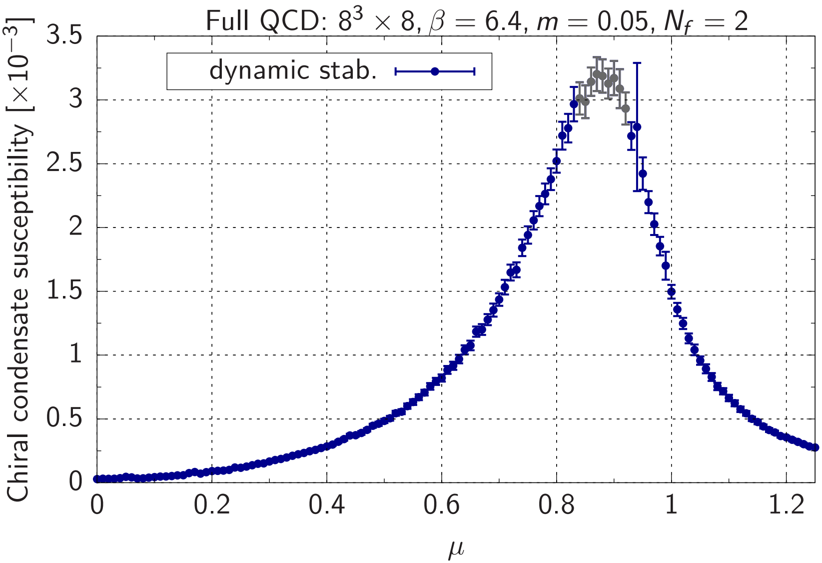
- Every Langevin update needs inversion of the Dirac operator. . .

Full (Staggered) QCD



- Unimproved Staggered quarks

Full (Staggered) QCD



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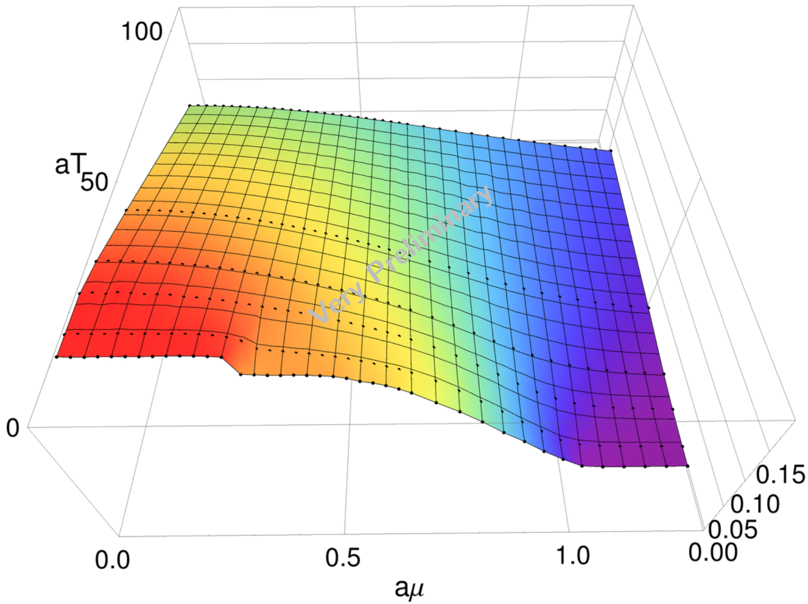
Full (Staggered) QCD

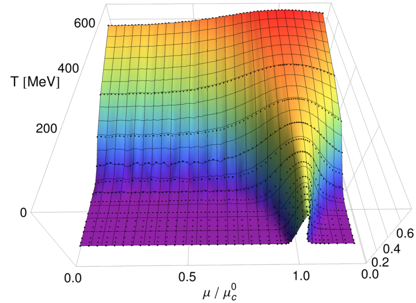


- Unimproved Staggered quarks

Future work

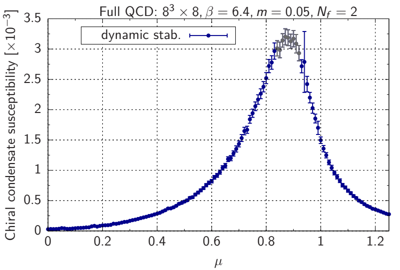
Conclusion

- Complex Langevin simulation can be used to study the QCD phase diagram
- Dynamical stabilisation improves convergence
- Work on the convergence, especially around μ_c.



Future work

- Start proper Full QCD simulations to identify phase structure of QCD.
- A lot of work to be done!



Thank you for your attention!