Spin Glasses in the Hypercube

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Why?

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   - Highly non trivial in spin glasses

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     - Spatial correlation functions, coherence lengths
     - Unusual in mean field

Overview

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3. Off-equilibrium results

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5. Conclusions

Edwards-Anderson Model

Degrees of Freedom

1. Dynamical: $\sigma_i = \pm 1$, con $i = 1, \ldots N$,
2. *Quenched*: lattice impurities
   - Connectivity matrix: $n_{ik} = n_{ki} = 1, 0$
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$$\mathcal{H} = -\sum_{i<k} J_{ik} n_{ik} \sigma_i \sigma_k$$

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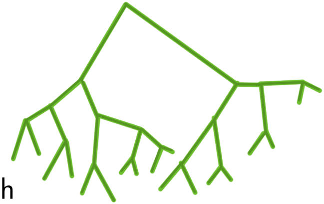
Mean Field

The exact solution in mean field approx. is known (Parisi, 1983):
$n_{ik} = 1 \ \forall\ i, k$, J_{ik} gaussian random var. ($\overline{J} = 0$ and $\overline{J^2} = 1/N$)

- Infinite degenerate states
- Ultrametric organization

Bethe Lattices

Spins are located on the nodes of a Poisson graph



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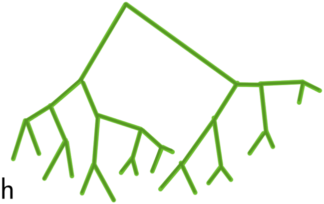

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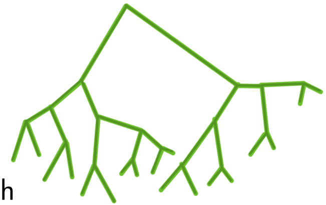

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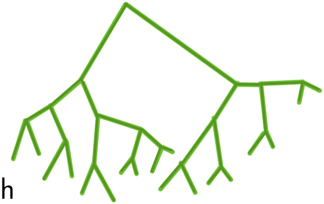

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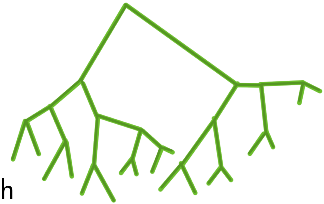

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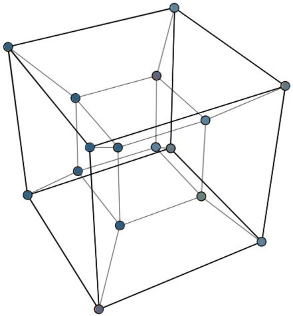
Poisson graphs still lack a notion of distance!

Hypercube Model

Hypercube Model

1. Spins are distributed on the vertex of a D-dimensional hypercube



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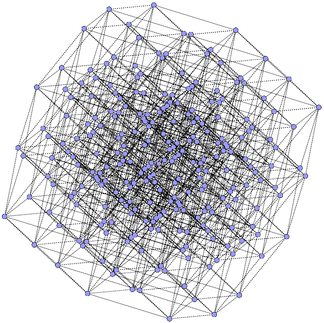

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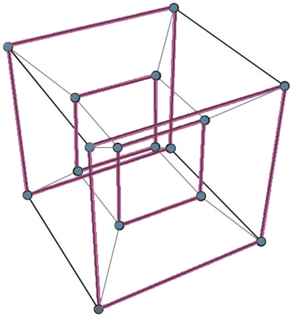

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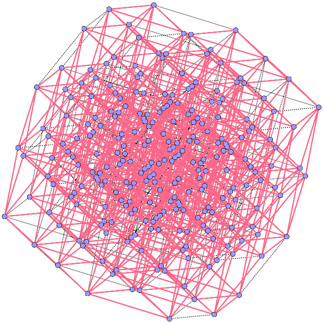

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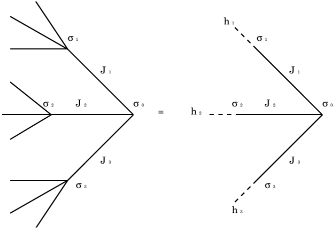

Bethe Approximation

Ferromagnet

$$K_c^{\mathrm{FM}} = \mathrm{atanh} \frac{1}{\langle z \rangle_1 - 1} \qquad (1)$$

SG

$$K_c^{\mathrm{SG}} = \mathrm{atanh} \frac{1}{\sqrt{\langle z \rangle_1 - 1}} \,, \qquad (2)$$

Ferromagnet: $B = \frac{\overline{\langle \mathcal{M}^4 \rangle}}{\overline{\langle \mathcal{M}^2 \rangle}^2}$, $\mathcal{M} = \sum_i \sigma_i$

Random connectivity model

$$\langle z \rangle_1 = 1 + z - \frac{z}{D}$$

Fixed connectivity model

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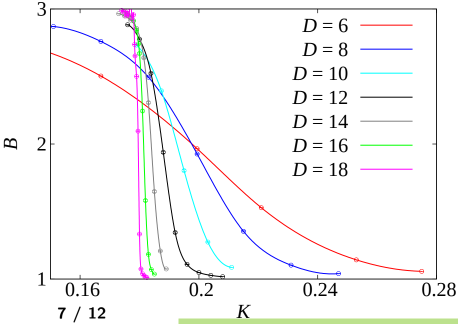

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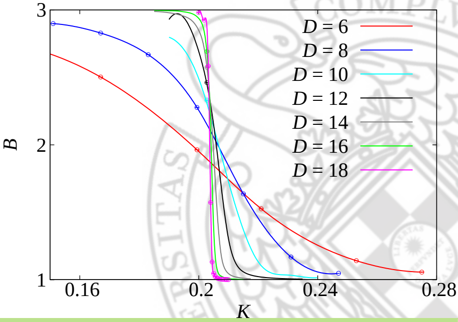
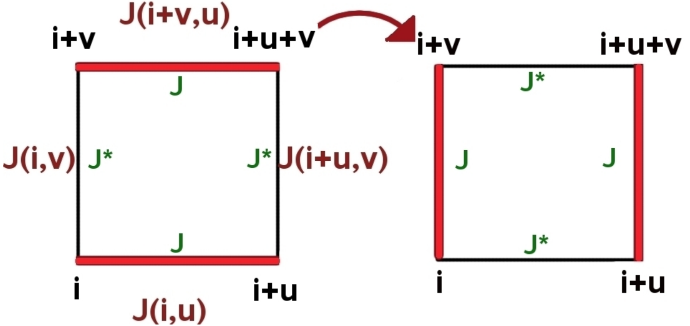


Generation of fixed connectivity graphs

Dynamic Monte Carlo method



Off-equibrium results

Time correlation function

tells about the memory at $t + t_{\mathrm{w}}$ of the configuration at t_{w}

$$C(t, t_{\mathrm{w}}) = \frac{1}{N}\overline{\sum_i \sigma_i(t+t_{\mathrm{w}})\sigma_i(t_{\mathrm{w}})} \Rightarrow \begin{cases} C = 1 & \longrightarrow \text{same config.} \\ C = 0 & \longrightarrow \text{no memory} \end{cases}$$

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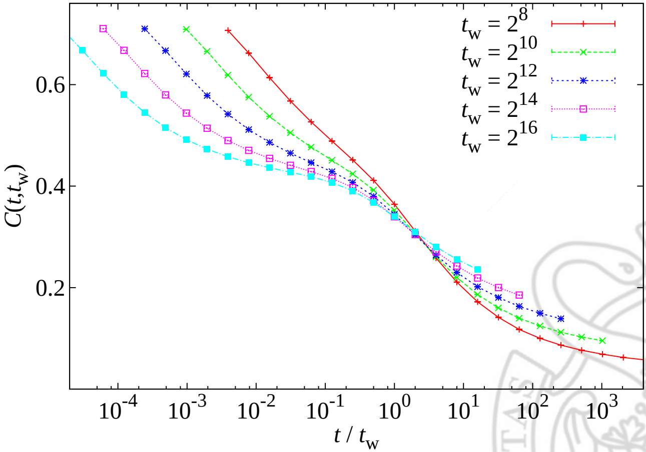
For $t_{\rm w}$ fixed $C(t, t_{\rm w}) \sim M(t, t_{\rm w})$: thermoremanent magnetization

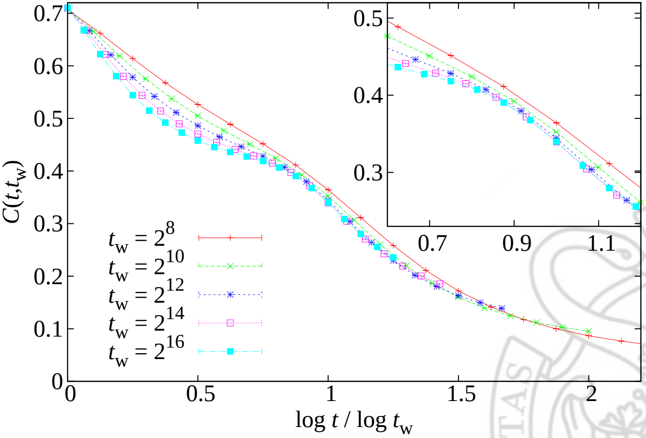
experimentally $M(t, t_{\rm w}) \sim f\left(\frac{t}{t_{\rm w}}\right)$ (Full Aging)

No Full Aging



Many time-sectors $C(t, t_{\rm w}) = \sum_i f_i \left(h_i(t_{\rm w})/h_i(t + t_{\rm w})\right)$!!

Yes, Bertin-Bouchead scaling



Infinite spectrum of time-sectors!!

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Link correlation function

$$C_\mathrm{link}(t, t_\mathrm{w}) = \frac{1}{DN}\overline{\sum_{ik} n_{ik}\ \sigma_i(t+t_\mathrm{w})\sigma_k(t+t_\mathrm{w})\sigma_i(t_\mathrm{w})\sigma_k(t_\mathrm{w})}$$

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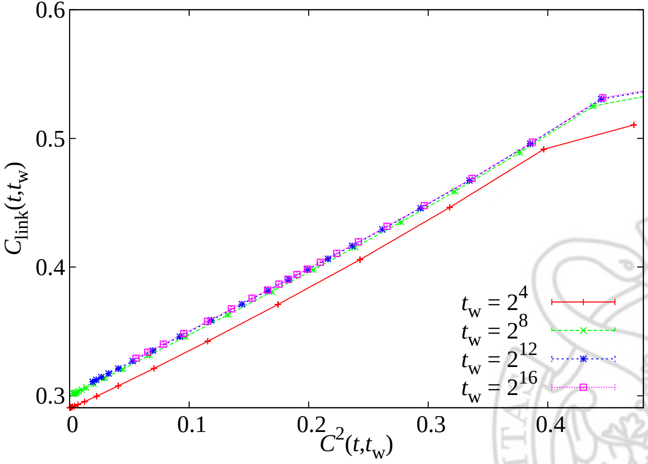
In Sherrington-Kirkpatrick $C_\mathrm{link}(t, t_\mathrm{w}) = C^2(t, t_\mathrm{w})$

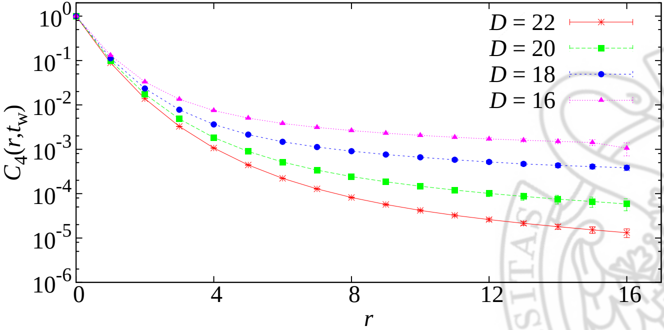
C_{link} vs. C^2



Spatial correlation function

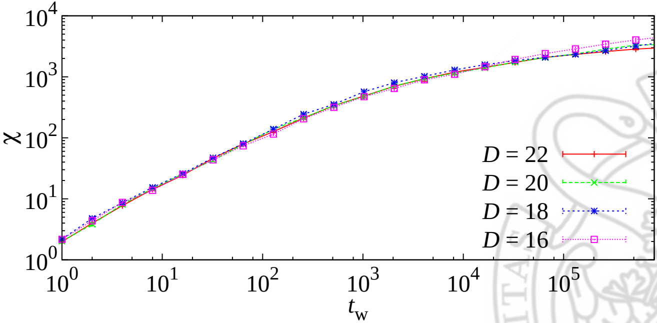
$$c_4(\boldsymbol{r}, t_{\mathrm{w}}) = \frac{1}{N}\overline{\sum_{\boldsymbol{x}} \sigma_{\boldsymbol{x}}^{(1)}(t_{\mathrm{w}})\sigma_{\boldsymbol{x}+\boldsymbol{r}}^{(1)}(t_{\mathrm{w}})\sigma_{\boldsymbol{x}}^{(2)}(t_{\mathrm{w}})\sigma_{\boldsymbol{x}+\boldsymbol{r}}^{(2)}(t_{\mathrm{w}})}$$

$$C_4(r, t_{\mathrm{w}}) = \frac{1}{N_r}\sum_{\boldsymbol{r}, |\boldsymbol{r}|=r} c_4(\boldsymbol{r}, t_{\mathrm{w}})$$



SG susceptibility

$$\chi_{\mathrm{SG}}(t_{\mathrm{w}}) = N\overline{q^2(t_{\mathrm{w}})} \text{ where } q(t_{\mathrm{w}}) = \sum_i \sigma_i^{(1)}(t_{\mathrm{w}})\sigma_i^{(2)}(t_{\mathrm{w}})$$



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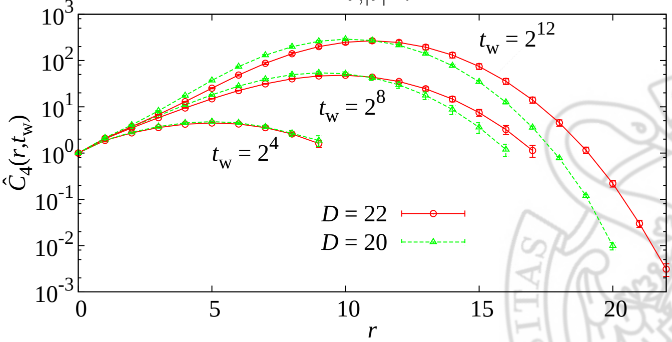

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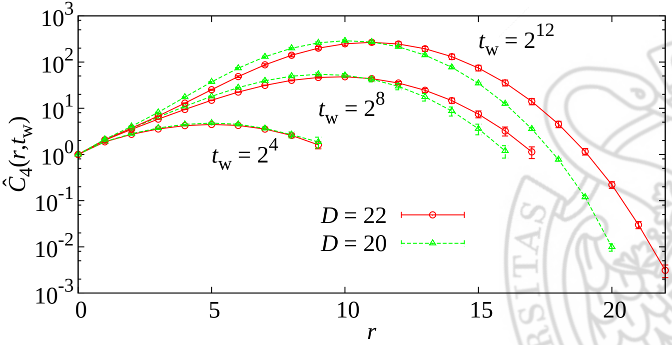
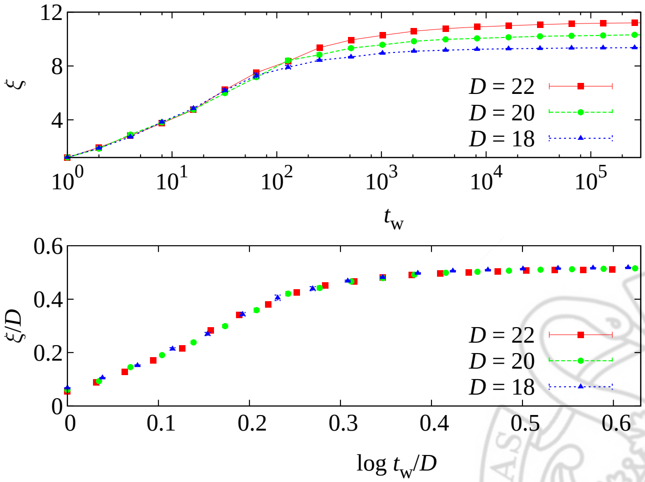
Coherence length

$$\xi_{0,1}(t_\mathrm{w}) = \frac{\int_0^\infty \mathrm{d}r \; r \; \hat{C}_4(r, t_\mathrm{w})}{\int_0^\infty \mathrm{d}r \; \hat{C}_4(r, t_\mathrm{w})}$$



Finite size effects



$$\xi_{D=\infty}(t_w) \propto \log t_w$$

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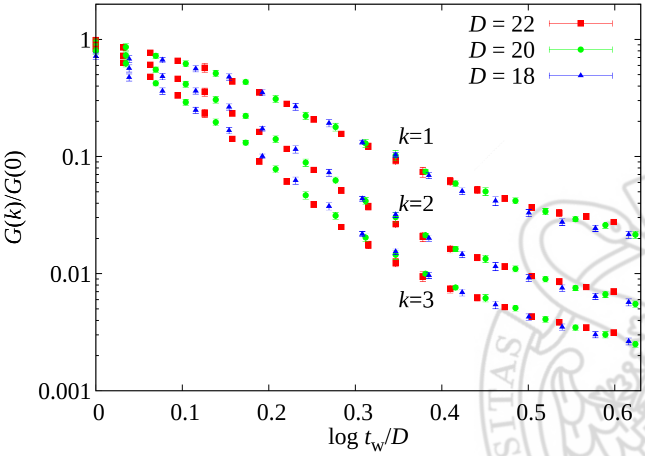

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4. We have studied finite size effects, finding that data follow a naive finite size scaling ansatz