

Trading leads to scale-free self- organization

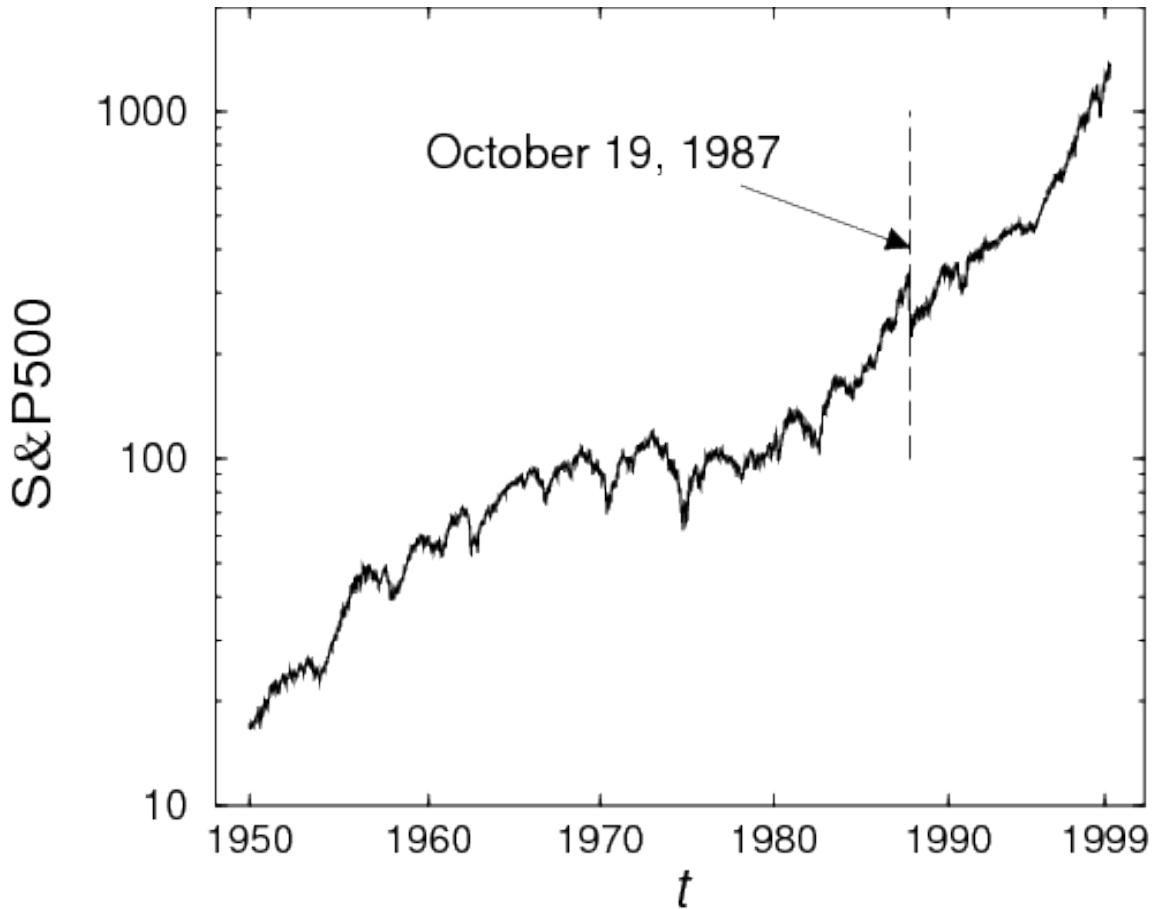
Wolfgang Paul
Institut für Physik
Martin Luther Universität
06099 Halle (Saale)

Marlon Ebert
Institut für Physik
Johannes-Gutenberg Universität
55099 Mainz

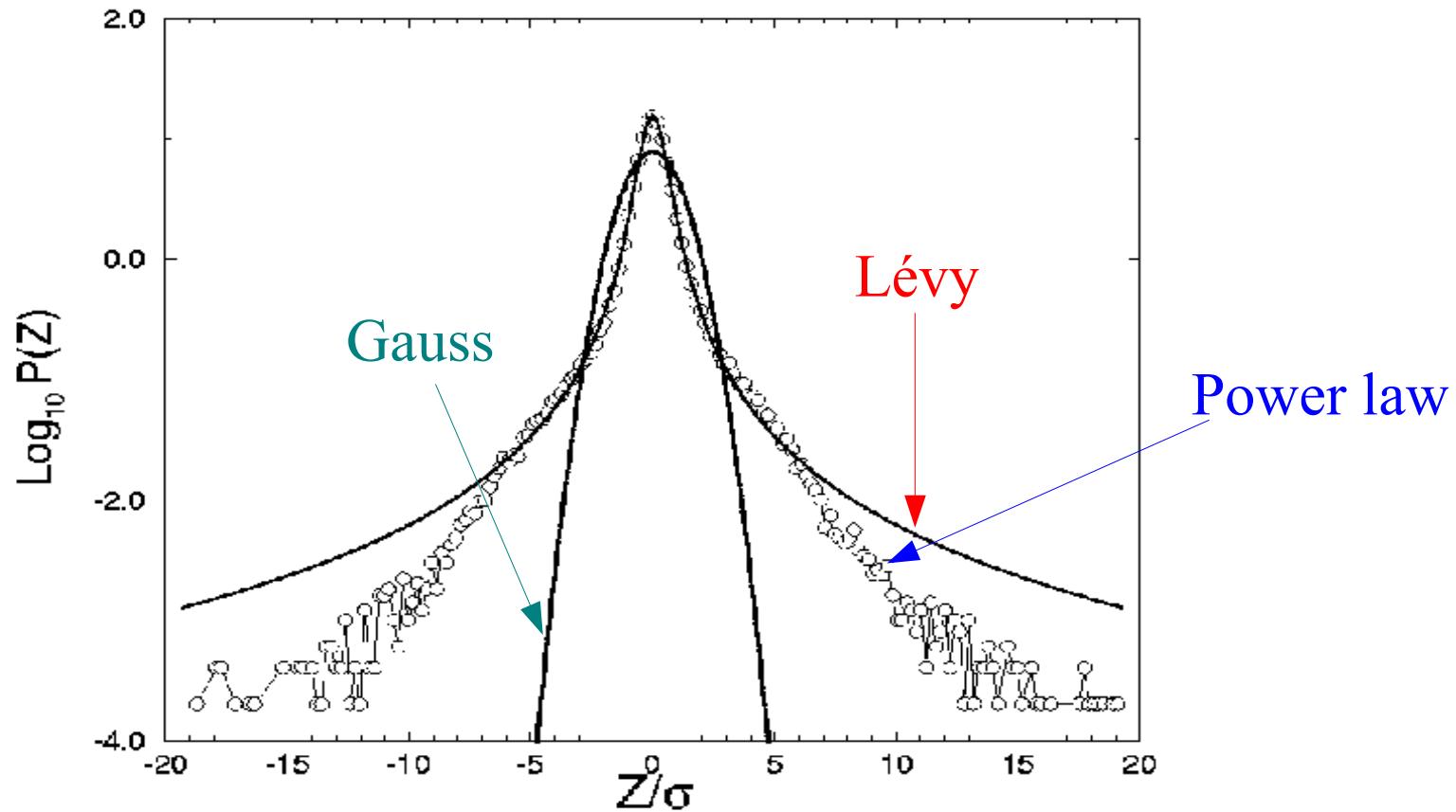


The stock-market for practical people

It would
be really
nice to
be able
to predict
those crashes!



The stock-market for physicists



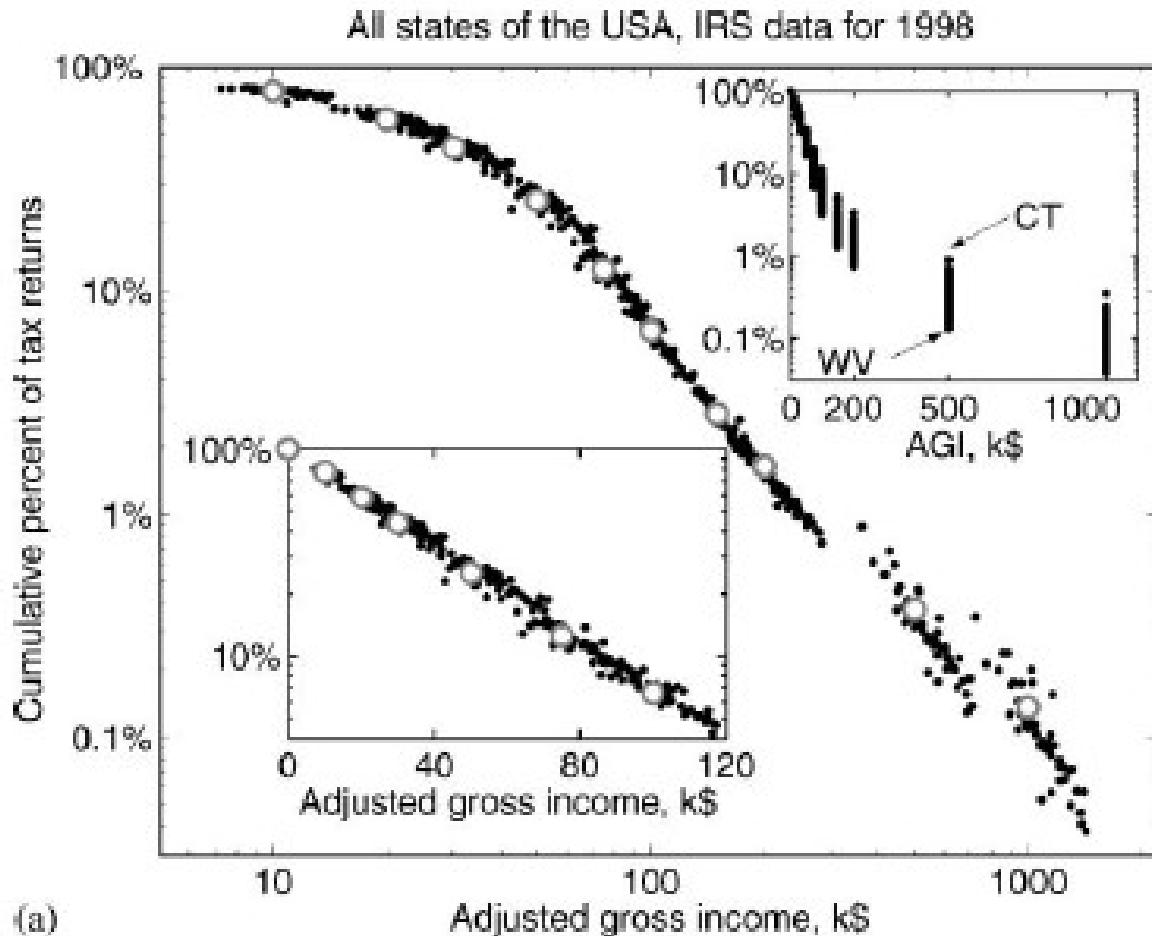
R. N. Mantegna, H. E. Stanley, Nature **376**, 46 (1995)



The Pareto-law

Boltzmann at
low income,
power-law at
large income

Multiplicative
stochastic process
for the income



A. Dragulescu, V. M. Yakovenko, Physica **299**, 213 (2001)



What “everybody” is willing to agree on

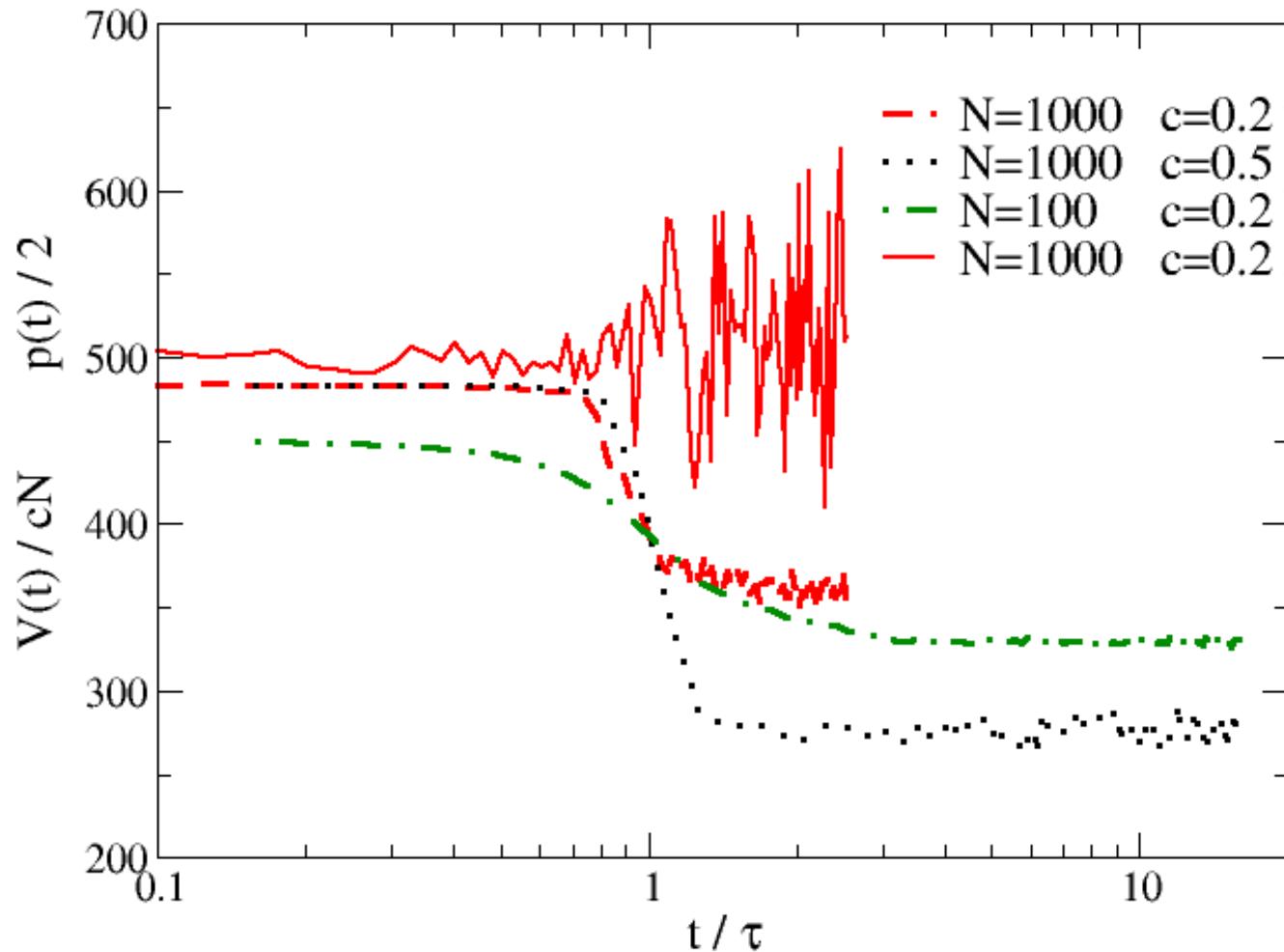
- Supply and demand determine the price
- Everybody trades according to his capacity

Take N agents, equip them with equal amounts of money and stocks and have them randomly buy or sell a fraction of their possession at each time step, limited by a maximum fraction, c.

Price update: $p(n+1) = p(n) * \left(1 + \frac{D-S}{D+2S}\right)$



What happens?

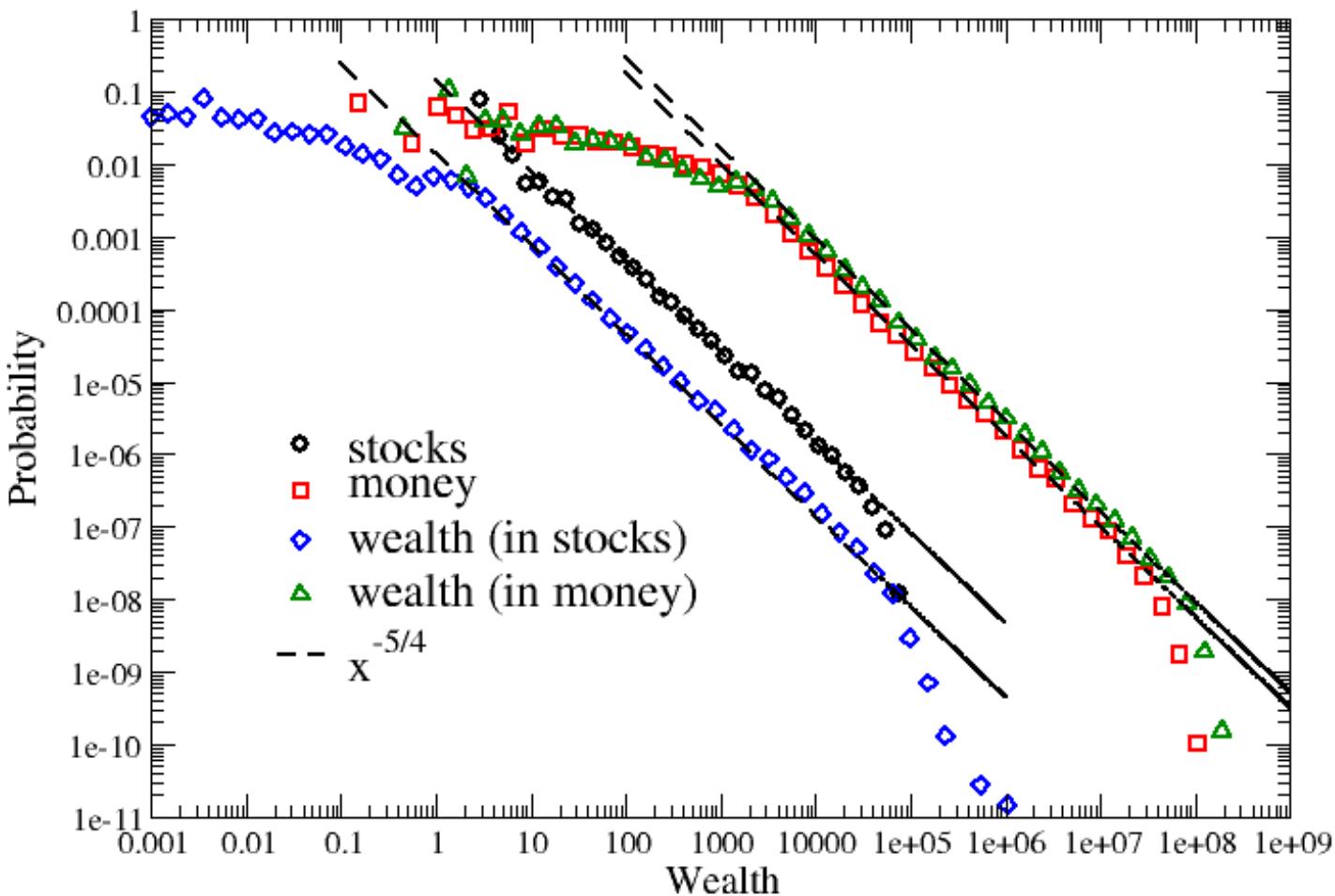


Dynamic
crossover

$$\tau \propto \left(\frac{N}{c} \right)^2$$



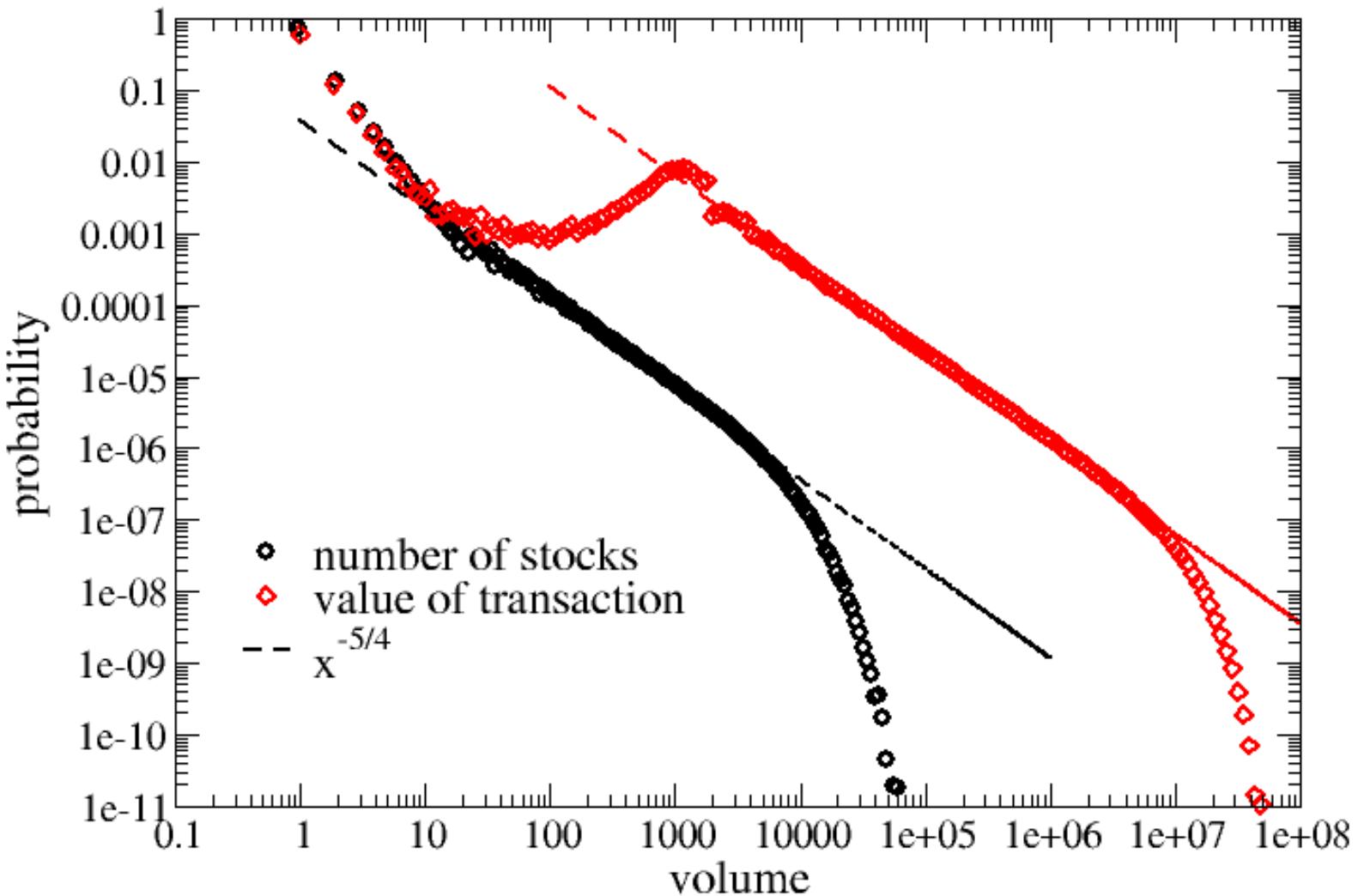
Stationary state



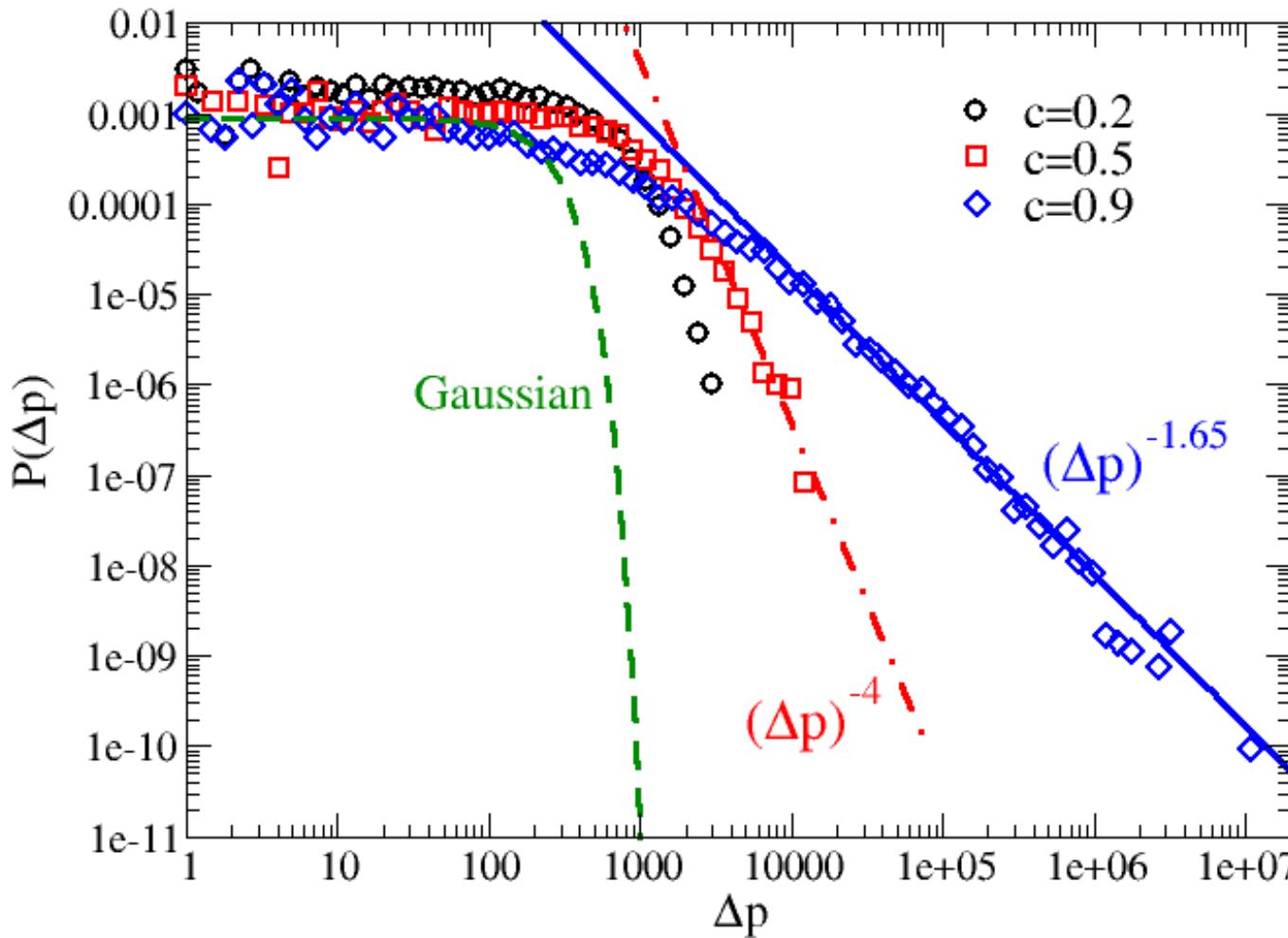
Pareto
wealth
distribution



Stationary trading



Stationary price fluctuations



Real stocks:
exponent 4



Stationary conclusions

- The interaction of price update and market participation proportional to wealth leads to **self-organization** into a **scale-free** state.
- We could have called money and stocks, fishes and Kauri shells, so **economic inequality** started when the first equivalent of money was invented.
- **Leverage** (banks do up to a factor of 50) increases the scale of **fluctuations**, which in turn topple the bank responsible for the fluctuations in the first place.



Master equation

$$\Pi(A, G, n+1) = (1 - q(n)) \Pi(A, G, n)$$

$$\frac{+q(n)}{2c} \sum_{\substack{A' = A - \frac{cG}{p(n)}}}^{A-1} \frac{\Pi(A', G + (A' - A)p(n), n)}{G + (A' - A)p(n)}$$

$$\frac{+q(n)}{2c} \sum_{\substack{A' = A + 1}}^{\frac{A}{1-c}} \frac{\Pi(A', G + (A' - A)p(n), n)}{A'}$$

$$q(n) = 1 - \left| \frac{\sum_i x_i(n) \left(A_i(n) - \frac{G_i(n)}{p(n)} \right)}{\sum_j x_j(n) \left(A_j(n) + \frac{G_j(n)}{p(n)} \right)} \right|$$

$x_i(n)$: random number
between 0 and c

