On the phase structure of 3D Abelian one-Higgs model on the lattice

Percolation of Vortex Networks

in the U(1) Lattice Higgs Model

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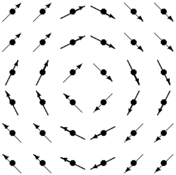
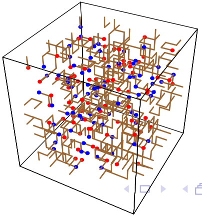
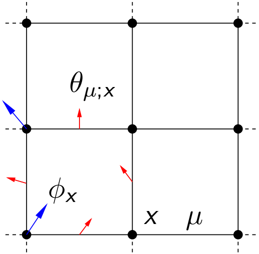
CompPhys07, Leipzig 2007





What is the U(1) lattice Higgs model?



Degrees of freedom:

- Higgs field $\phi(x) = \rho(x)\mathrm{e}^{\mathrm{i}\varphi(x)}$, $\varphi(x) \in [-\pi, \pi)$ on sites x
- Gauge field $\theta_\mu(x) \in [-\pi, \pi)$ on links
  - plaquette angle $\theta_{\mu\nu}(x) = \partial_\mu \theta_\nu(x) - \partial_\nu \theta_\mu(x)$

Action:

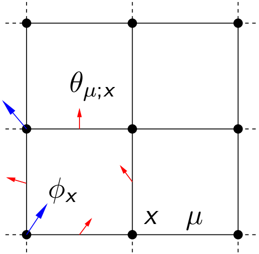
$$S = S_g + S_\phi$$

$$S_g = \beta \sum_{x,\mu<\nu} [1 - \cos \theta_{\mu\nu}(x)]$$

$$S_\phi = -\kappa \sum_{x,\mu} \rho(x)\rho(x+\mu) \cos[\Delta_\mu \varphi(x) - \theta_\mu(x)] + \sum_x \left\{ \rho^2(x) + \lambda \left[\rho^2(x) - 1\right]^2 \right\}$$

κ controls inter-coupling! λ controls Higgs self-coupling!

What is the U(1) lattice Higgs model?



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Objectives and Method

- Goals:
  - phase structure and nature of transitions
  - study and contrast behaviour of

    geometric (cluster) properties

    vs.

    thermodynamic observables

- Method: Monte Carlo
  - Metropolis, Heat-Bath update
  - Multicanonical Method
- fix β and trace out (λ - κ) phase diagram

Standard Observables

- energy (densities)

$$E_l = \frac{1}{L^3}\langle -\kappa \sum_{x,\mu} \rho(x)\rho(x+\mu)\cos\left[\Delta_\mu \varphi(x) - \theta_\mu(x)\right]\rangle$$

- mean Higgs amplitude square

$$\langle \rho^2 \rangle \equiv \frac{1}{L^3} \sum_x \rho^2(x)$$

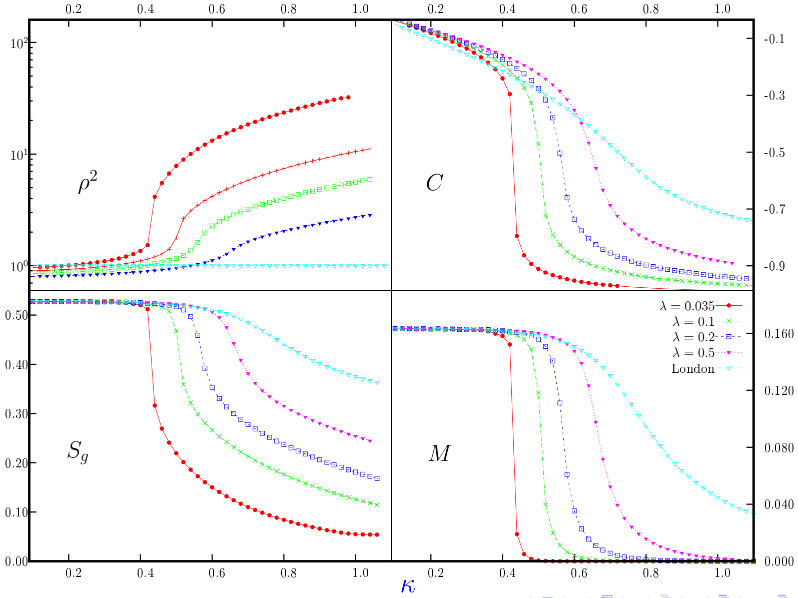
- susceptibilities

$$\chi_{\mathcal{O}} = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$$

- Binder ratios

$$B = \frac{\langle \mathcal{O}^4 \rangle}{\langle \mathcal{O}^2 \rangle^2}$$

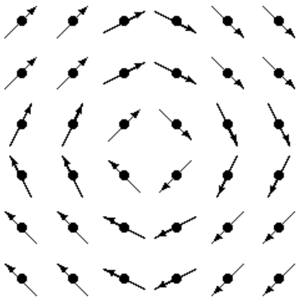
Example for Standard Observables



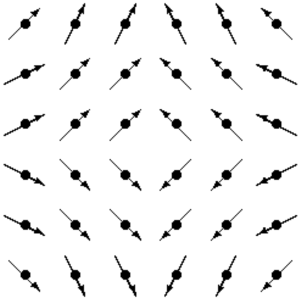
Definition of Vorticity – Continuum Case

- consider a vector field $\theta(\mathbf{x})$
- define vortex with charge m:

$$\mathbf{m} \sim \nabla \times \theta \qquad \oint_{\mathcal{C}} \mathrm{d}\theta = 2\pi m$$



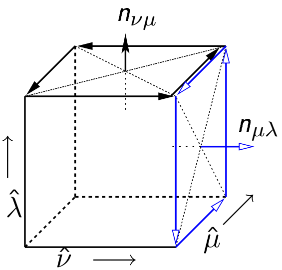
$m = 1$



$m = -1$

pictures by O. Kapikranian

Definition of Vorticity – Lattice Case



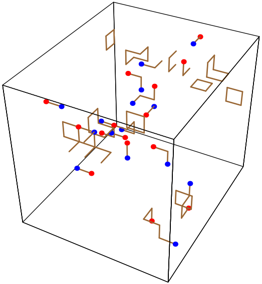
- vortex (flux line) $n = \nabla \times A$
- on lattice $\nabla \times A$ is just plaquette angle $\theta_{\mu\nu}$

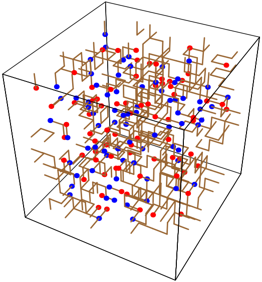
- flux comes in naturally quantized units of 2π since

$$\theta_{\mu\nu} = \alpha + 2\pi n \quad \alpha \in (-\pi, \pi]$$

  $n \in [0, \pm 1, \dots]$ is called excitation
- can also define monopoles $M = \nabla \cdot n$
- generalization to gauge invariant definition possible

Networks and Network-Observables





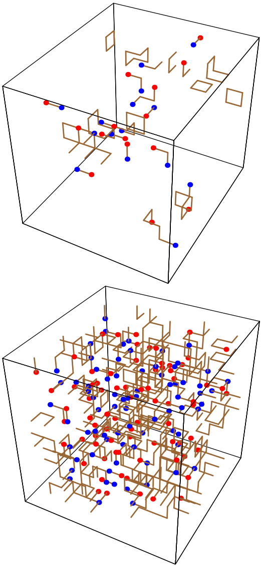
Definitions:

- cluster = set of all **connected** vortex segments
- network = set of all clusters

Observables:

- P probability that one cluster spans whole system (percolation)
- P_∞ largest cluster size of network
- can just ask for the density of vortex lines (no cluster observable)

Networks and Network-Observables



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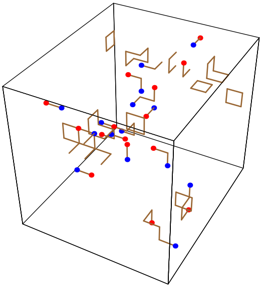

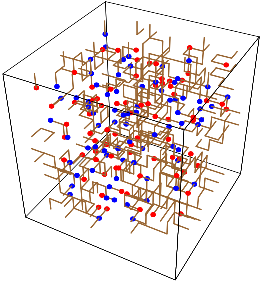

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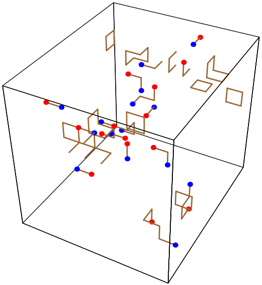

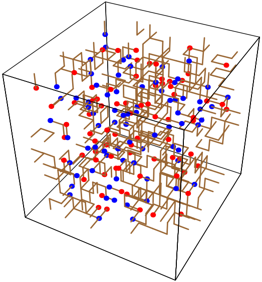

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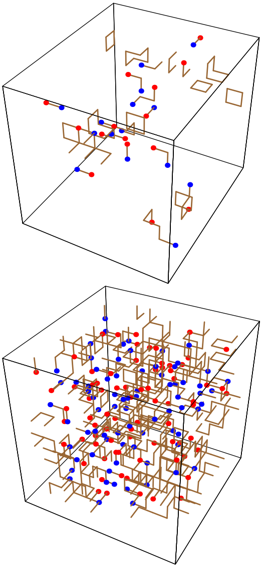

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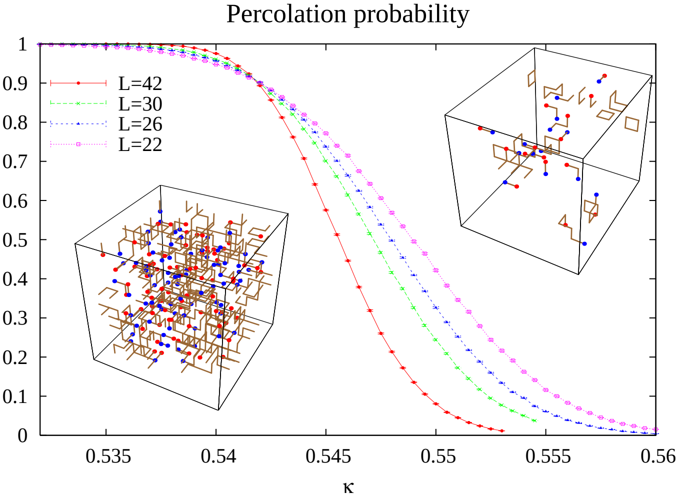
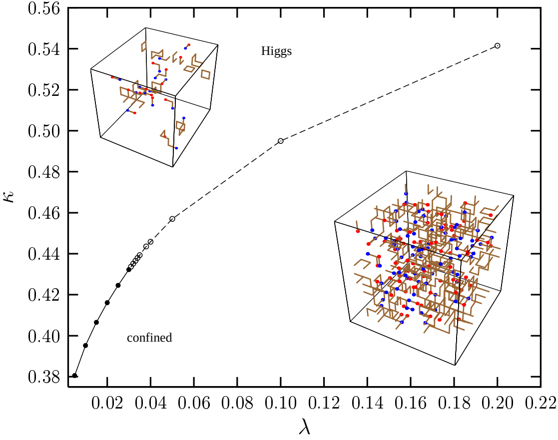
Example for Percolation Probability ($\lambda = 0.2$)



Phase Diagram

β fixed (lattice spacing), $\lambda - \kappa$ plane



- question what is characteristic of transition line? Will discuss two cases (blue lines)

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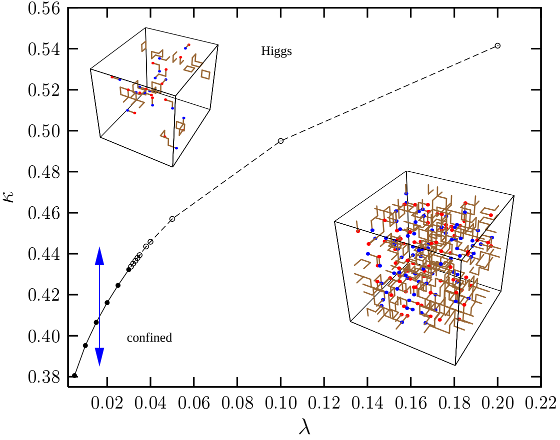

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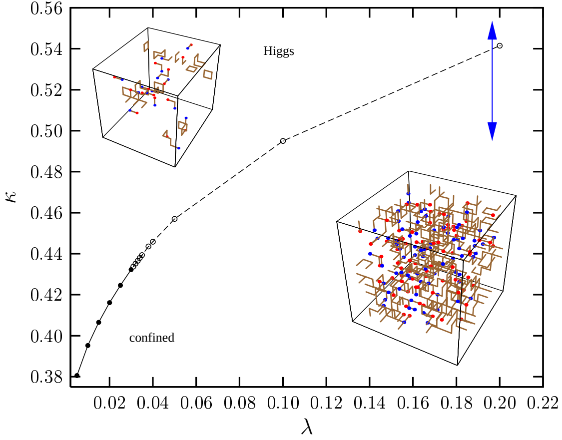

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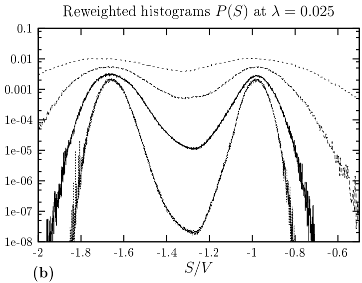
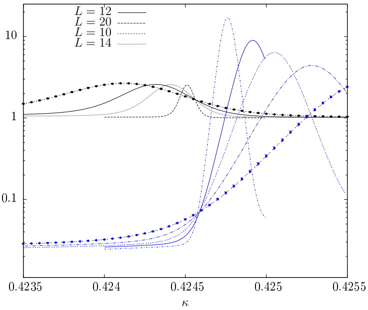
Case 1: First-Order Region



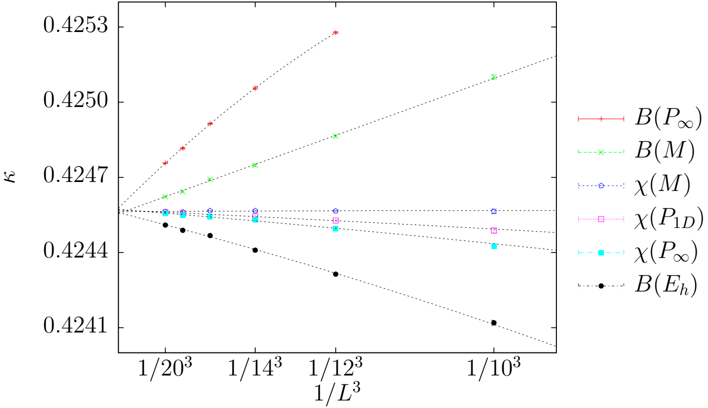


- metastability!
- first order region for $\lambda <= 0.03$

Look at scaling of Binder ratios:

- ordinary energy (black)
- maximal cluster size
- both scale!

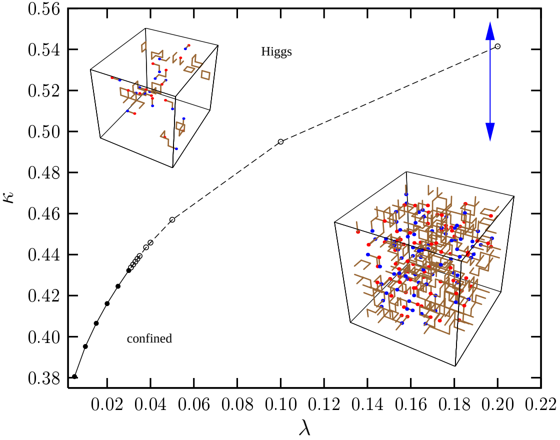
First-Order Region(2), Finite-Size Scaling



- within error bars, infinite volume critical point agrees for both normal and cluster observables

Case 2: Percolation Region

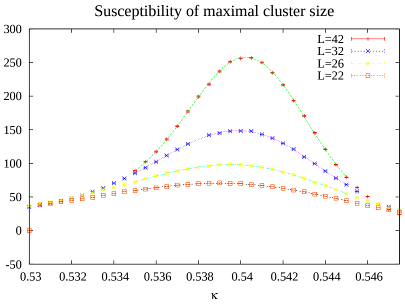
now change parameter $\lambda = 0.2$



Percolation Region, Finite-Size Scaling ($\lambda = 0.2$)

compare scaling of susceptibilities of cluster observables vs. density observables for different lattice sizes:

cluster observable:



have diverging quantity

percolation transition

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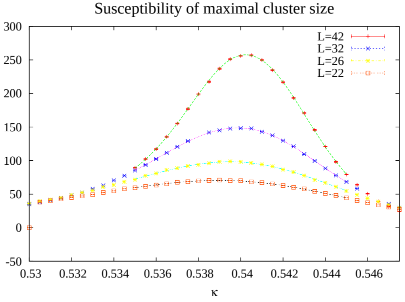
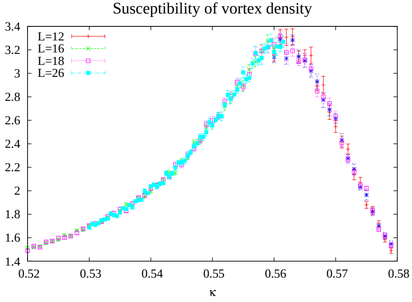


have diverging quantity

percolation transition

density observable:



no diverging quantity

no phase transition

Critical Exponents of Percolation Transition

- what are the exponents describing the percolation transition?

- we determine critical exponents by collapsing data onto a universal curve

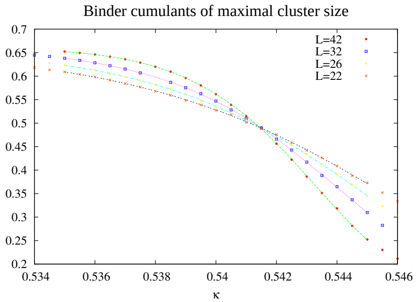
$$\mathcal{O}_L(\kappa) = L^{\lambda_{\mathcal{O}}/\nu} f_{\mathcal{O}}\left(xL^{1/\nu}\right), \quad x \equiv \frac{\kappa - \kappa_c}{\kappa_c}.$$

- exponents: $\nu_{\rm per}$ correlation length

  $\beta_{\rm per}$ $\quad P_\infty = L^{-\beta_{\rm per}/\nu_{\rm per}} f\left(xL^{1/\nu_{\rm per}}\right)$

  $\gamma_{\rm per}$ $\quad \chi(P_\infty) = L^{\gamma_{\rm per}/\nu_{\rm per}} g\left(xL^{1/\nu_{\rm per}}\right)$

Exponent ν from Data Collapse



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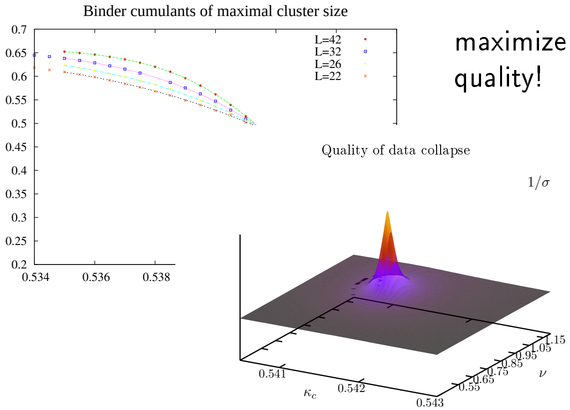
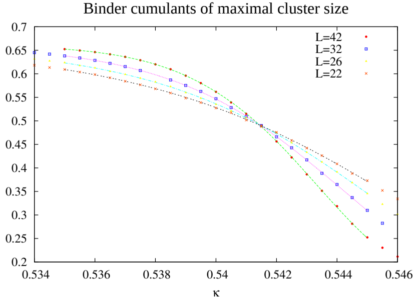


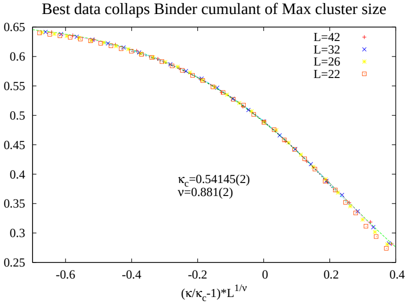
maximize collapse quality!

Exponent ν from Data Collapse

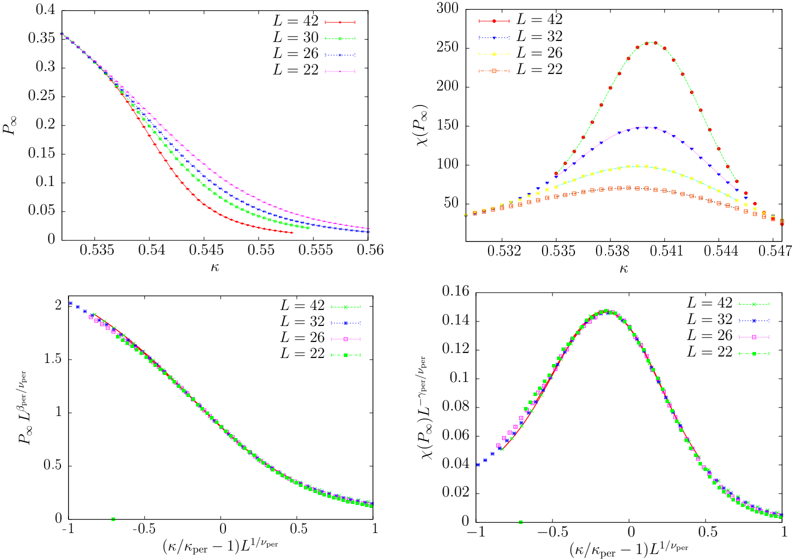


best collapse $\Longrightarrow$

$\nu = 0.881(2)$



Exponents β and γ



$\beta = 0.43(2)$

$\gamma = 1.76(2)$

Critical Exponents – Results

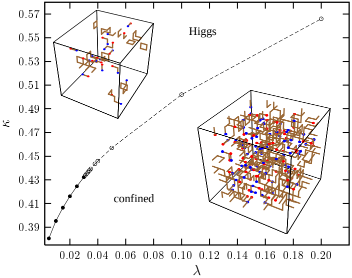
	$\nu_{\rm per}$	$\beta_{\rm per}$	$\gamma_{\rm per}$
	0.881(2)	0.43(2)	1.76(2)
reference[1]:	0.879	0.44	1.8

vortex network has ordinary 3D percolation exponents!

[1] taken from Stauffer & Aharony

Summary and Interpretation

- throughout phase diagram have two **distinct phases** which are separated by a line.
- for small λ **first-order line**
- line then continuous as a **percolation line** of networks with no thermal phase transition



A Kertész Line!

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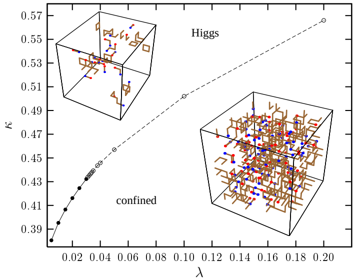

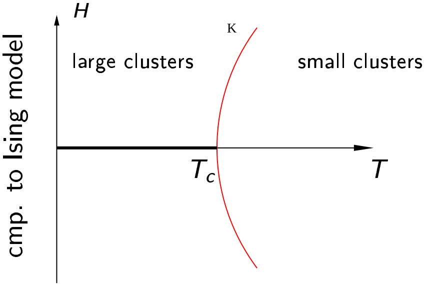

A Kertész Line!

Conclusion

- analysed behaviour of vortex network in U(1) Lattice Higgs model
- clarified nature of transitions
- in phase diagram can distinguish two cases

	local observables	**cluster observables**
First-Order region	both types show typical scaling. Percolation threshold agrees with therm. critical point	
Kertész line	no scaling, no singularities, crossover effect	diverging quantities, percolation exponents

Thank You

literature:

- S. Wenzel, E. Bittner, W. Janke, A.M.J. Schakel, A. Schiller, Phys. Rev. Lett. **95** (2005) 051601.
- S. Wenzel, E. Bittner, W. Janke, A.M.J. Schakel, Nucl. Phys. B (2007), in press.
- E. Bittner, A. Krinner, W. Janke, Phys. Rev. **B72** (2005) 094511.

Remaining Problems