Numerical investigation of the aging of the $2d$ Fully-Frustrated XY model

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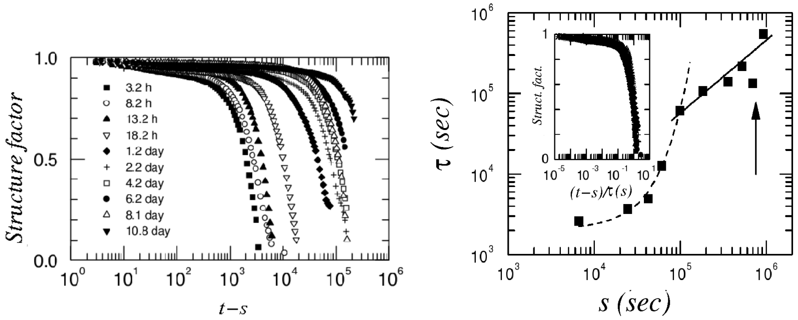
Experimental example

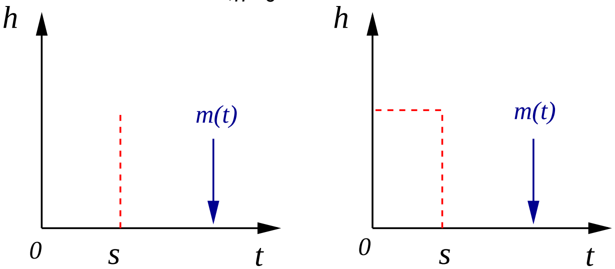
- Cipelletti *et al*, *Phys. Rev. Lett.* **84**, 2275 (2000)



$$C_q(t,s) \sim \langle I_q(t) I_q(s) \rangle \neq C_q(t-s)$$

Autoresponse and integrated autoresponse

$$R(t,s) = \left.\frac{\delta\langle m_i(t)\rangle}{\delta h_i(s)}\right|_{h=0} \qquad \rho_{TRM}(t,s) = \int_0^s du R(t,u)$$

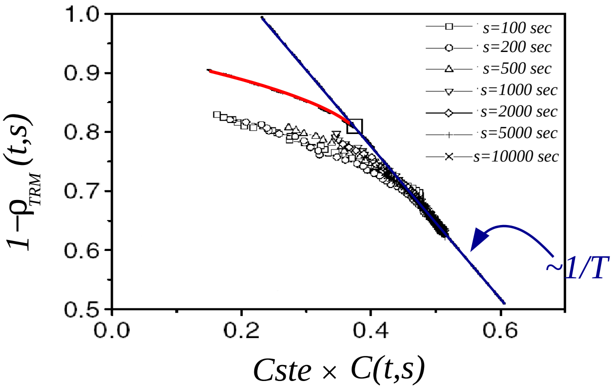


- Fluctuation-dissipation theorem :

$$R(t,s) = \frac{1}{k_B T}\partial_s C(t,s) \qquad \rho_{TRM}(t,s) = \frac{1}{k_B T}(C(t,s) - C(t,0))$$

Experimental example

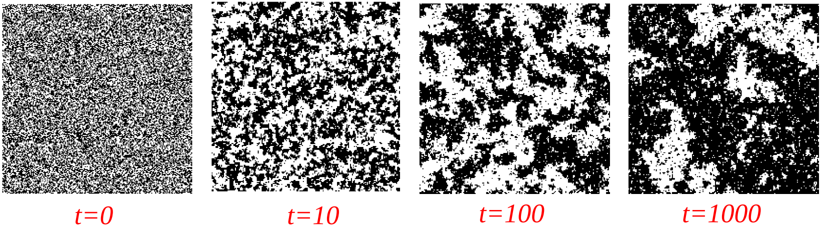
- Hérisson & Ocio, *Phys. Rev. Lett.* **88**, 257202 (2002)



Features of aging

- Slow, non-exponential relaxation
- Two-time functions depend on t and s
- Two-time quantities depend on scale-invariant functions
- Deviation from the fluctuation-dissipation theorem

Domain growth

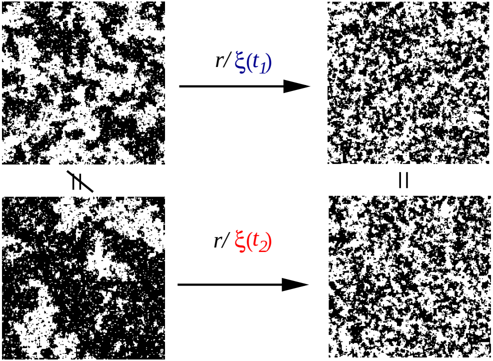


- Inside the domains : reversible fluctuations (spins equilibrated)
- Domains walls : non-reversible growth

Scaling hypothesis

- Only one characteristic length at each time :

$$\xi(t) \sim t^{1/z_c}$$



The autocorrelation

- At equilibrium :

$$C(t,s) = \langle S_i(t) S_i(s) \rangle \sim (t-s)^{-a_c}$$

where $a_c = 2\beta/(\nu z_c) \underset{d=2}{=} \eta/z_c$.

- Out-of-equilibrium :

$$\begin{aligned} C(t,s) &\sim (t-s)^{-a_c} f(\xi(t)/\xi(s)) \\ &\underset{t,s\to\infty}{\sim} (t-s)^{-a_c} (t/s)^{-\phi} \\ &\underset{t,s\to\infty}{\sim} s^{-a_c} (t/s)^{-\lambda_c/z_c} \end{aligned}$$

where λ_c is a new exponent ($\phi = \lambda_c/z_c - a_c$).

The fluctuation-dissipation ratio

$$R(t,s) = \left.\frac{\delta\langle S_i(t)\rangle}{\delta h_i(s)}\right|_{h=0}$$

- At equilibrium :

$$k_B T R(t,s) = \partial_s C(t,s)$$

- Out-of-equilibrium :

$$k_B T R(t,s) = X(t,s)\partial_s C(t,s)$$

Because $R(t,s) \sim s^{-a_c-1}(t/s)^{-\lambda_r/z_c}$, we get for $\lambda_r = \lambda_c$:

$$\lim_{t,s\to\infty} X(t,s) = X_\infty$$

- X_∞ : universal

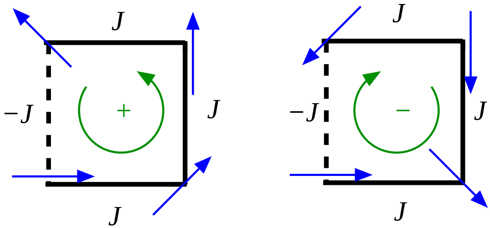
The 2d fully-frustrated XY model

- Hamiltonian on a square lattice (FFXY) :

$$\mathcal{H}_{FFXY} = -\sum_{\langle i,j\rangle} J_{ij}\vec{S}_i \cdot \vec{S}_j$$

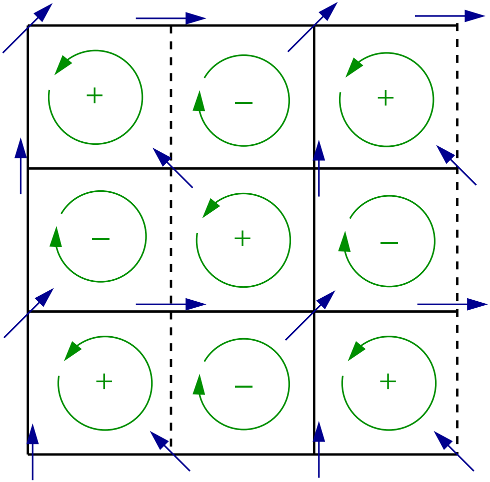
  where $\vec{S}_i$ is a planar spin and the J_{ij} ensure the full frustration of the lattice
- Hamiltonian of an array of Josephson junctions in a tranverse field

Plaquette in the ground state



- Symmetry $U(1)$ for angles
- Symmetry Z_2 for chiralities

Ground state structure



Equilibrium properties

- Two transitions at different temperatures for the angles (T_{BKT}) and the chiralities (T_{Ch}) where $T_{BKT} < T_{Ch}$
- Angles : Berezinskiĭ-Kosterlitz-Thouless transition (BKT) with a critical line
- Low-temperature : spin-waves approximation
- Chirality : second order phase transition in the $2d$ Ising universality class

Quench from $T = 0$ to $T \leq T_{BKT}$

- Langevin equation :

$$\frac{\partial \theta_i(t)}{\partial t} = -\frac{\delta \mathcal{H}^{SW}[\theta]}{\delta \theta_i} + \Omega_i(t),$$

where

$$\mathcal{H}^{SW}[\theta] = \frac{J}{2\sqrt{2}} \sum_{\langle i,j \rangle} (\theta_i - \theta_j)^2 ,$$

is the FFXY hamiltonian in SW approximation.
Ω is a gaussian noise

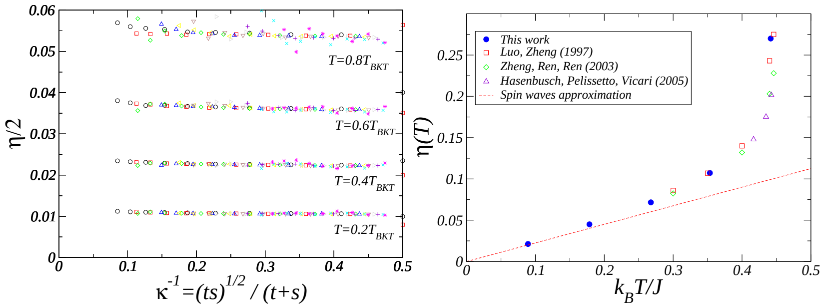
- We get the autocorrelation :

$$C(t,s) = \langle \cos(\theta_i(t) - \theta_j(s)) \rangle \sim (t-s)^{-\eta/2} \left(\frac{t+s}{\sqrt{ts}} \right)^{\eta/2} ,$$

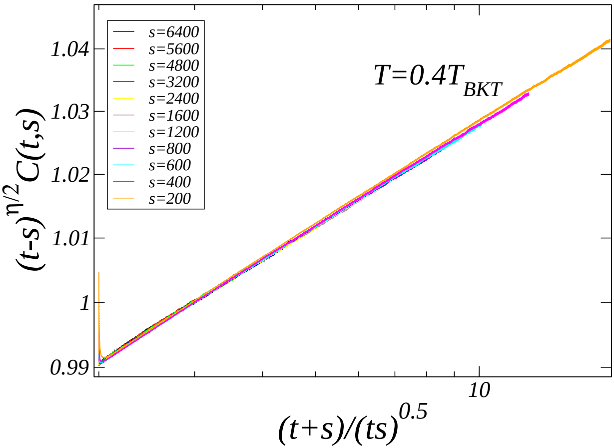
where $\eta(T) = \frac{k_B T \sqrt{2}}{2\pi J}$

Quench from $T = 0$ to $T \leq T_{BKT}$

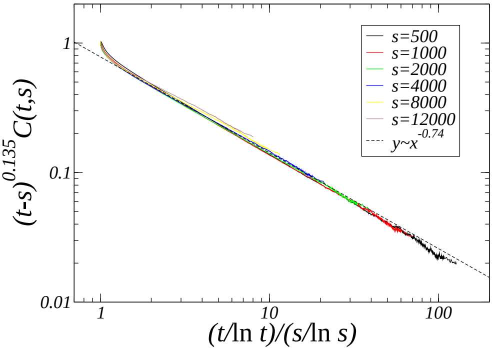
$$C(t,s) \sim (t-s)^{-\eta/2}\left(\frac{t+s}{\sqrt{ts}}\right)^{\eta/2}$$



Quench from $T = 0$ to $T \leq T_{BKT}$

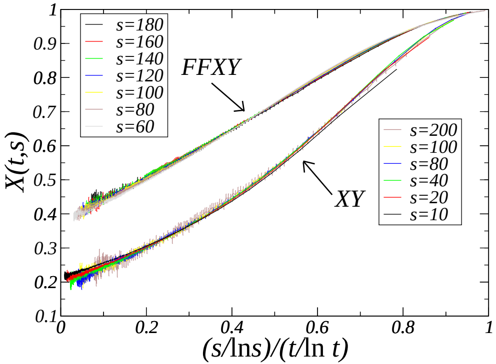


Quench from $T = \infty$ to $T = T_{BKT}$



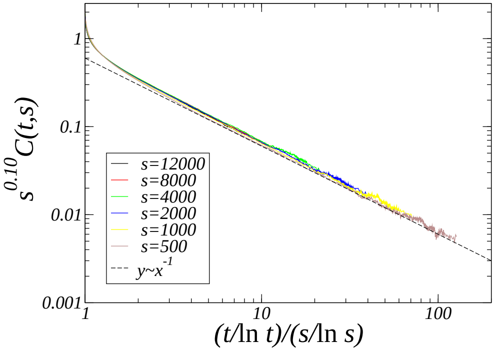
- $\phi = 0.74(3) \neq$ XY model ($\phi = 0.613(4)$)

Quench from $T = \infty$ to $T = T_{BKT}$



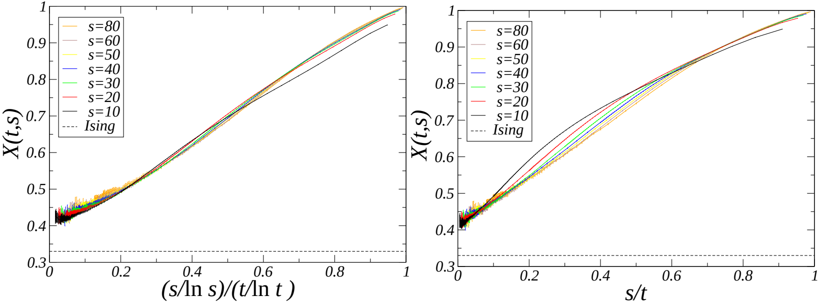
- FFXY : $X_\infty = 0.385(15) \neq$ XY model ($X_\infty = 0.215(15)$)
- Log. corrections in both cases : BKT-like transition

Quench from $T = \infty$ to $T = T_{Ch}$



- $\lambda_c / z_c = 0.98(5) \neq$ Ising model ($\lambda_c / z_c = 0.738(21)$)

Quench from $T = \infty$ to $T = T_{Ch}$



- $X_\infty = 0.410(10) \neq$ Ising model ($X_\infty = 0.328(1)$)
- Log. corrections : influence of topological defects

Aging of the FFXY

- Aging of angles
  - Quench from the ground state to the critical line :
    - Estimation of $\eta(T)$
    - Evidences of the validity of SW approximation
  - Quench from HT phase to T_{BKT} :
    - Universal quantities $\Phi = 0.74(3)$ and $X_\infty = 0.385(15)$.
    - Logarithmic corrections in the scaling variables

- Aging of the chirality from HT phase to T_{Ch}
  - Universal quantities $\lambda_c/z_c = 0.98(5)$ and $X_\infty = 0.410(10)$ incompatible with the Ising model. Cross-over ?
  - Influence of topological defects on the scaling variables.

Work available at : *J. Stat. Mech.* (2009) P10017
& cond-mat/0907.1474