SCALING ANALYSIS OF THE SITE-DILUTED ISING MODEL IN TWO DIMENSIONS

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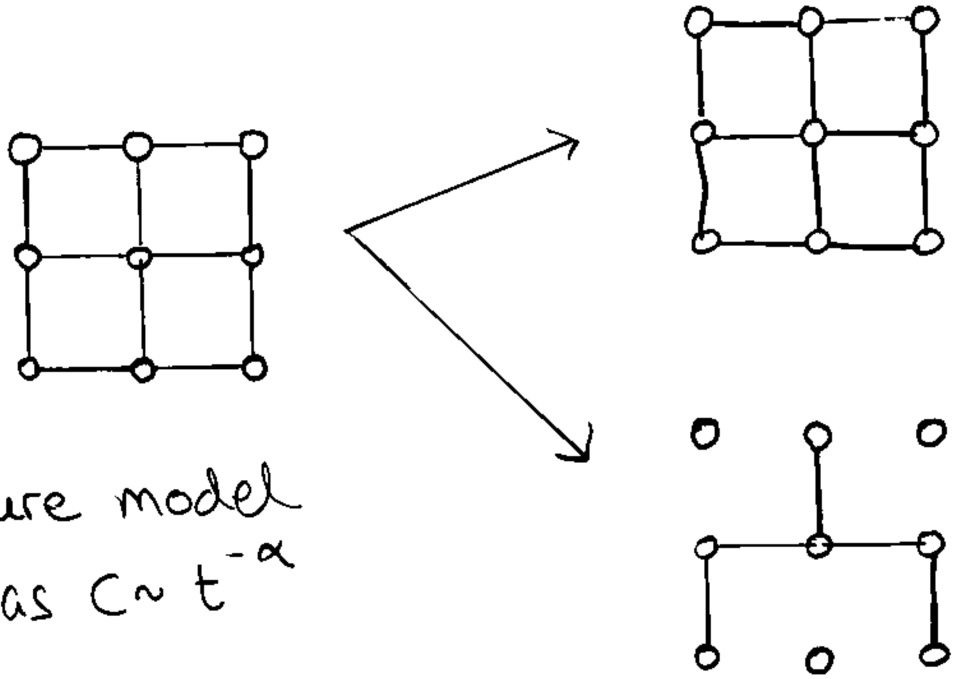
HARRIS CRITERION



Pure model has $C \sim t^{-\alpha}$

If $\alpha > 0$ disorder is relevant

If $\alpha < 0$ " " irrelevant

If $\alpha = 0$????

WEAK UNIVERSALITY HYPOTHESIS

$\Rightarrow \alpha \rightarrow \alpha(p)$, $\beta \rightarrow \beta(p)$, $\gamma \rightarrow \gamma(p)$ etc

where $p \leftrightarrow$ dilution.

In fact supporters of this hypothesis claim $\alpha(p) < 0$ so that C is finite.

STRONG UNIVERSALITY HYPOTHESIS

$$\Rightarrow \alpha(p) = \alpha \,, \quad \beta(p) = \beta, \quad \text{etc}$$

but there are LOGARITHMIC CORRECTIONS

$$C \sim t^{-\alpha} |\ln t|^{\hat{\alpha}}$$

$$m \sim t^{\beta} |\ln t|^{\hat{\beta}}$$

$$\chi \sim t^{\gamma} |\ln t|^{\hat{\gamma}}$$

etc.

Shalaev, Shankar, Ludwig, Jug $\Rightarrow$ [SSLJ]

$\alpha = 0$ $\quad \hat{\alpha} = 0 \rightarrow C \sim \ln|\ln t|$

[Dotsenko + Dotsenko]

$\beta = 1/8$ $\quad \hat{\beta} = -1/16$

$\gamma = 7/4$ $\quad \hat{\gamma} = 7/8$

$\delta = 15$ $\quad \hat{\delta} = 0$

$\nu = 1$ $\quad \hat{\nu} = 1/2$

$\eta = 1/4$ $\quad \hat{\eta} = 0$

obey scaling relations

obey scaling relations for log corrections

[R.K., D. Johnston, W. Janke, 2006]

Roder, Adler, Janke (1999) $\Rightarrow$ clear direct quantitative and conclusive confirmatio[n] that

$$\gamma = 7/4 \text{ and } \hat{\gamma} = 7/8$$

in RBIM.

But couldn't really confirm $\hat{\alpha} = 0$.

Numerical support for $C \sim \ln \ln t$ $(\alpha = 0, \hat{\alpha} = 0)$ claimed by

Andreichenko, Dotsenko, Selke, Wang (1990)
Talapov, Shchur (1994)
Wiseman, Domany (1995)
Aarão Reis, de Queiroz, dos Santos, Stauffer (1997)
Hasenbusch, Toldin, Pelissetto, Vicari (2008)

RBIM

Ballesteros, Fernández, Martín-Mayor,
Muñoz Sudupe, Parisi, Ruiz-Lorenzo (1997)
Selke, Shchur, Vasilyev (1998)

RSIM

But Kim (2000) argues

"double log FSS behaviour of the specific heat does not imply its divergence"

T

i.e., cannot distinguish

$$C \sim A + B \ln(1 + C \ln L)$$

from

$$C \sim D + E L^{\alpha/\nu} \qquad [\alpha/\nu < 0]$$

unless have enormous lattices.

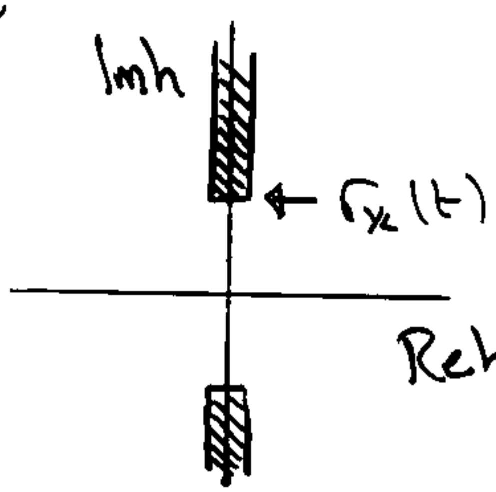
Indeed ... Hadjiagapiou, Malakis + Martinos (2008) $\Rightarrow$

α is negative !!

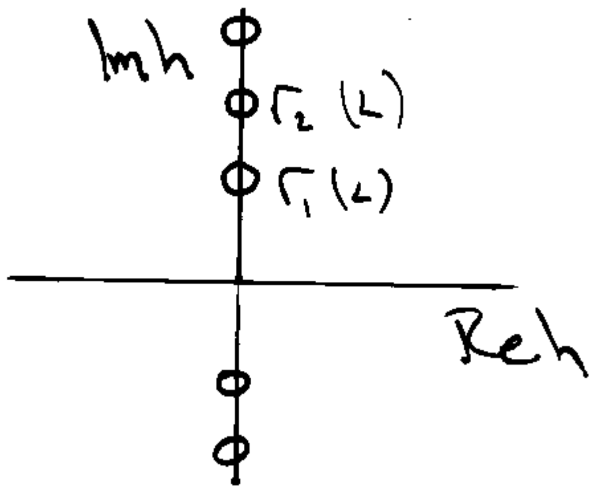
LEE-YANG ZEROS APPROACH

$$Z(\beta, h) = 0 \quad \text{at certain } h \in \mathbb{C}$$

In fact



$L = \infty$



$L < \infty$

YANG-LEE EDGE: $\quad r_{YL}(t) \sim t^{\Delta} \, |\ln t|^{\hat{\Delta}}$

$$\Delta = \beta + \gamma = \frac{15}{8} \quad \text{for RSIM}$$

$$\hat{\Delta} = \hat{\beta} - \hat{\gamma} = -\frac{15}{16}$$ [from RK, Johnston, Janke 2006

DENSITY OF ZEROS: $\quad g(r) \sim r^{a_2 - 1} \, |\ln r|^{\hat{a}_2}$

Integrate the density of zeros

$$G(r) = \int_{r_{YL}}^{r} g(s)\, ds$$

$$\sim r^{a_2} \, |\ln r|^{\hat{a}_2}$$

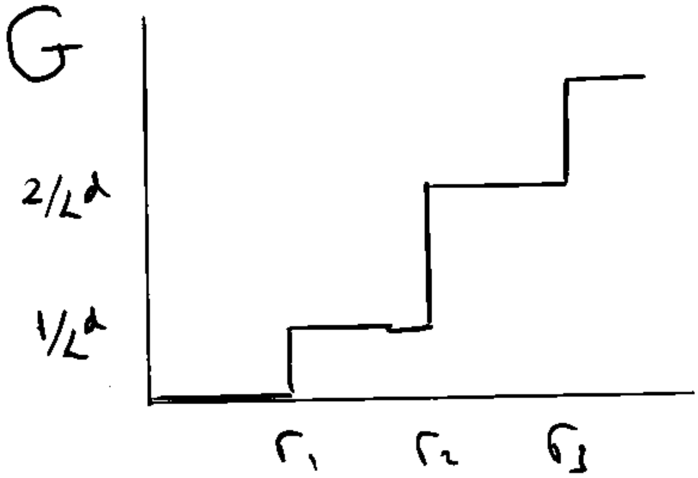
Here

$$a_2 = \frac{2-\alpha}{\Delta}$$

$$\hat{a}_2 = \hat{\alpha} - 2\,\frac{\hat{\Delta}}{\Delta} - 1$$ [RK, Johnston, Janke 200

Numerically, the integrated density of zeros for finite systems is [Janke, Johnston, RK, 2004]

$$G = \frac{j \cdot 1/2}{L^d}$$



Simulate

$$Z_L(\beta,h) = \sum_{\{\sigma_i\}} e^{\beta \underbrace{\sum_{\langle ij \rangle} \epsilon_i \epsilon_j \sigma_i \sigma_j}_{S} + h \underbrace{\sum_i \epsilon_i \sigma_i}_{M}}$$

$$\epsilon_i = \text{quenched random variables} = \begin{cases} 1 & \text{with prob. } p \\ 0 & \text{" " } 1-p \end{cases}$$

Use square lattice, periodic boundary conditions

$$Z_L(\beta,h) = \sum_{S,M} \rho(S,M) e^{\beta S + hM}$$

$$= \sum_M \rho(M) e^{hM}$$

$$= \sum_M \rho(M) e^{irM} \quad \text{if } h = ir$$

$$= Z_L(\beta,0) \langle \cos rM + i \underbrace{\sin rM}_{\to 0} \rangle$$

$\Rightarrow$ Zeros given by $\langle \cos r_j M \rangle = 0$.

Find zeros for each sample and then average over samples

(1000 samples for $L=32$ to 160 for $L=256$)

Use

$p = 0.88889$ — weak randomness

$p = 0.75$ — moderate

$p = 0.66661$ — strong

RESULTS WILL TURN OUT TO BE INDEP OF p.

Expect $\quad \chi_L \sim L^{\gamma/\nu} (\ln L)^{\frac{\nu\hat{\gamma} - \gamma\hat{\nu}}{\nu}}$

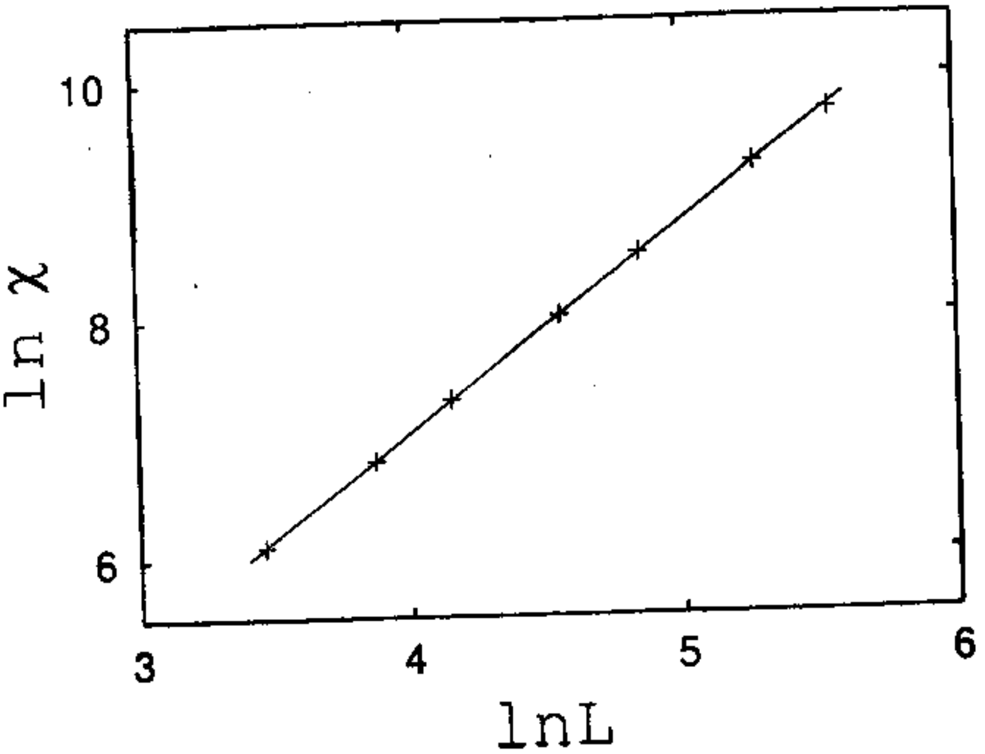
SSLJ $\Rightarrow$ $\nu\hat{\gamma} - \gamma\hat{\nu} = 0$ for $RSIM_2$

This serves as check that $\hat{\gamma} = 7/8 \quad \hat{\nu} = 1/2$

We find

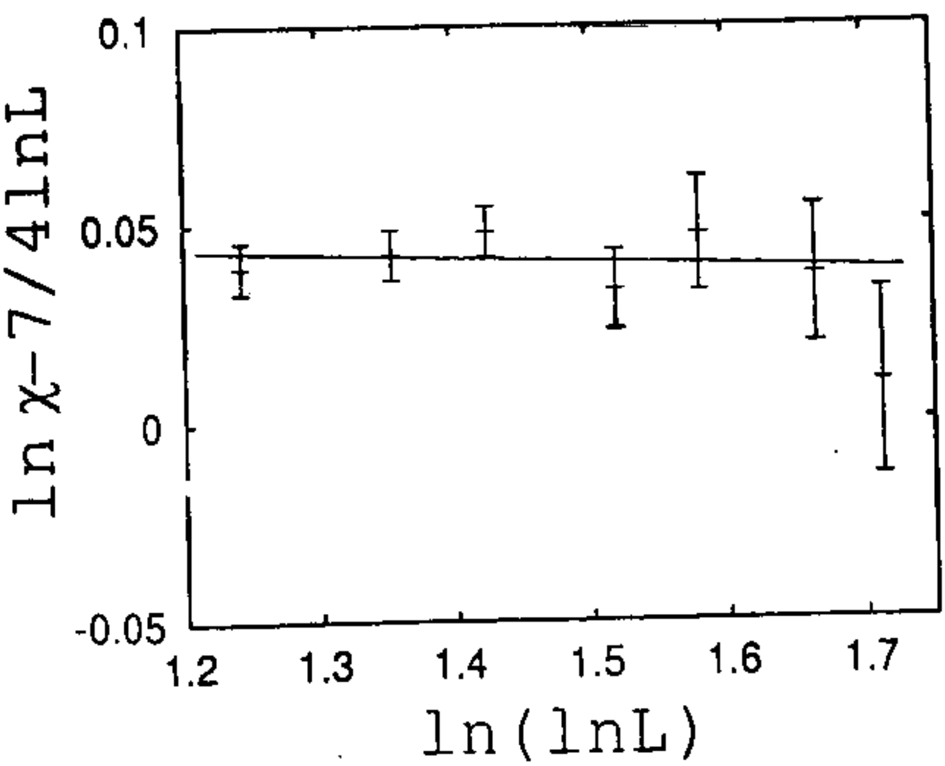
$$\nu\hat{\gamma} - \gamma\hat{\nu} = -0.01(3), \quad 0.02(3), \quad 0.01(3)$$

for the three dilutions.



$\Rightarrow \frac{\gamma}{\nu} = 1.747(7)$

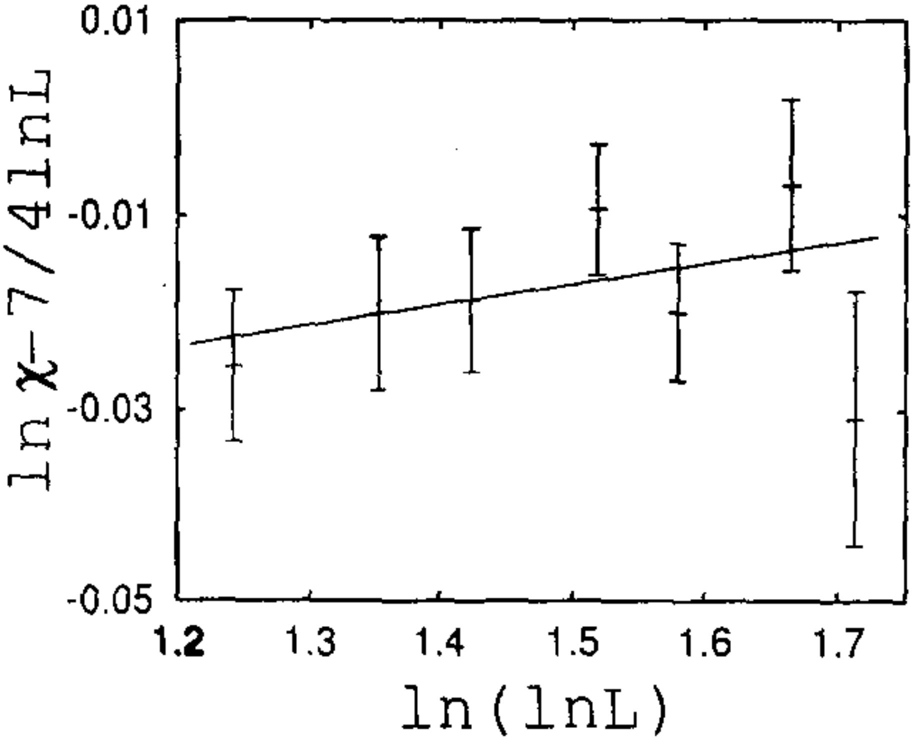
cf 1.75 from theory



$\Rightarrow \nu\hat{\gamma} - \gamma\hat{\nu}$

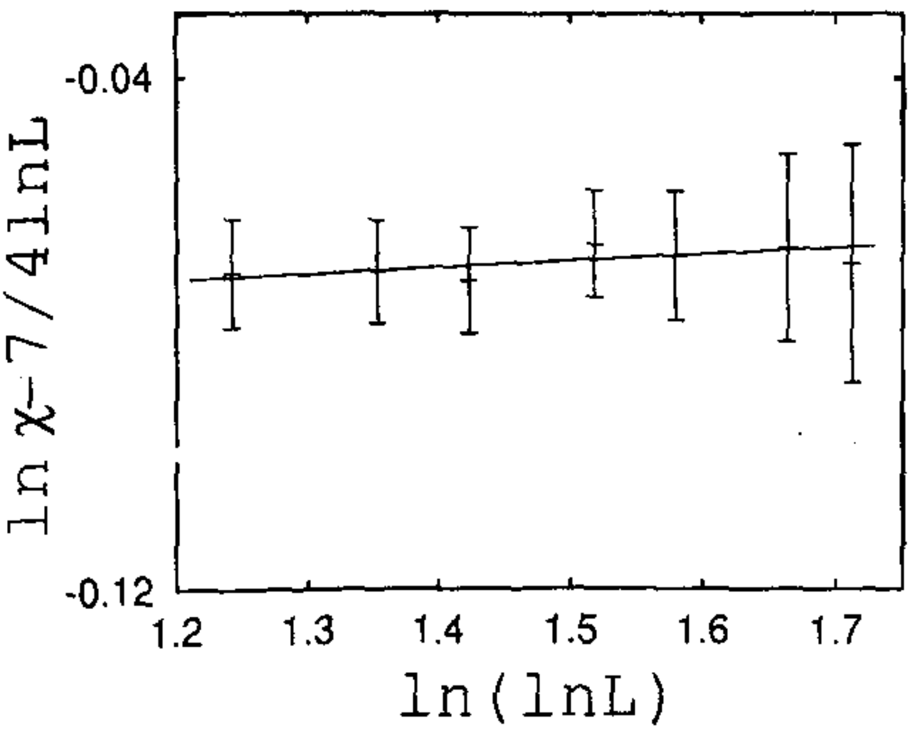
$= -0.01(3)$

cf 0 from theory



Slope = 0.02(1)

p = 0.6666 |



Slope = 0.01(1)

Expect $r_1 \sim L^{-\frac{\Delta}{\nu}} (\ln L)^{\frac{\nu \hat{\Delta} + \Delta \hat{\nu}}{\nu}}$

Theory $\Rightarrow$ $\nu \hat{\Delta} + \Delta \hat{\nu} = 0$

$$\Rightarrow \hat{\Delta} = -\frac{\Delta \hat{\nu}}{\nu} = -\frac{15}{16} = -0.9375$$

we find

$$\hat{\Delta} = -0.95(2), \quad -0.95(3), \quad -0.95(3)$$

Expect $G = a_1 r^{\frac{2-\alpha}{\Delta}} (\ln r)^{\hat{\alpha}} + a_3$

For transition, require $a_3 = 0$.

we find $a_3 = 0.0000002(4)$

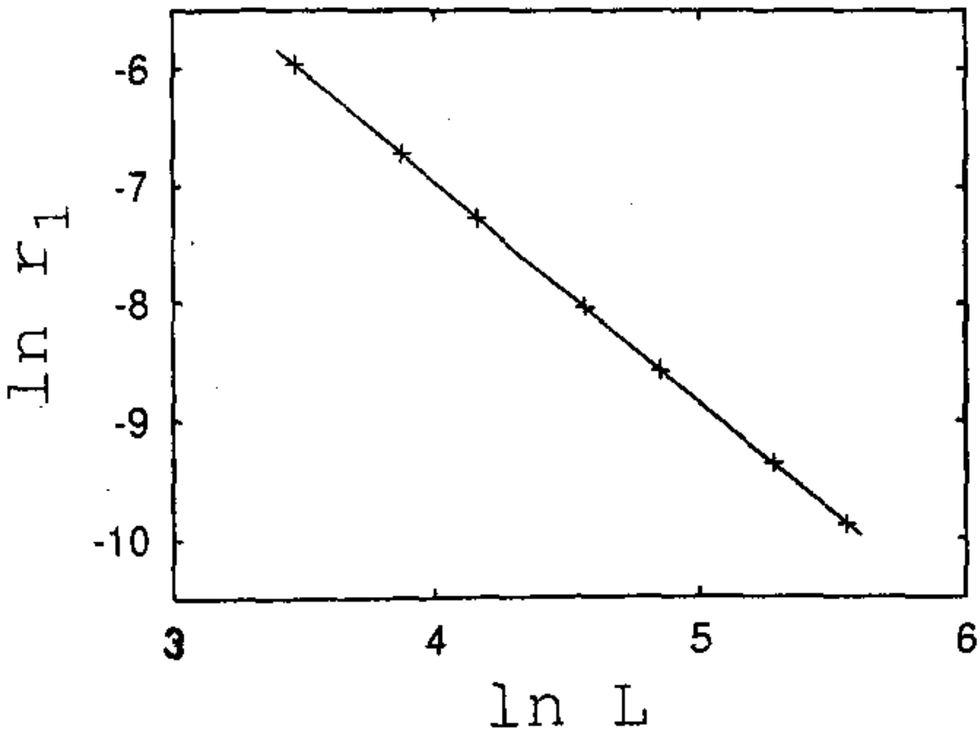
Theory $\Rightarrow$ $\frac{2-\alpha}{\Delta} = \frac{16}{15} = 1.067$

we find $1.076(16) \Rightarrow \alpha = -0.02(3)$

Theory $\Rightarrow$ $\hat{\alpha} = 0$

Find $\hat{\alpha} = -0.02(5), \quad -0.01(3), \quad -0.04(5)$

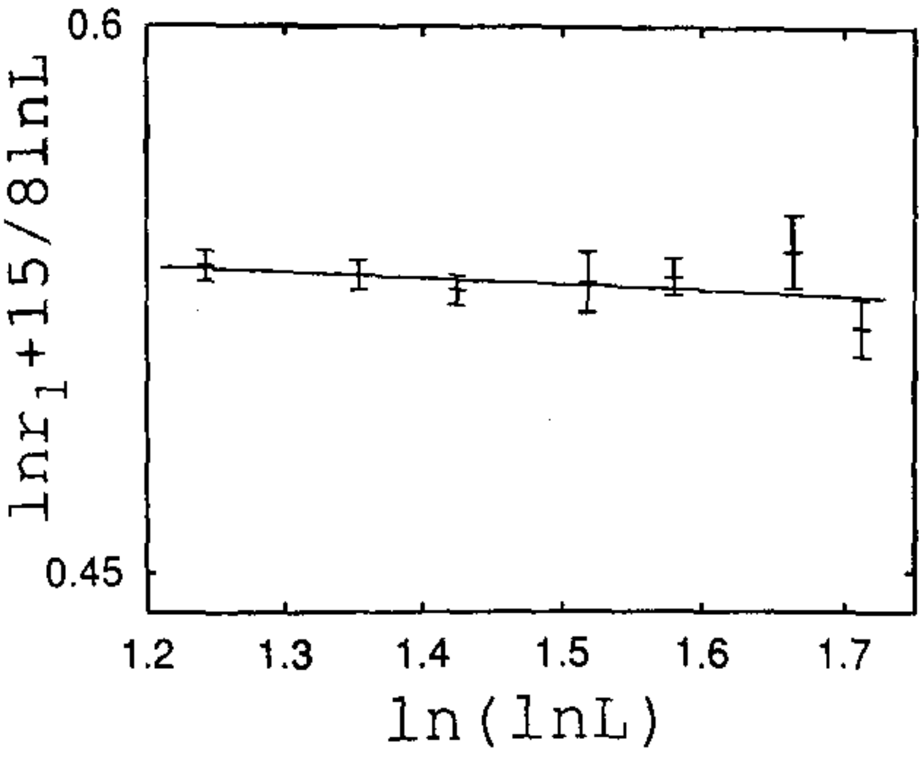
at the three dilution values



$\Rightarrow \frac{\Delta}{\nu} = 1.878(4)$

cf 1.875

from theory



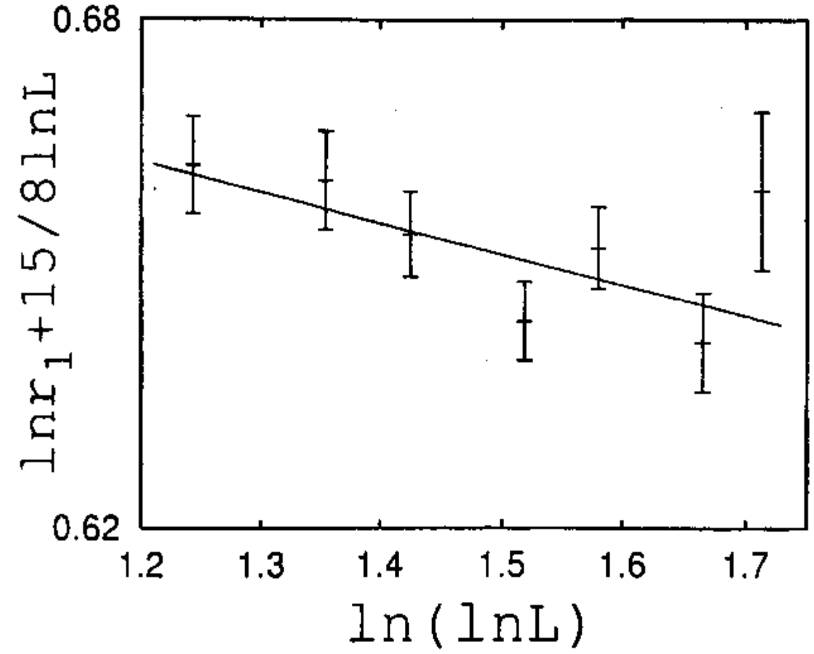
Slope =

$(\nu\hat{\Delta} + \Delta\hat{\nu})/\nu$

$= -0.015\ (14)$

cf 0

from theory



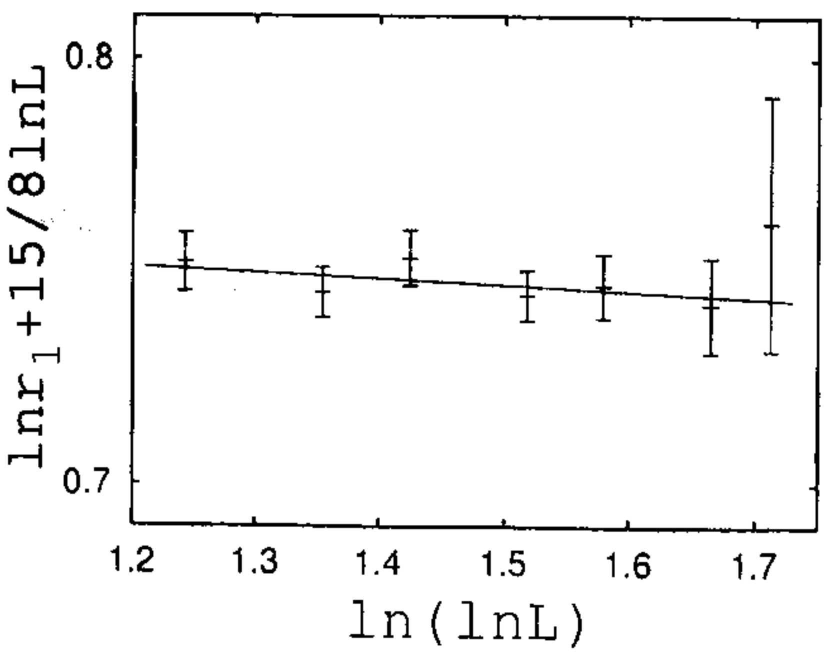
Slope

$= -0.026\,(15)$

cf 0

from theory

$p = 0.66661$

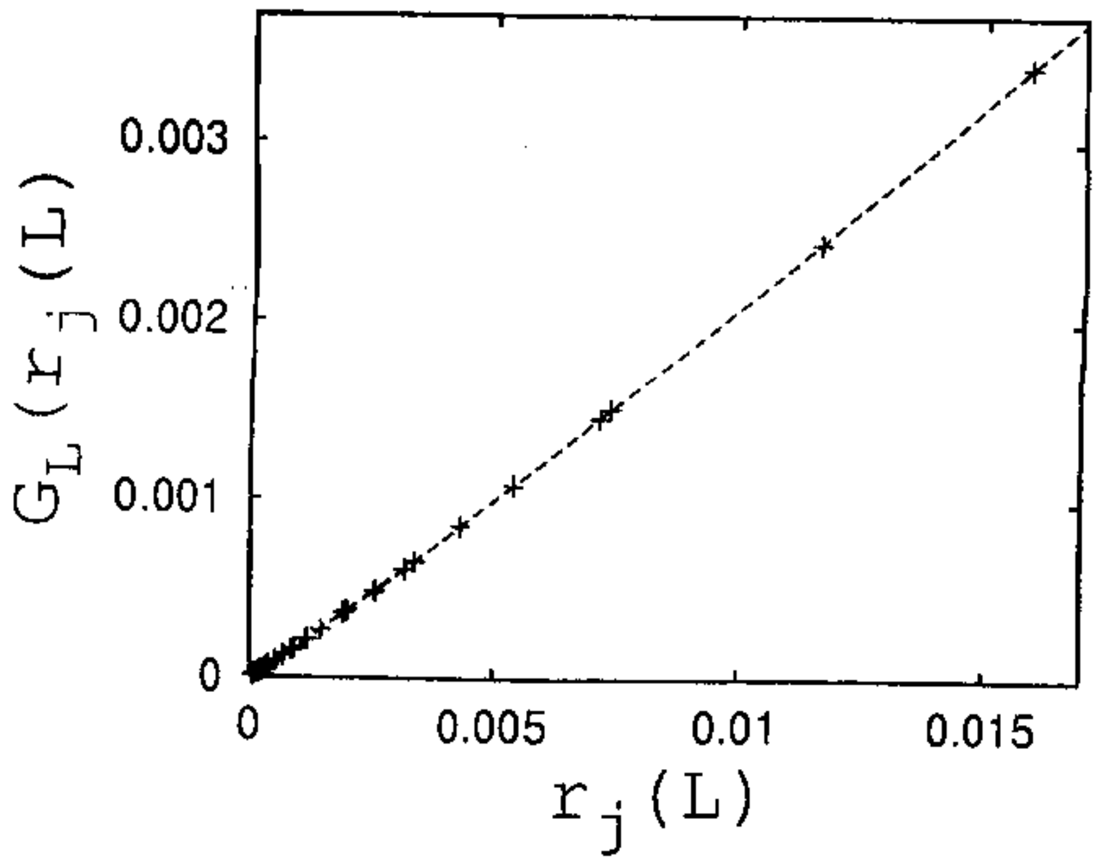


Slope =

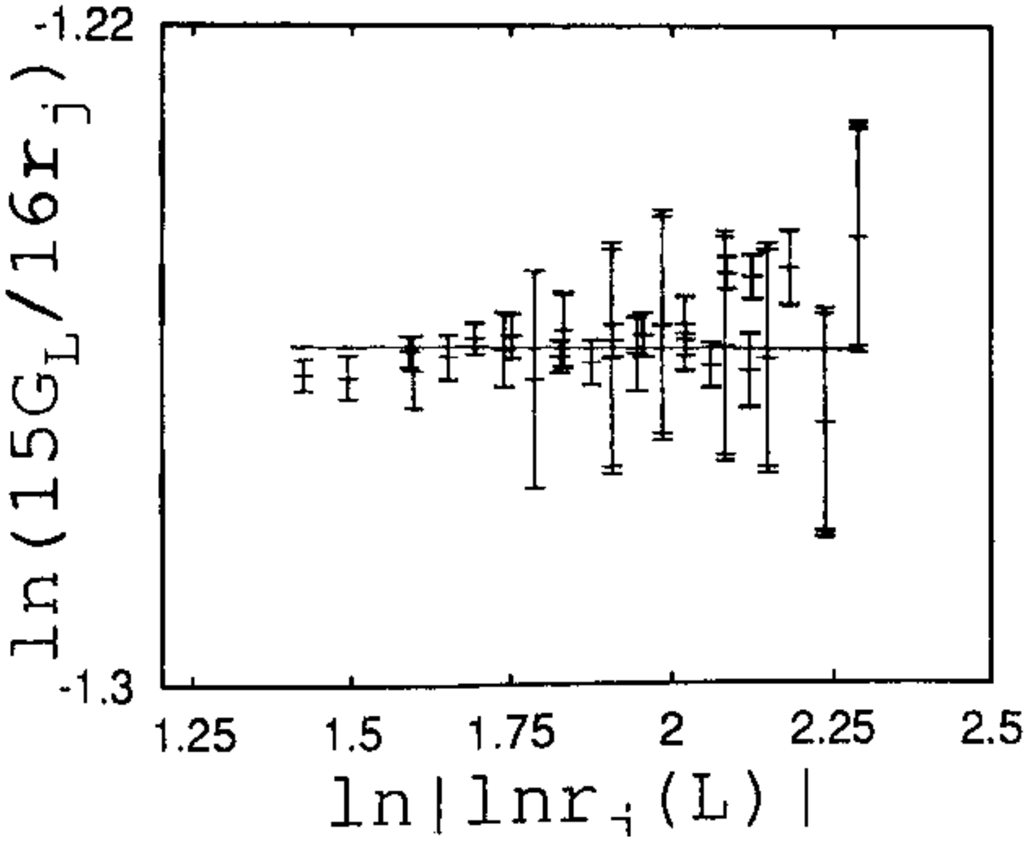
$-0.015\,(22)$

cf 0

from theory



0.003
0.002
0.001
0
$G_L(r_j(L))$
0
0.005
0.01
0.015
$r_j(L)$



-1.22
-1.3
$\ln(15G_L/16r_j)$
1.25
1.5
1.75
2
2.25
2.5
$\ln|\ln r_j(L)|$

CONCLUSIONS

1. Can numerically determine zeros + log corrections for quenched random systems

2. SSLJ values for $\hat{\nu}$, $\hat{\gamma}$ supported

3. Value for $\hat{\Delta}$ from scaling relations for log corrections [RK, D Johnston, W. Janke] verified

4. Density analysis $\Rightarrow$ $\hat{\alpha} \simeq 0$.

5. Results show no systematic dependence on dilution.

   $\Rightarrow$ Strong universality hypothesis supported

NEXT:

Verify $C \sim \ln \ln t$.