Heat flux and Casimir forces in non-equilibrium scenarios

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Dispersion forces and dissipation – Nov 2017

Outline

- Introduction
- Model, equations of motion and long-time limit
- Expectation values of the energy-momentum tensor
- Two plates configuration
- Casimir force
- Heat flux: Some cases
- Heat flux: Numerics
- Conclusions

Introduction

Two parallel plates

Dispersion phenomena

Casimir-Lifshitz Forces

Heat flux

Equlibrium

Non-Eq

Equlibrium

Non-Eq

Lifshitz Formula

$Q = 0$

Landauer Formula

First-principles quantum approach

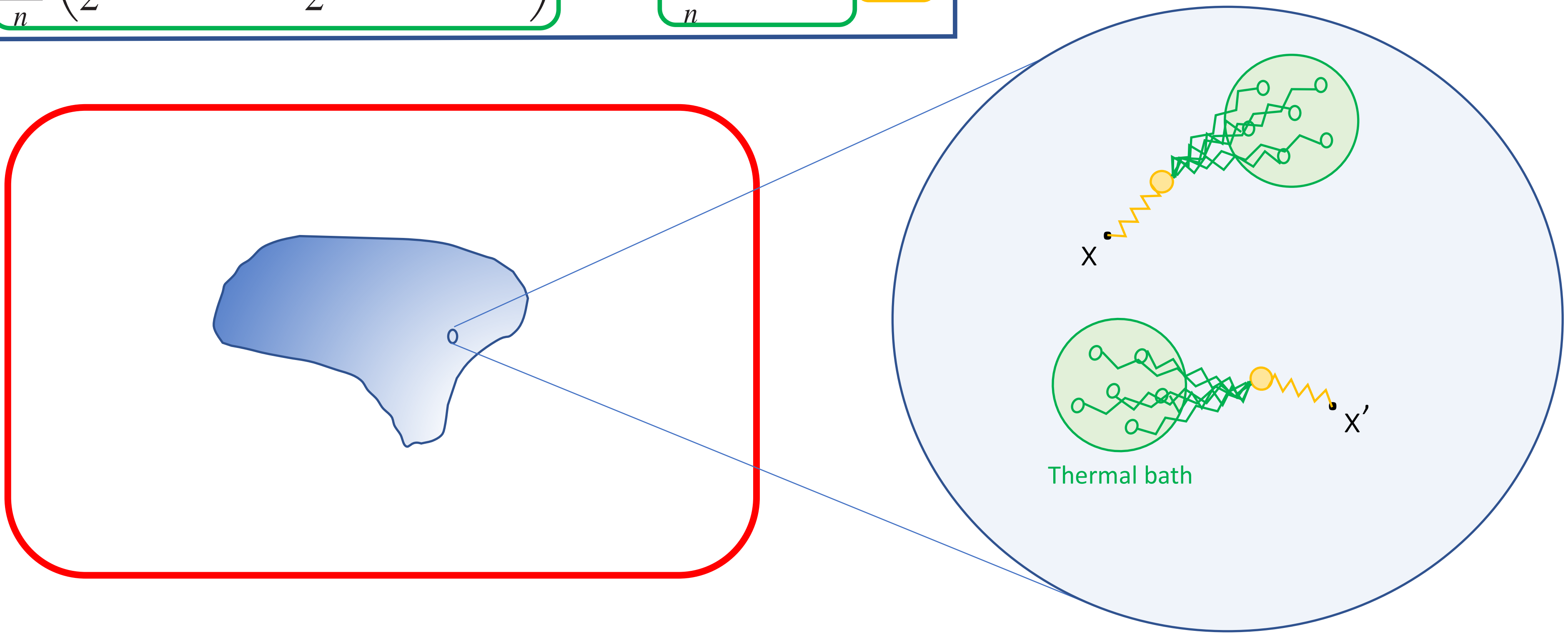
Formula applications

Technological applications

Refs: 1 – AERL, Phys. Rev. D 95, 025009 (2017).
2 – AERL, Poggi P. M., Lombardo F. C., Giannini V., arXiv:1709.09277 (Oct2017). Submitted to PRA.

Model, Eqs. Motion, Long-time Limit

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_{\phi-S} + \mathcal{L}_B + \mathcal{L}_{S-B} \\
&= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 4\pi\eta\left(\frac{1}{2}m\dot{r}_x^2(t) - \frac{1}{2}m\omega_0^2 r_x^2(t)\right) + 4\pi\eta e\phi(x,t)\dot{r}_x(t) \\
&\quad + 4\pi\eta\sum_n\left(\frac{1}{2}m_n\dot{q}_{n,x}^2(t) - \frac{1}{2}m_n\omega_n^2 q_{n,x}^2(t)\right) - 4\pi\eta\sum_n \lambda_n q_{n,x}(t)\, r_x(t)
\end{aligned}
$$

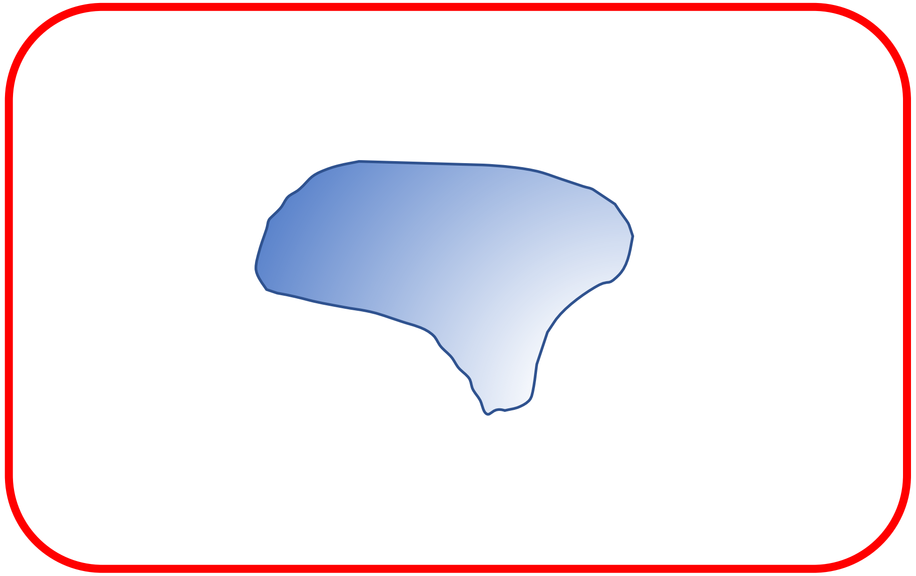


Model, Eqs. Motion, Long-time Limit

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_{\phi-S} + \mathcal{L}_B + \mathcal{L}_{S-B}$$

$$= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 4\pi\eta\left(\frac{1}{2}m\dot{r}_x^2(t) - \frac{1}{2}m\omega_0^2 r_x^2(t)\right) + 4\pi\eta e\phi(x,t)\dot{r}_x(t)$$

$$+ 4\pi\eta\sum_n\left(\frac{1}{2}m_n\dot{q}_{n,x}^2(t) - \frac{1}{2}m_n\omega_n^2 q_{n,x}^2(t)\right) - 4\pi\eta\sum_n\lambda_n q_{n,x}(t)r_x(t)$$



- Interactions begin at $t = t_0$
- For $t < t_0$, each part is free
- Canonical quantization (Heisenberg equations)
- Composite Hilbert space: $H = H_\phi \otimes H_\mathrm{A} \otimes H_\mathrm{B}$
- Uncorrelated initial state:

$$\hat{\rho}(t_0) = \hat{\rho}_\mathrm{IC}(t_0) \otimes \hat{\rho}_\mathrm{A}(t_0) \otimes \hat{\rho}_\mathrm{B}$$

- Effective field equation **(1+1)**:

$$\Box\hat{\phi} + \frac{\partial^2}{\partial t^2}\left[\int_{t_0}^{t} d\tau\chi_x(t-\tau)\hat{\phi}(x,\tau)\right] = 4\pi\eta e C(x)\left[\dot{G}_1(t-t_0)\hat{r}_x(t_0) + \dot{G}_2(t-t_0)\frac{\hat{p}_x(t_0)}{m} + \int_{t_0}^{t} d\tau\dot{G}_2(t-\tau)\frac{\hat{F}_x(\tau-t_0)}{m}\right]$$

$$\hat{\phi}(x,t_0) = \int dk\left[\frac{1}{\omega_k}\right]^{\frac{1}{2}}\left(\hat{a}_k(t_0)e^{i(kx-\omega_k t_0)} + \hat{a}_k^\dagger(t_0)e^{-i(kx-\omega_k t_0)}\right)$$

$$\dot{\hat{\phi}}(x,t_0) = \int dk\left[\frac{1}{\omega_k}\right]^{\frac{1}{2}} i\omega_k\left(-\hat{a}_k(t_0)e^{i(kx-\omega_k t_0)} + \hat{a}_k^\dagger(t_0)e^{-i(kx-\omega_k t_0)}\right)$$

Model, Eqs. Motion, Long-time Limit

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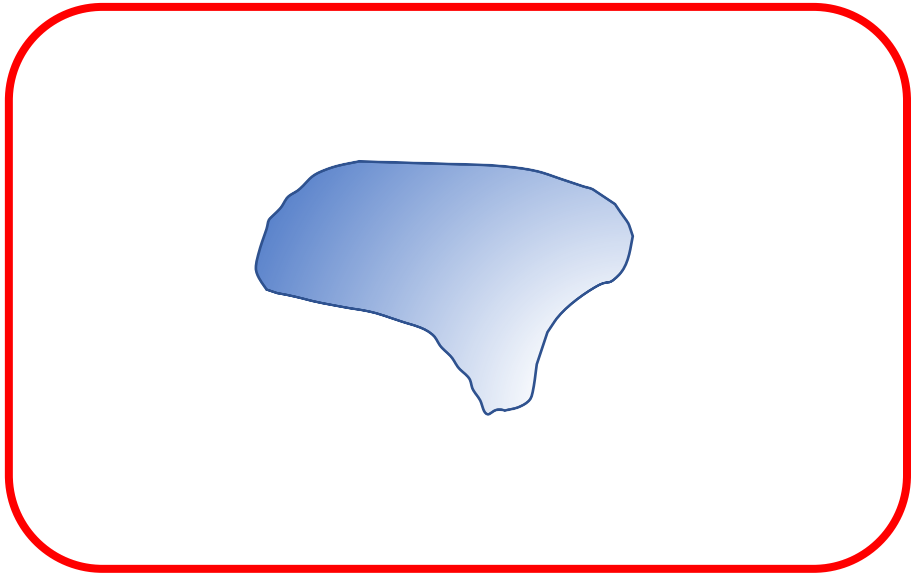

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- Effective field equation **(1+1)**:

$$\hat{\phi}(x,t) = -\int dx'\dot{\mathfrak{G}}_{\mathrm{Ret}}(x,x',t-t_0)\hat{\phi}(x',t_0) - \int dx'\mathfrak{G}_{\mathrm{Ret}}(x,x',t-t_0)\frac{\partial\hat{\phi}}{\partial t}(x',t_0)$$
$$-\int_{t_0}^{t} dt'\int dx'\mathfrak{G}_{\mathrm{Ret}}(x,x',t-t')4\pi\eta e C(x')\left(\dot{G}_1(t'-t_0)\hat{r}_{x'}(t_0) + \dot{G}_2(t'-t_0)\frac{\hat{p}_{x'}(t_0)}{m}\right)$$
$$-\int_{t_0}^{t} dt'\int dx'\mathfrak{G}_{\mathrm{Ret}}(x,x',t-t')4\pi\eta e C(x')\int_{t_0}^{t'} d\tau\dot{G}_2(t'-\tau)\frac{\hat{F}_{x'}(\tau-t_0)}{m},$$

Retarded Green function

Model, Eqs. Motion, Long-time Limit

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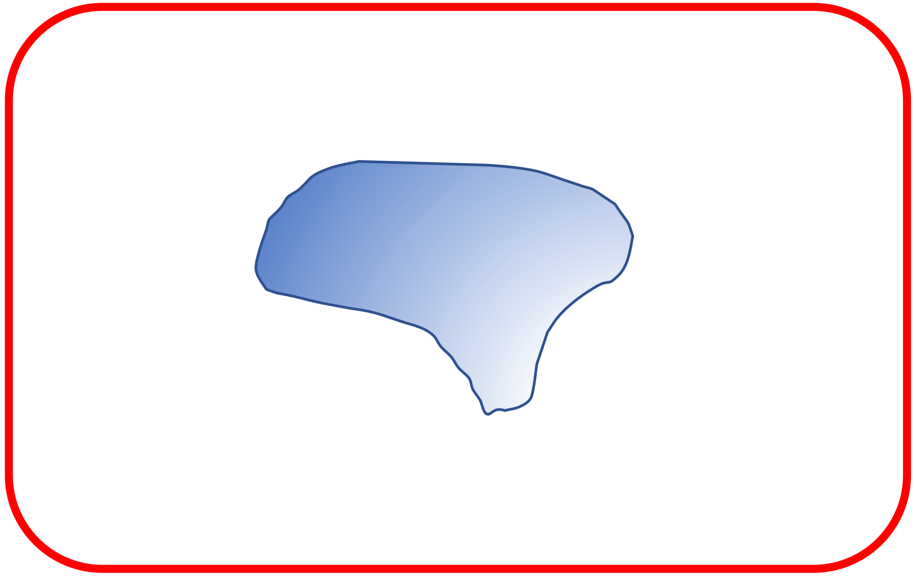

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$$\Big\Downarrow \quad t_0 \to -\infty \text{ and poles analysis}$$

$$\hat{\phi}^\infty(x,t) = \hat{\phi}_\mathrm{IC}^\infty(x,t) \otimes \mathbb{I}_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}_\mathrm{A}^\infty \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \mathbb{I}_\mathrm{A} \otimes \hat{\phi}_\mathrm{B}^\infty(x,t)$$

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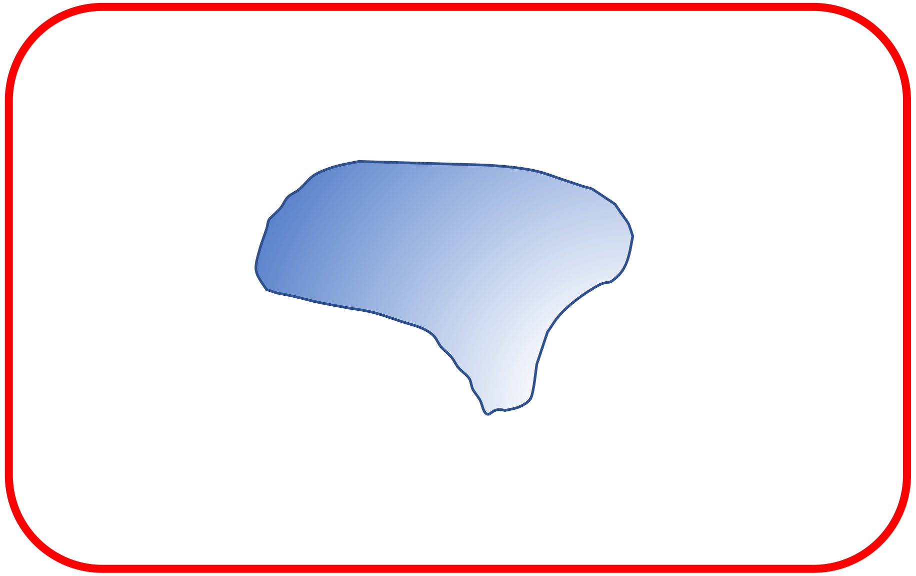
$$+ 4\pi\eta\sum_n\left(\frac{1}{2}m_n\dot{q}_{n,x}^2(t) - \frac{1}{2}m_n\omega_n^2 q_{n,x}^2(t)\right) - 4\pi\eta\sum_n\lambda_n q_{n,x}(t)r_x(t)$$



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$t_0 \rightarrow -\infty$ and poles analysis

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Independent of x and t
(damped dynamics of r)

NO contribution to $T_{\mu\nu}$

Model, Eqs. Motion, Long-time Limit

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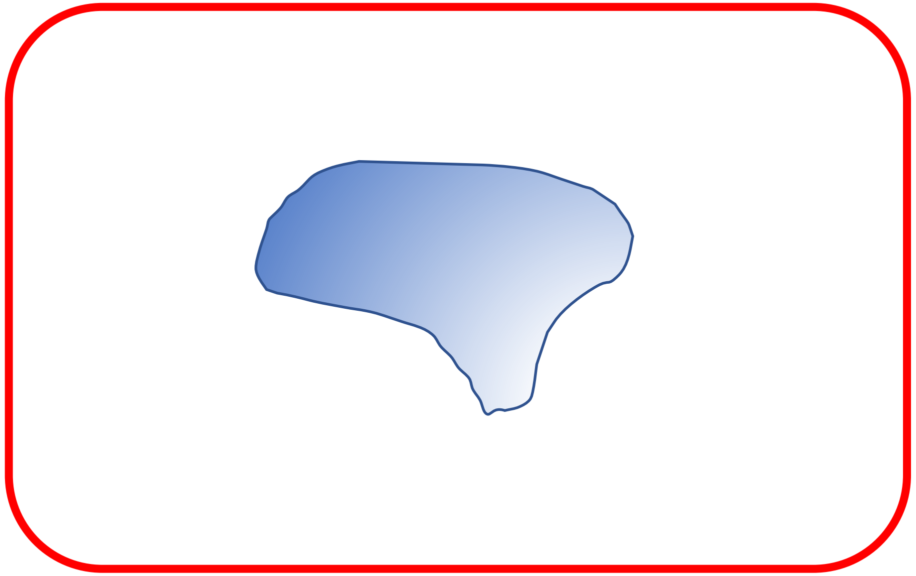

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$$\hat{\phi}(x,t) = \hat{\phi}_\mathrm{IC}(x,t) \otimes \mathbb{I}_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}_\mathrm{A}(x,t) \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \mathbb{I}_\mathrm{A} \otimes \hat{\phi}_\mathrm{B}(x,t)$$

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$$\hat{\phi}^\infty(x,t) = \hat{\phi}^\infty_\mathrm{IC}(x,t) \otimes \mathbb{I}_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}^\infty_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \mathbb{I}_\mathrm{A} \otimes \hat{\phi}^\infty_\mathrm{B}(x,t)$$

$$\widehat{\phi}^\infty_\mathrm{B}(x,t) = \int dx' \, \frac{4\pi\eta e C(x')}{m} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t} \, i\omega \, \overline{G}_2(\omega) \, \overline{\mathfrak{G}}_\mathrm{Ret}(x,x',\omega) \, \widehat{\overline{F}}^\infty_{x'}(\omega)$$

Inhomogeneous solution in FQED framework

Model, Eqs. Motion, Long-time Limit

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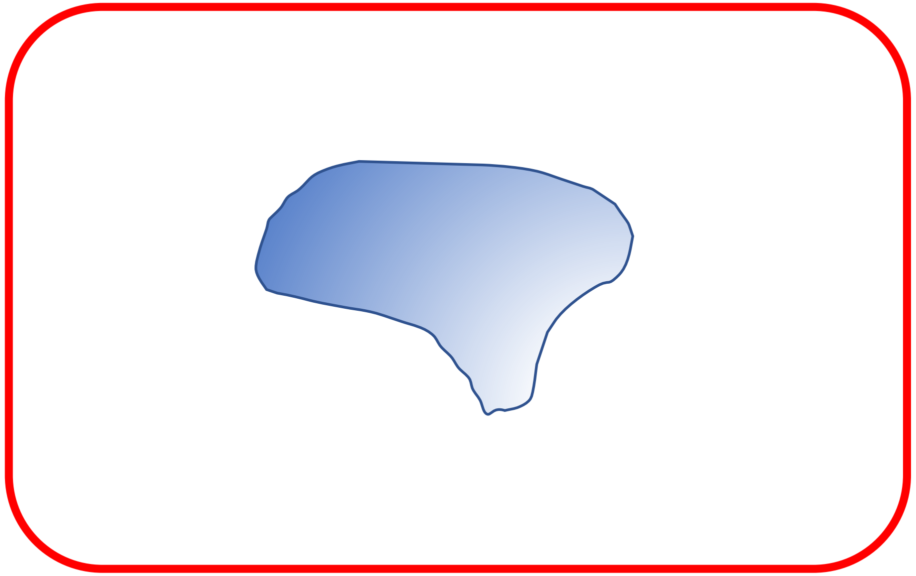

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Radiated field by the bodies

Baths' fluctuations generate field (Undamped dynamics of the baths' oscillators)

Model, Eqs. Motion, Long-time Limit

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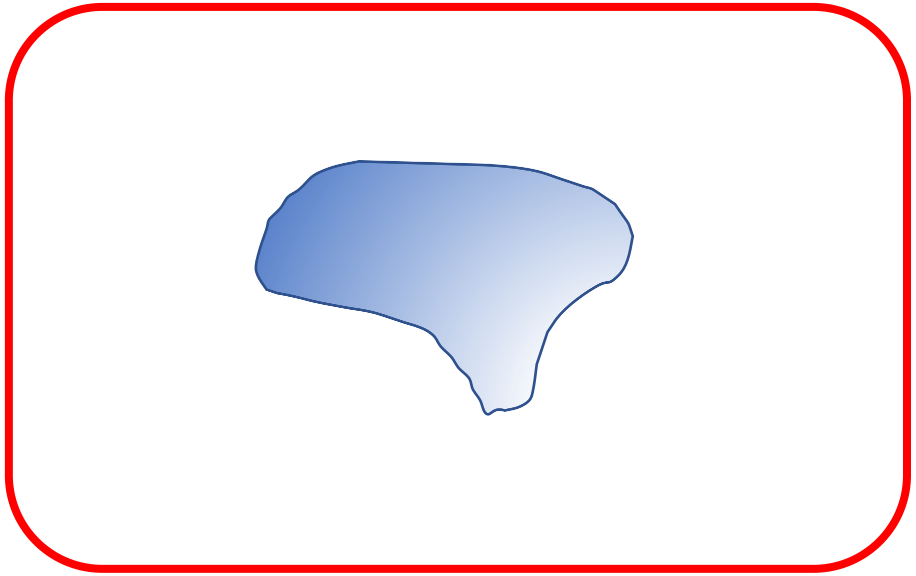
$$+ 4\pi\eta\sum_n\left(\frac{1}{2}m_n\dot{q}_{n,x}^2(t) - \frac{1}{2}m_n\omega_n^2 q_{n,x}^2(t)\right) - 4\pi\eta\sum_n\lambda_n q_{n,x}(t)r_x(t)$$



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Depends on the material configuration → Infinite-size “dissipationless” regions → Dissipation VS. free fluctuations in the different regions

Model, Eqs. Motion, Long-time Limit

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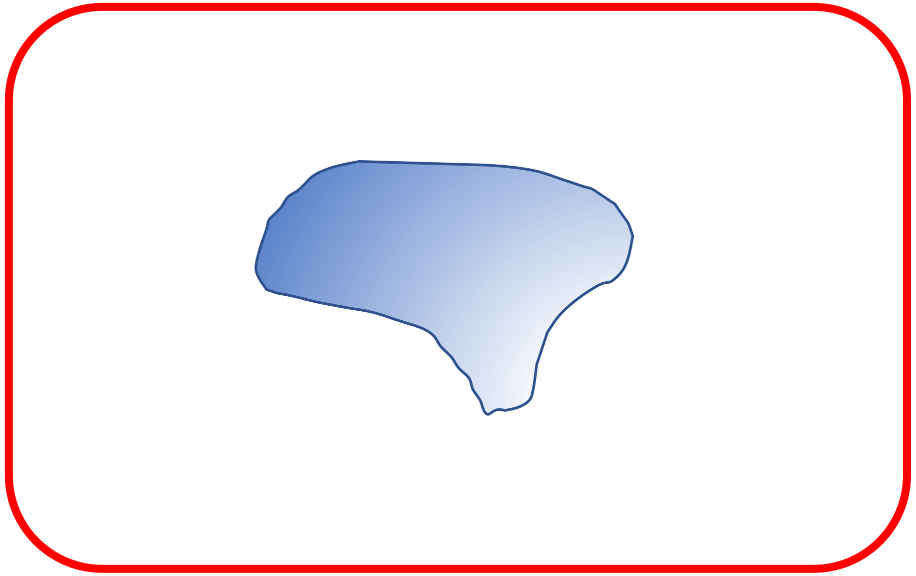

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Depends on the material configuration

Infinite-size "dissipationless" regions

Dynamical adaptation of the free field modes to the presence of the bodies

Model, Eqs. Motion, Long-time Limit

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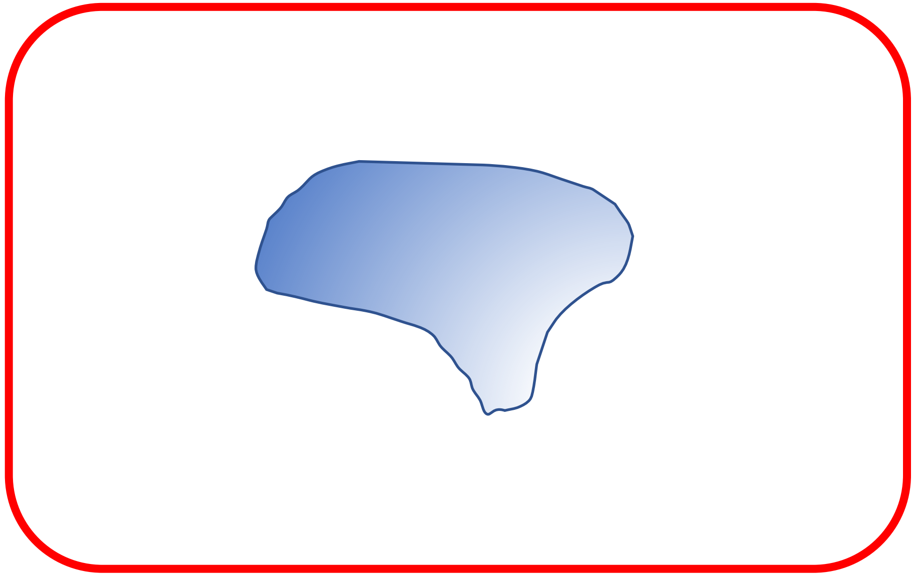

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$t_0 \rightarrow -\infty$ and poles analysis

$$\hat{\phi}^\infty(x,t) = \hat{\phi}^\infty_{\mathrm{IC}}(x,t)\otimes \mathbb{I}_\mathrm{A}\otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}^\infty_\mathrm{A}\otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi\otimes\mathbb{I}_\mathrm{A}\otimes\hat{\phi}^\infty_\mathrm{B}(x,t)$$

$$\int dk\left[\frac{1}{\omega_k}\right]^{\frac{1}{2}}\left[\hat{a}_k(-\infty)e^{-i\omega_k t}\Phi_k(x) + \hat{a}_k^\dagger(-\infty)e^{i\omega_k t}(\Phi_k(x))^*\right]$$

Homogeneous solution

Model, Eqs. Motion, Long-time Limit

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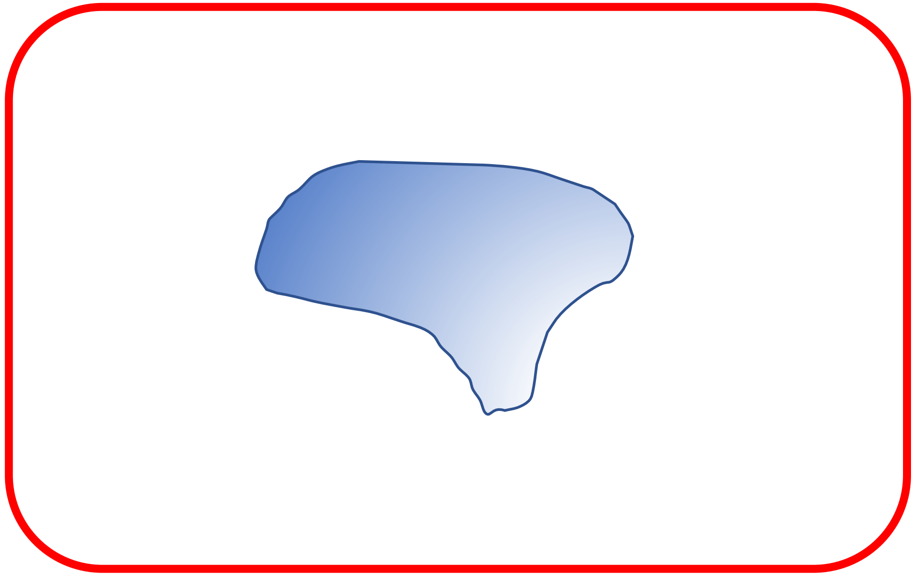

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$$\hat{\rho}(t_0) = \hat{\rho}_\mathrm{IC}(t_0) \otimes \hat{\rho}_\mathrm{A}(t_0) \otimes \hat{\rho}_\mathrm{B}$$

- Effective field equation **(1+1)**:

$$\hat{\phi}(x,t) = \hat{\phi}_\mathrm{IC}(x,t) \otimes \mathbb{I}_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}_\mathrm{A}(x,t) \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \mathbb{I}_\mathrm{A} \otimes \hat{\phi}_\mathrm{B}(x,t)$$

$t_0 \rightarrow -\infty$ and poles analysis

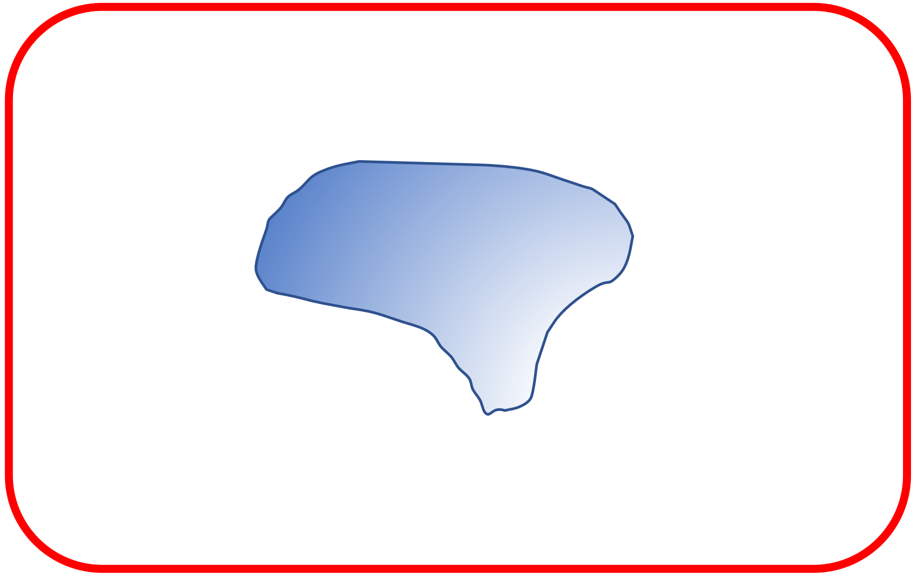
$$\hat{\phi}^\infty(x,t) = \hat{\phi}_\mathrm{IC}^\infty(x,t) \otimes \mathbb{I}_\mathrm{A} \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \hat{\phi}_\mathrm{A}^\infty \otimes \mathbb{I}_\mathrm{B} + \mathbb{I}_\phi \otimes \mathbb{I}_\mathrm{A} \otimes \hat{\phi}_\mathrm{B}^\infty(x,t)$$

Defined in the free field Hilbert space

Homogeneous solution

Model, Eqs. Motion, Long-time Limit

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_{\phi-S} + \mathcal{L}_B + \mathcal{L}_{S-B}$$
$$= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 4\pi\eta\left(\frac{1}{2}m\dot{r}_x^2(t) - \frac{1}{2}m\omega_0^2 r_x^2(t)\right) + 4\pi\eta e\phi(x,t)\dot{r}_x(t)$$
$$+ 4\pi\eta\sum_n\left(\frac{1}{2}m_n\dot{q}_{n,x}^2(t) - \frac{1}{2}m_n\omega_n^2 q_{n,x}^2(t)\right) - 4\pi\eta\sum_n\lambda_n q_{n,x}(t) r_x(t)$$



- Interactions begin at $t = t_0$
- For $t < t_0$, each part is free
- Canonical quantization (Heisenberg equations)
- Composite Hilbert space: $H = H_\phi \otimes H_{\mathrm{A}} \otimes H_{\mathrm{B}}$
- Uncorrelated initial state:

$$\hat{\rho}(t_0) = \hat{\rho}_{\mathrm{IC}}(t_0) \otimes \hat{\rho}_{\mathrm{A}}(t_0) \otimes \hat{\rho}_{\mathrm{B}}$$

- Effective field equation **(1+1)**:

$$\hat{\phi}(x,t) = \hat{\phi}_{\mathrm{IC}}(x,t) \otimes \mathbb{I}_{\mathrm{A}} \otimes \mathbb{I}_{\mathrm{B}} + \mathbb{I}_\phi \otimes \hat{\phi}_{\mathrm{A}}(x,t) \otimes \mathbb{I}_{\mathrm{B}} + \mathbb{I}_\phi \otimes \mathbb{I}_{\mathrm{A}} \otimes \hat{\phi}_{\mathrm{B}}(x,t)$$

$t_0 \rightarrow -\infty$ and poles analysis

$$\hat{\phi}^\infty(x,t) = \hat{\phi}_{\mathrm{IC}}^\infty(x,t) \otimes \mathbb{I}_{\mathrm{A}} \otimes \mathbb{I}_{\mathrm{B}} + \mathbb{I}_\phi \otimes \hat{\phi}_{\mathrm{A}}^\infty \otimes \mathbb{I}_{\mathrm{B}} + \mathbb{I}_\phi \otimes \mathbb{I}_{\mathrm{A}} \otimes \hat{\phi}_{\mathrm{B}}^\infty(x,t)$$

- Each part of the system with undamped dynamics contribute
- **Non-thermal initial states** can be considered for each part, defined at the initial time, when all the parts are not interacting

Expectation values of $T_{\mu\nu}$

$$\widehat{T}_{\mu\nu}(x_1^\sigma, t_0) \equiv \left(\delta_\mu{}^\gamma \delta_\nu{}^\alpha - \frac{1}{2}\,\eta_{\mu\nu}\eta^{\gamma\alpha}\right)\frac{1}{2}\left(\partial_\gamma\widehat{\phi}(x_1^\sigma)\;\partial_\alpha\widehat{\phi}(x_1^\sigma) + \partial_\alpha\widehat{\phi}(x_1^\sigma)\;\partial_\gamma\widehat{\phi}(x_1^\sigma)\right)$$

$$\left\langle \widehat{T}^{\infty}_{\mu\nu}(x_1^\sigma)\right\rangle = \left\langle \widehat{T}^{\mathrm{IC},\infty}_{\mu\nu}(x_1^\sigma)\right\rangle_\phi + \left\langle \widehat{T}^{\mathrm{B},\infty}_{\mu\nu}(x_1^\sigma)\right\rangle_{\mathrm{B}}$$

Initial free field fluctuations:

$$\left\langle \widehat{a}_k(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi, \; \left\langle \widehat{a}^\dagger_k(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi$$

Fluctuation-dissipation for the sources:

$$\langle\{\widehat{\widehat{F}}^{\infty}_{x'}(\omega'), \widehat{\widehat{F}}^{\infty}_{x''}(\omega'')\}\rangle_{\mathrm{B}} = (2\pi)^2\delta(x'-x'')\,\frac{J(\omega')}{2\eta}\coth\left(\frac{\beta_{\mathrm{B}}\omega'}{2}\right)\delta(\omega'+\omega'')$$

Casimir force

$$F_{\mathrm{C}} = \langle \widehat{T}^{\mathrm{Free}}_{xx}\rangle_\phi - \langle \widehat{T}^{\mathrm{IC},\infty}_{xx}\rangle^{\mathrm{Int}}_\phi - \langle \widehat{T}^{\mathrm{B},\infty}_{xx}\rangle^{\mathrm{Int}}_{\mathrm{B}}$$

Casimir's prescription for regularization

Expectation value for initial state of the field in absence of material bodies **(Thermal or not)**

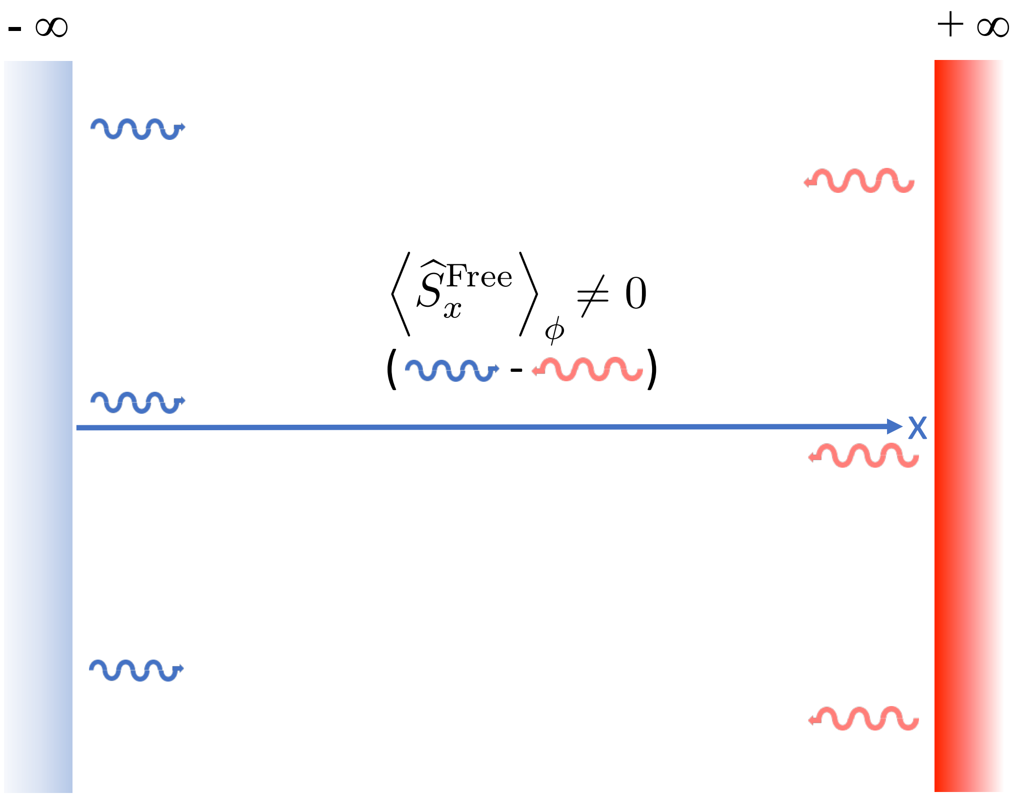
Heat flux

$$Q_\infty \equiv \left\langle \widehat{S}^{\infty}_{x}\right\rangle = -\left\langle \widehat{T}^{\infty}_{x0}\right\rangle$$

No regularization required

Two plates configuration

Initial scenario:



- 1+1
- Initial state field:

Modes traveling from right to left (k<0)

$$\left\langle \widehat{a}_k(-\infty)\widehat{a}_{k'}(-\infty) \right\rangle_\phi = 0$$

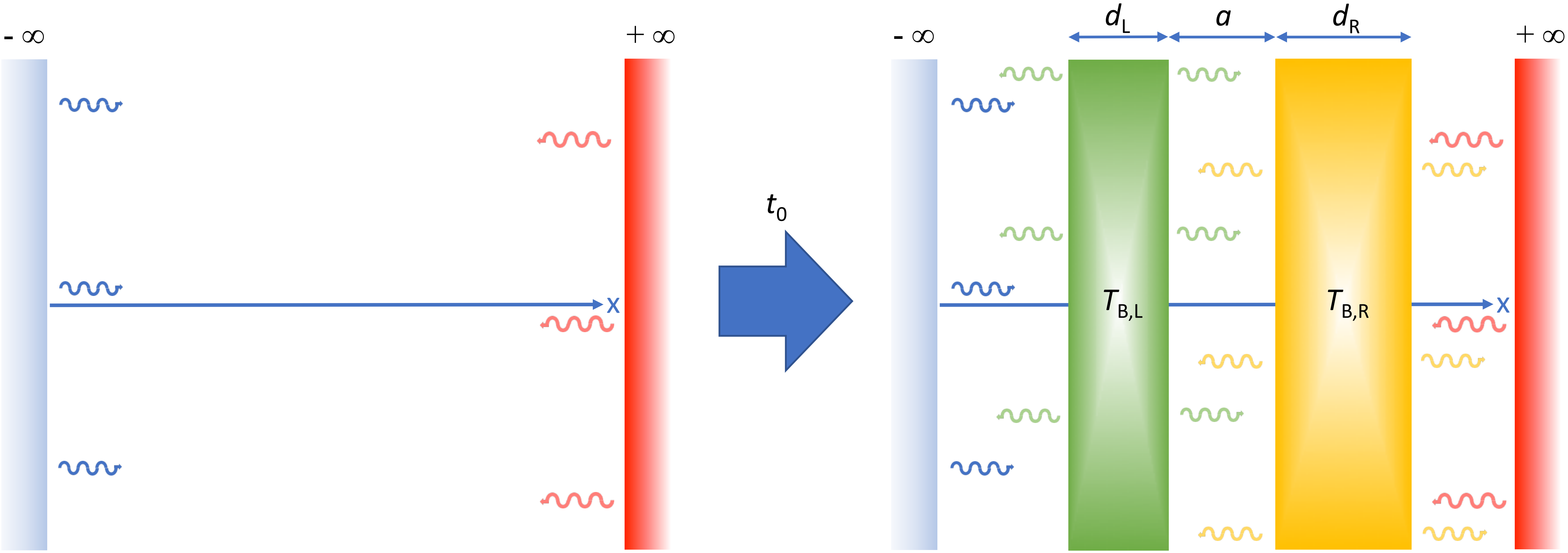
Modes traveling from left to right (k>0)

$$\left\langle \widehat{a}_k^\dagger(-\infty)\widehat{a}_{k'}(-\infty) \right\rangle_\phi = \left[\Theta(k)\, N_{\phi,\mathrm{L}}(\omega_k) + \Theta(-k)\, N_{\phi,\mathrm{R}}(\omega_k)\right] \delta(k-k')$$

Two plates configuration

Initial scenario:

Time Evolution:



Retarded Green function $\mathfrak{G}_{\mathrm{Ret}}$

- 1+1
- Initial state field:

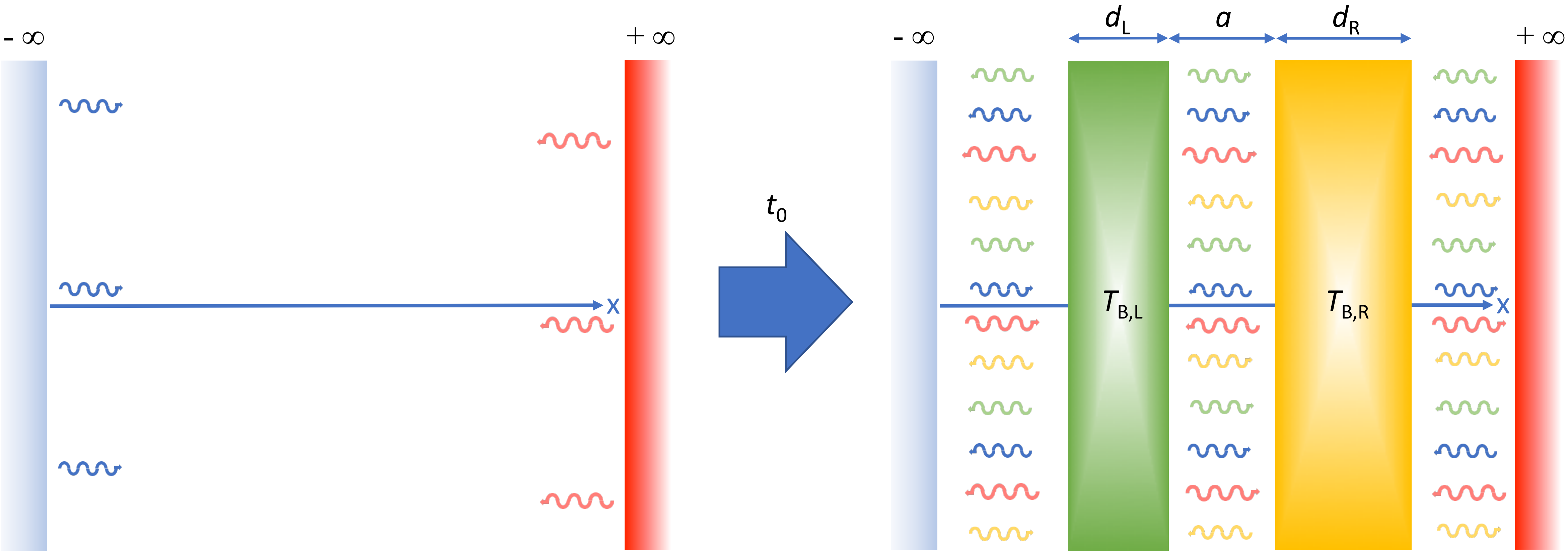
$$\left\langle \widehat{a}_k(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi = 0$$

$$\left\langle \widehat{a}^\dagger_k(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi = \left[\Theta(k)\ N_{\phi,\mathrm{L}}(\omega_k) + \Theta(-k)\ N_{\phi,\mathrm{R}}(\omega_k)\right]\delta(k-k')$$

Two plates configuration

Initial scenario:

Steady situation:



- 1+1
- Initial state field:

$$\left\langle \widehat{a}_k(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi = 0$$

$$\left\langle \widehat{a}_k^\dagger(-\infty)\widehat{a}_{k'}(-\infty)\right\rangle_\phi = [\Theta(k)\ N_{\phi,\mathrm{L}}(\omega_k) + \Theta(-k)\ N_{\phi,\mathrm{R}}(\omega_k)]\,\delta(k-k')$$

Retarded Green function $\mathfrak{G}_{\mathrm{Ret}}$

(Initial scenario)

$$F_{\mathrm{C}} = \langle \widehat{T}_{xx}^{\mathrm{Free}}\rangle_\phi - \langle \widehat{T}_{xx}^{\mathrm{IC},\infty}\rangle_\phi^{\mathrm{Int}}[a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\phi,\mathrm{L}}, \beta_{\phi,\mathrm{R}}] - \langle \widehat{T}_{xx}^{\mathrm{B},\infty}\rangle_{\mathrm{B}}^{\mathrm{Int}}[a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\mathrm{B,L}}, \beta_{\mathrm{B,R}}]$$

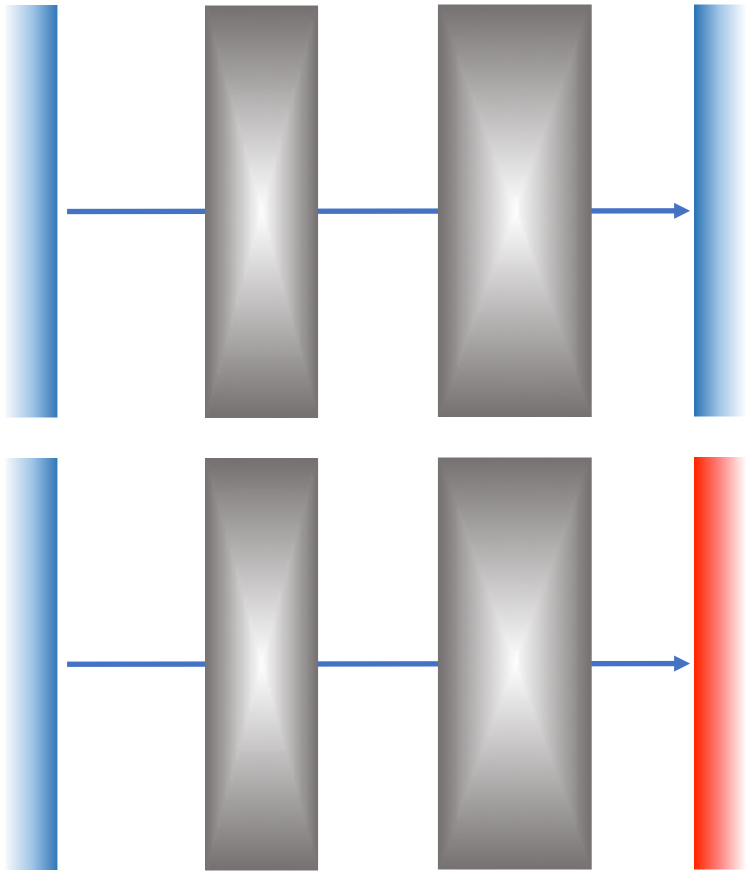
$$Q_\infty \equiv \left\langle \widehat{S}_x^\infty \right\rangle = -\left\langle \widehat{T}_{x0}^\infty \right\rangle = Q_\infty^{\mathrm{IC}}(a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\phi,\mathrm{L}}, \beta_{\phi,\mathrm{R}}) + Q_\infty^{\mathrm{B}}(a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\mathrm{B,L}}, \beta_{\mathrm{B,R}})$$

Casimir force

$$F_{\mathrm{C}} = \langle \widehat{T}^{\mathrm{Free}}_{xx} \rangle_{\phi} - \langle \widehat{T}^{\mathrm{IC},\infty}_{xx} \rangle^{\mathrm{Int}}_{\phi} - \langle \widehat{T}^{\mathrm{B},\infty}_{xx} \rangle^{\mathrm{Int}}_{\mathrm{B}}$$

		Reg. IC	IC	B	F_{TOTAL}
NO dissipation	Eq.	$\neq 0$ Eq.	$\neq 0$	0	Lifshitz Finite
	Non-Eq.	$\neq 0$ Non-Eq	$\neq 0$		Non-Eq. Finite

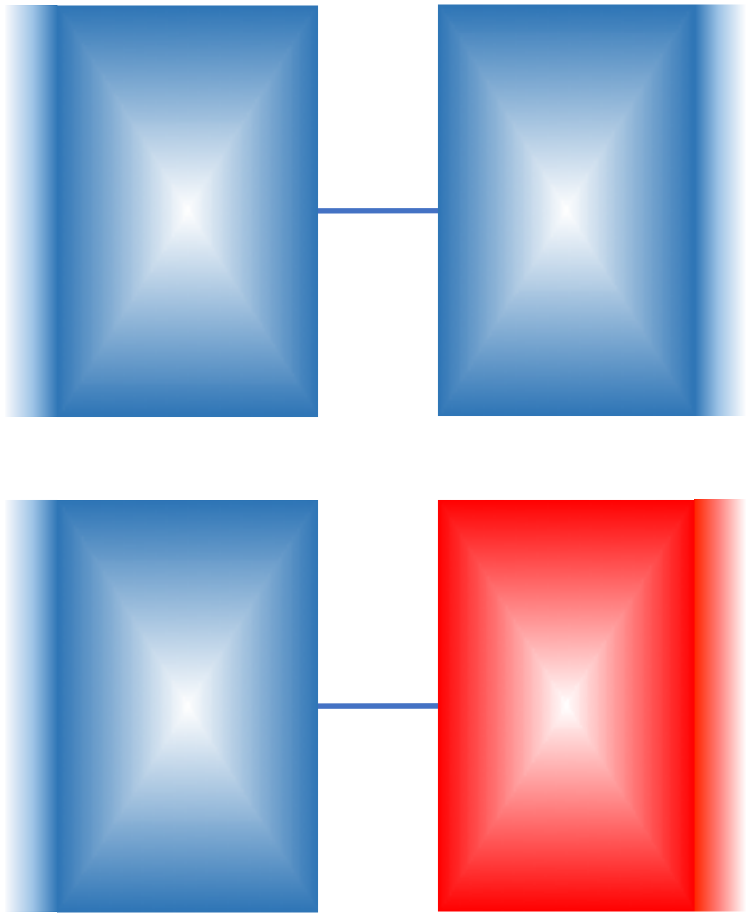
$$\mathrm{Im}(n_i) \equiv 0$$



Casimir force

$$F_{\mathrm{C}} = \langle \widehat{T}^{\mathrm{Free}}_{xx} \rangle_{\phi} - \langle \widehat{T}^{\mathrm{IC},\infty}_{xx} \rangle^{\mathrm{Int}}_{\phi} - \langle \widehat{T}^{\mathrm{B},\infty}_{xx} \rangle^{\mathrm{Int}}_{\mathrm{B}}$$

		Reg. IC	IC	B	F_{TOTAL}
NO dissipation	**Eq.**	≠ 0 Eq.	≠ 0	0	Lifshitz Finite
	Non-Eq.	≠ 0 Non-Eq	≠ 0		Non-Eq. Finite
$d_{L,R} \to +\infty$	**Eq.**	≠ 0 Eq.	0	≠ 0	Lifshitz
	Non-Eq.	≠ 0 Non-Eq.		≠ 0	Non-Eq. Lifshitz



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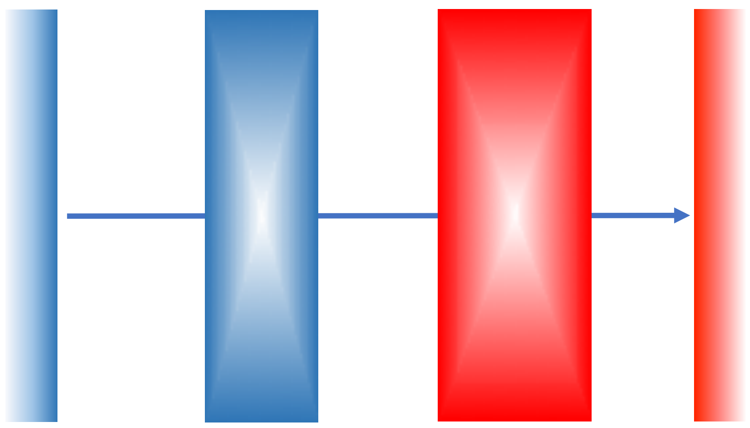
$$F_{\mathrm{C}} = \langle \widehat{T}^{\mathrm{Free}}_{xx} \rangle_{\phi} - \langle \widehat{T}^{\mathrm{IC},\infty}_{xx} \rangle^{\mathrm{Int}}_{\phi} - \langle \widehat{T}^{\mathrm{B},\infty}_{xx} \rangle^{\mathrm{Int}}_{\mathrm{B}}$$

		Reg. IC	IC	B	F_{TOTAL}
NO dissipation	Eq.	≠ 0 Eq.	≠ 0	0	Lifshitz Finite
	Non-Eq.	≠ 0 Non-Eq	≠ 0		Non-Eq. Finite
$d_{\mathrm{L,R}} \to +\infty$	Eq.	≠ 0 Eq.	0	≠ 0	Lifshitz
	Non-Eq.	≠ 0 Non-Eq.		≠ 0	Non-Eq. Lifshitz
$d_{\mathrm{L,R}}$	Non-Eq.	≠ 0 Non-Eq.	≠ 0	≠ 0	



Casimir force

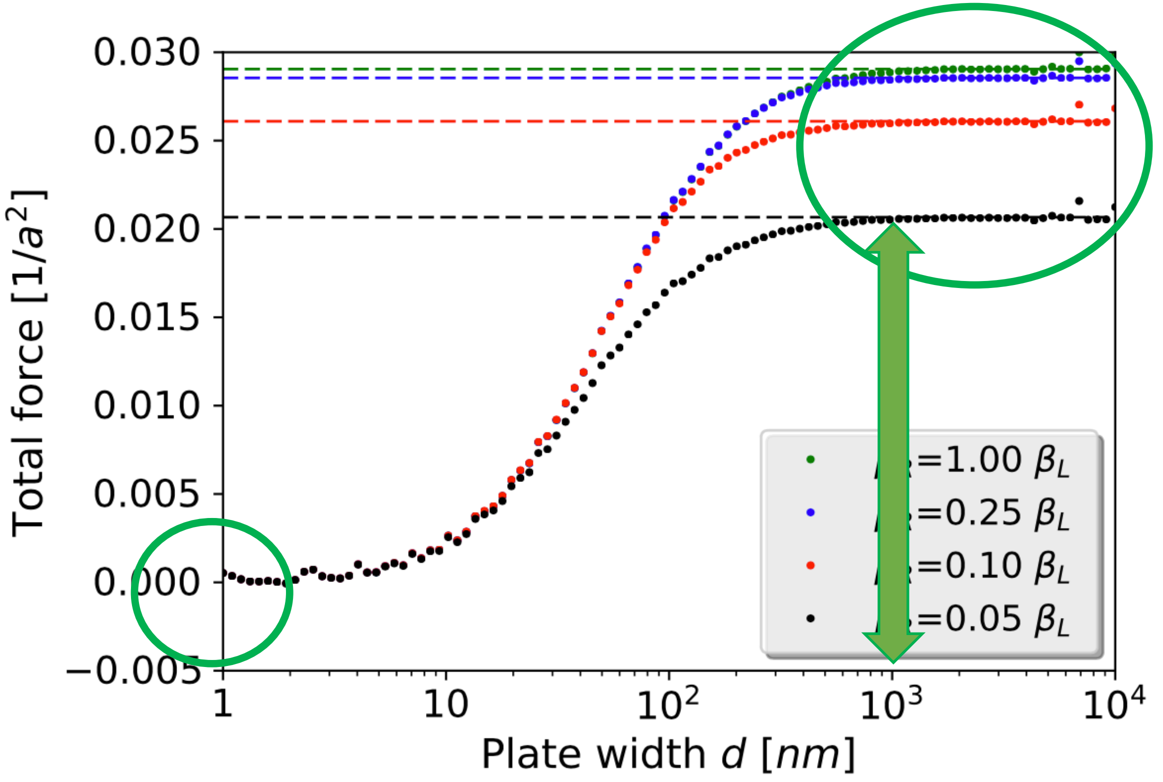
At which scale the force for is approximately the infinite-thickness value?

What means "infinite-thickness" in an experiment?

$d \approx 1\mu\text{m} = 10\ a$

		Reg. IC	IC	B	F_{TOTAL}
NO dissipation	**Eq.**	≠ 0 Eq.	≠ 0	0	Lifshitz Finite
	Non-Eq.	≠ 0 Non-Eq	≠ 0		Non-Eq. Finite
$d_{L,R} \to +\infty$	**Eq.**	≠ 0 Eq.	0	≠ 0	Lifshitz
	Non-Eq.	≠ 0 Non-Eq.		≠ 0	Non-Eq. Lifshitz
$d_{L,R}$	**Non-Eq.**	≠ 0 Non-Eq.	≠ 0	≠ 0	

$$F_{\mathrm{C}} = \langle \widehat{T}^{\mathrm{Free}}_{xx} \rangle_{\phi} - \langle \widehat{T}^{\mathrm{IC},\infty}_{xx} \rangle^{\mathrm{Int}}_{\phi} - \langle \widehat{T}^{\mathrm{B},\infty}_{xx} \rangle^{\mathrm{Int}}_{\mathrm{B}}$$



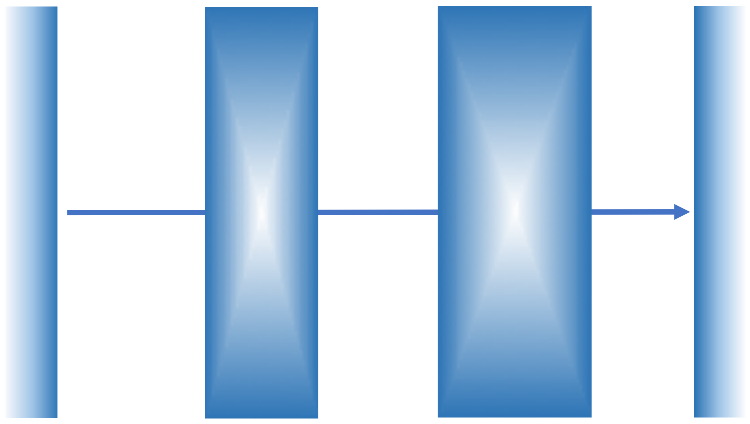
- Dielectric, a = 100nm
- $F_{\text{TOTAL}}(d = 0) = 0$
- Maximum value at equilibrium (extra momentum exchange)

Heat flux: Some cases

For which situations, Q_{TOTAL} is Landauer-like?

$$Q_\infty \equiv \left\langle \widehat{S}_x^\infty \right\rangle = -\left\langle \widehat{T}_{x0}^\infty \right\rangle = Q_\infty^{\mathrm{IC}}(a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\phi,\mathrm{L}}, \beta_{\phi,\mathrm{R}}) + Q_\infty^{\mathrm{B}}(a, d_{\mathrm{L}}, d_{\mathrm{R}}, \beta_{\mathrm{B,L}}, \beta_{\mathrm{B,R}})$$

		Q_{IC}	Q_{B}	Q_{TOTAL}
Eq.	$T_{\mathrm{B},j} = T_{\phi,k}$	$\neq 0$	$\neq 0$	0



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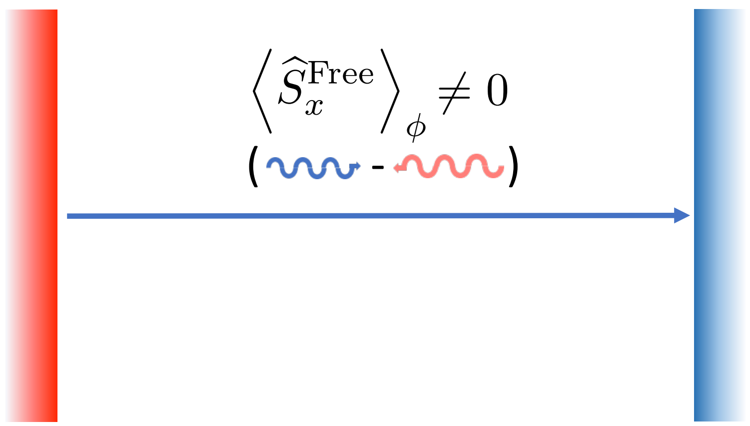
		Q_{IC}	Q_{B}	Q_{TOTAL}
Eq.	$T_{\text{B},j} = T_{\phi,k}$	$\neq 0$	$\neq 0$	0
d = 0		$\neq 0$	0	$\neq 0$ (L)



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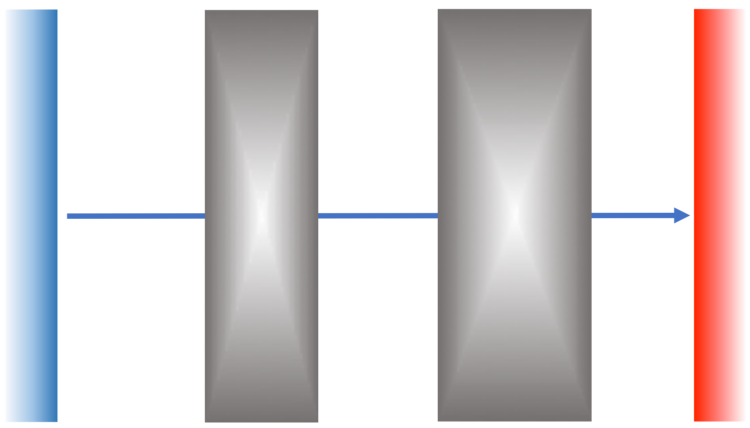
		Q_{IC}	Q_B	Q_{TOTAL}
Eq.	$T_{B,j} = T_{\phi,k}$	≠ 0	≠ 0	0
d = 0		≠ 0	0	≠ 0 (L)
NO diss.		≠ 0	0	≠ 0 (LC)

$$\mathrm{Im}(n_i) \equiv 0$$



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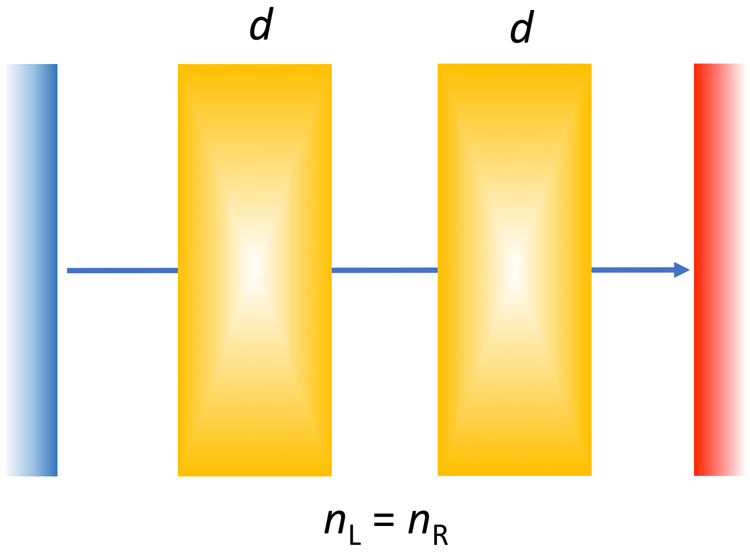
		Q_{IC}	Q_{B}	Q_{TOTAL}
Eq.	$T_{\mathrm{B},j} = T_{\phi,k}$	$\neq 0$	$\neq 0$	0
d = 0		$\neq 0$	0	$\neq 0$ (L)
NO diss.		$\neq 0$	0	$\neq 0$ (LC)
Identical	$T_{\mathrm{B,L}} = T_{\mathrm{B,R}}$	$\neq 0$	0	$\neq 0$ (L)



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		Q_{IC}	Q_{B}	Q_{TOTAL}
Eq.	$T_{\text{B},j} = T_{\phi,k}$	$\neq 0$	$\neq 0$	0
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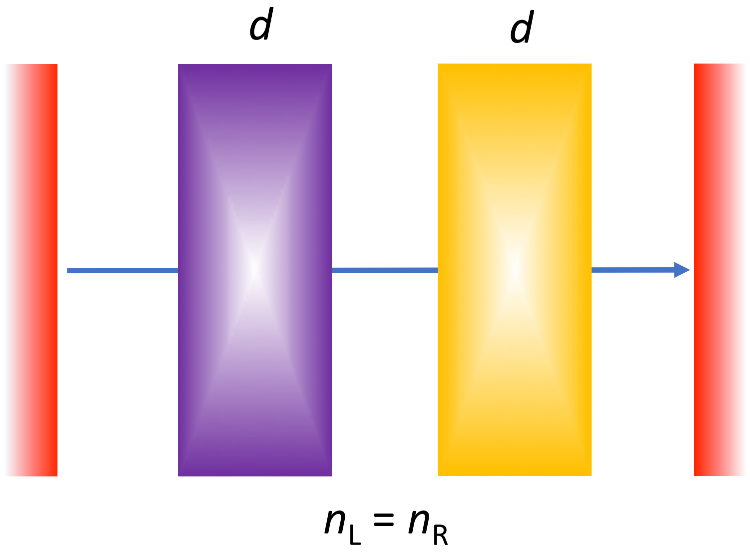

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NO diss.		$\neq 0$	0	$\neq 0$ (LC)
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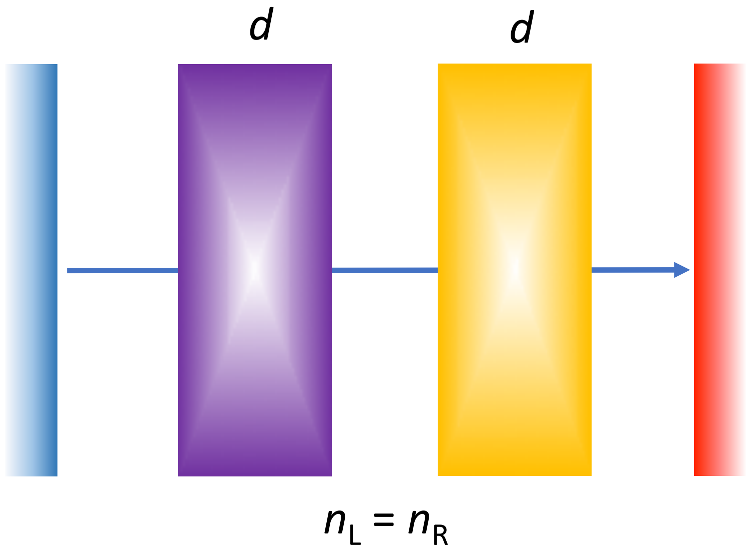
	$T_{\phi,\mathrm{L}} = T_{\phi,\mathrm{R}}$	0	$\neq 0$	$\neq 0$ (L)
	$T_{\mathrm{B},j} \neq T_{\phi,k}$	$\neq 0$	$\neq 0$	$\neq 0$ (LC)



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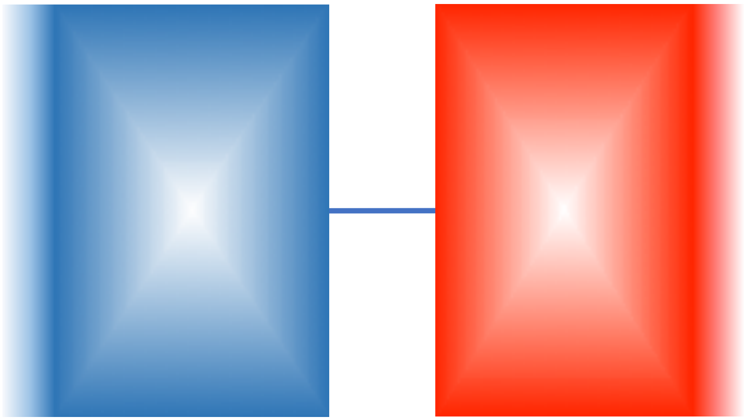
		Q_{IC}	Q_B	Q_{TOTAL}
Eq.	$T_{B,j} = T_{\phi,k}$	≠ 0	≠ 0	0
d = 0		≠ 0	0	≠ 0 (L)
NO diss.		≠ 0	0	≠ 0 (LC)
Identical	$T_{B,L} = T_{B,R}$	≠ 0	0	≠ 0 (L)
	$T_{\phi,L} = T_{\phi,R}$	0	≠ 0	≠ 0 (L)
	$T_{B,j} \neq T_{\phi,k}$	≠ 0	≠ 0	≠ 0 (LC)
d → + ∞		0	≠ 0	≠ 0 (L)



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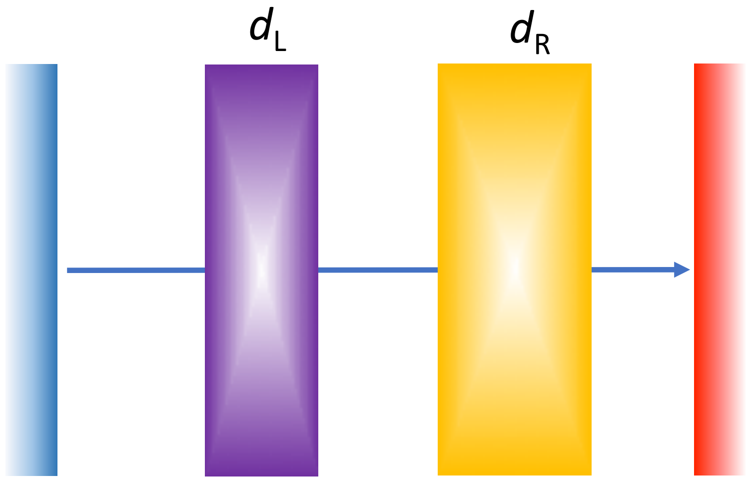
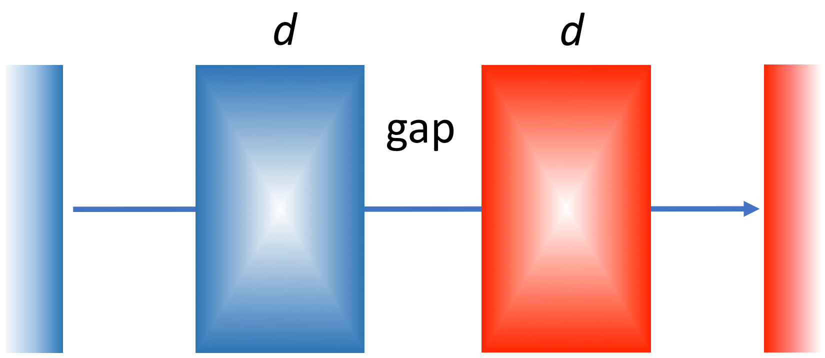
		Q_{IC}	Q_{B}	Q_{TOTAL}
Eq.	$T_{\mathrm{B},j} = T_{\phi,k}$	≠ 0	≠ 0	0
d = 0		≠ 0	0	≠ 0 (L)
NO diss.		≠ 0	0	≠ 0 (LC)
Identical	$T_{\mathrm{B,L}} = T_{\mathrm{B,R}}$	≠ 0	0	≠ 0 (L)
	$T_{\phi,\mathrm{L}} = T_{\phi,\mathrm{R}}$	0	≠ 0	≠ 0 (L)
	$T_{\mathrm{B},j} \neq T_{\phi,k}$	≠ 0	≠ 0	≠ 0 (LC)
d → + ∞		0	≠ 0	≠ 0 (L)
General		≠ 0	≠ 0	≠ 0

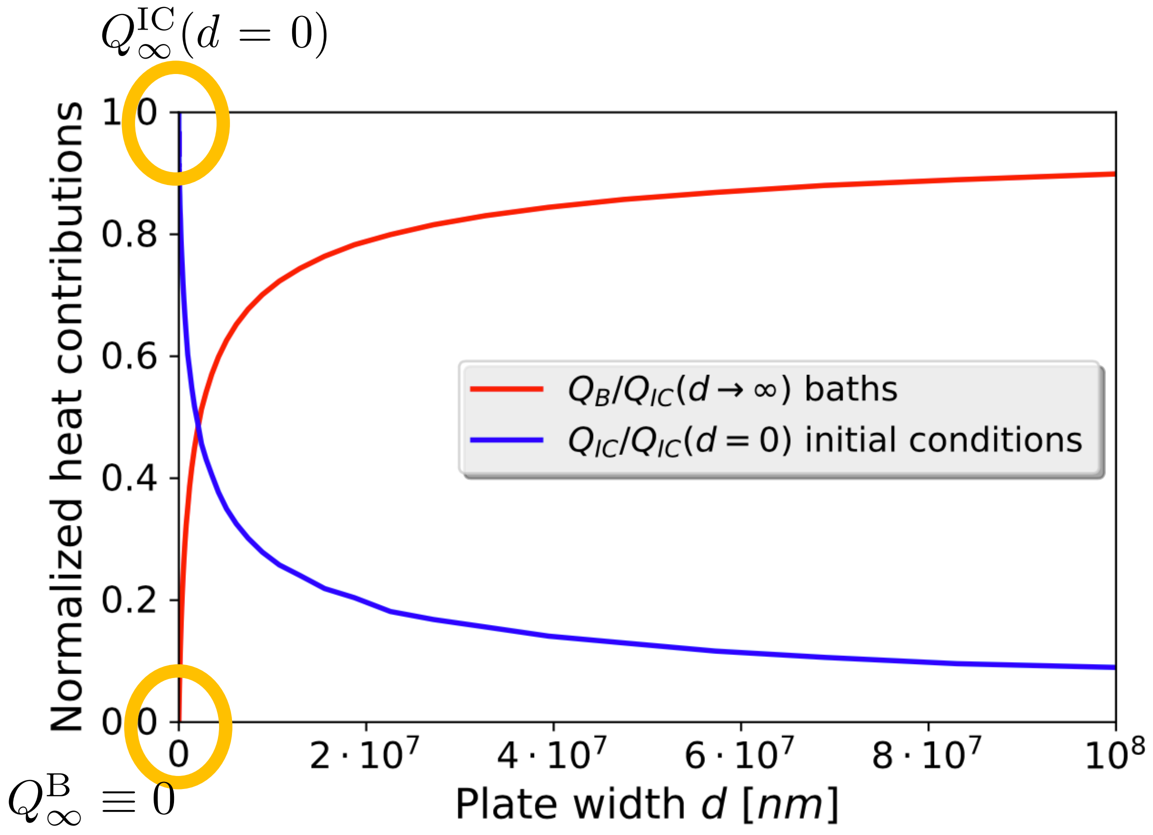


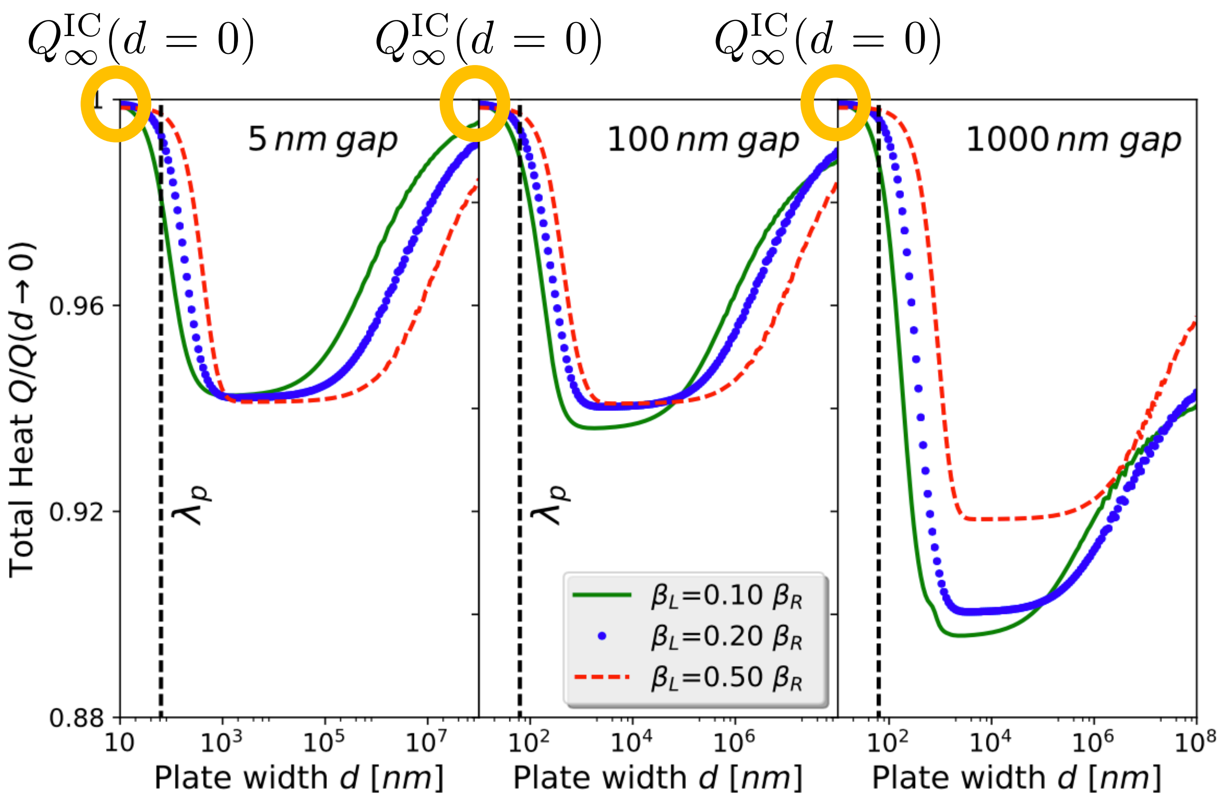
Heat flux: Numerics

Identical plates:

- Same (dissipative) material
- Same thickness *d*
- 2 "THERMAL SIDES"







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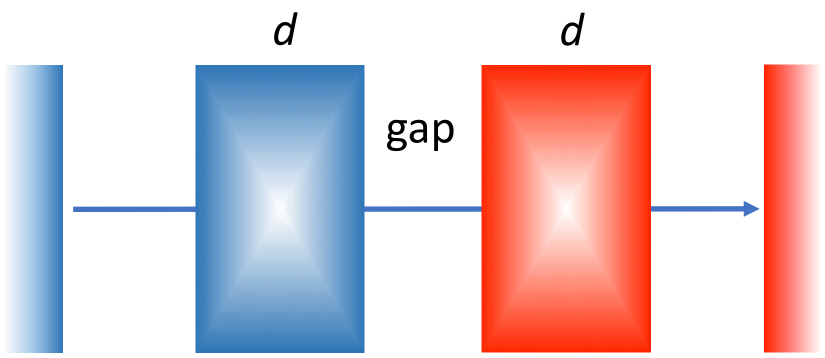
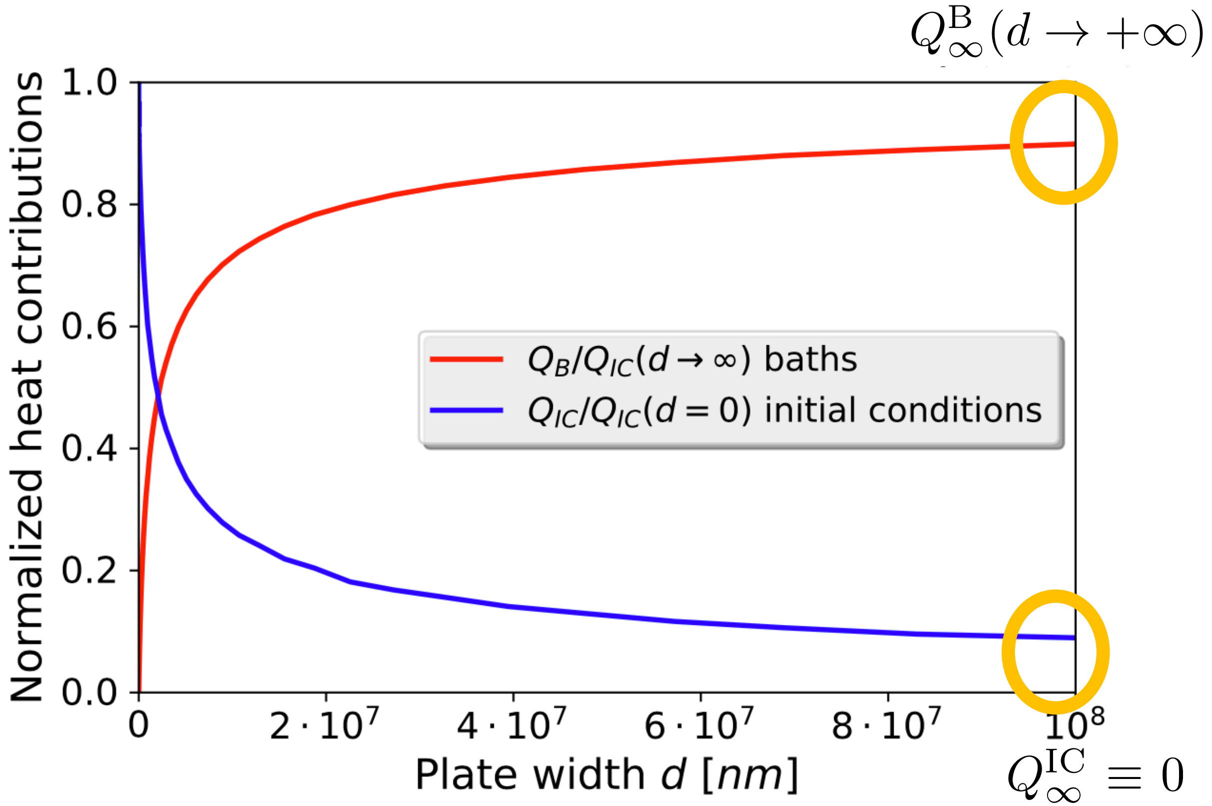
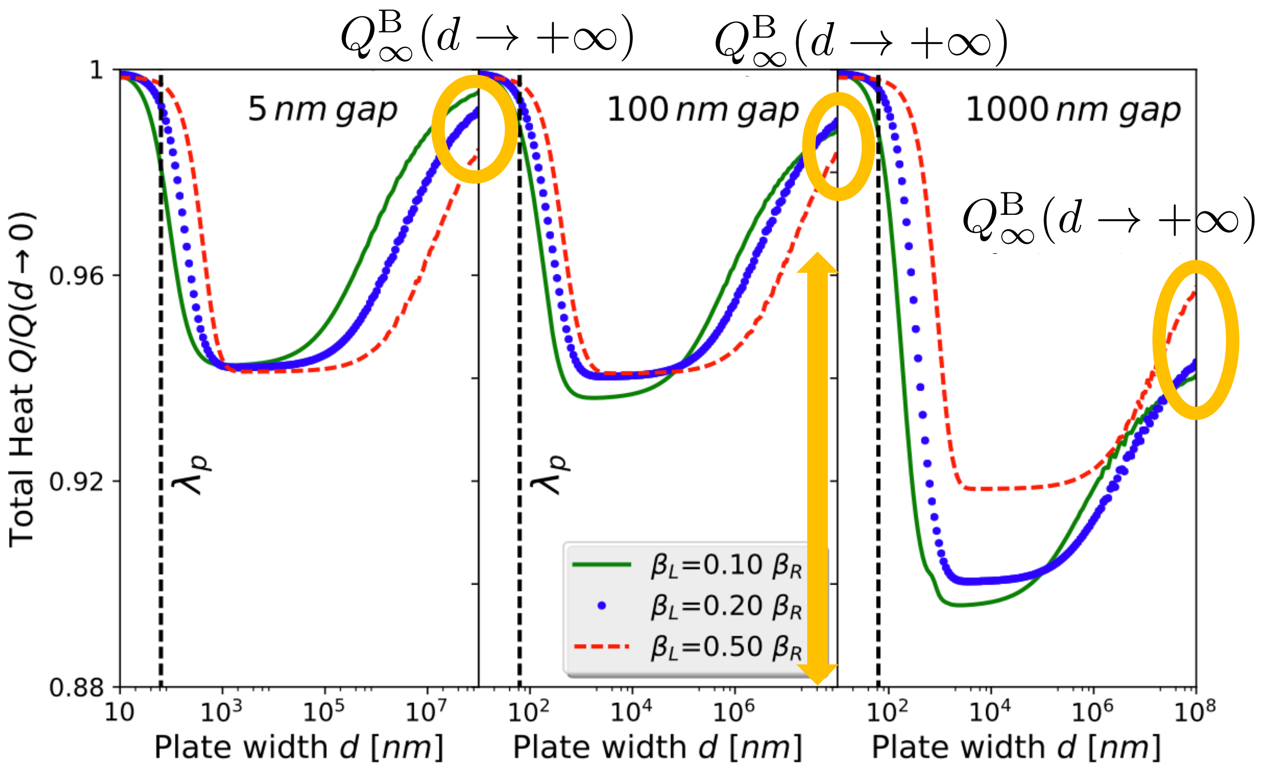


Convergence
d ≈ 1cm = 10^5 *a*
(greater than for F)

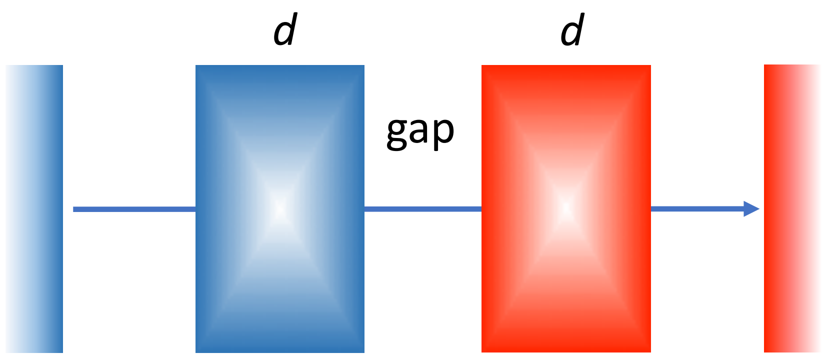




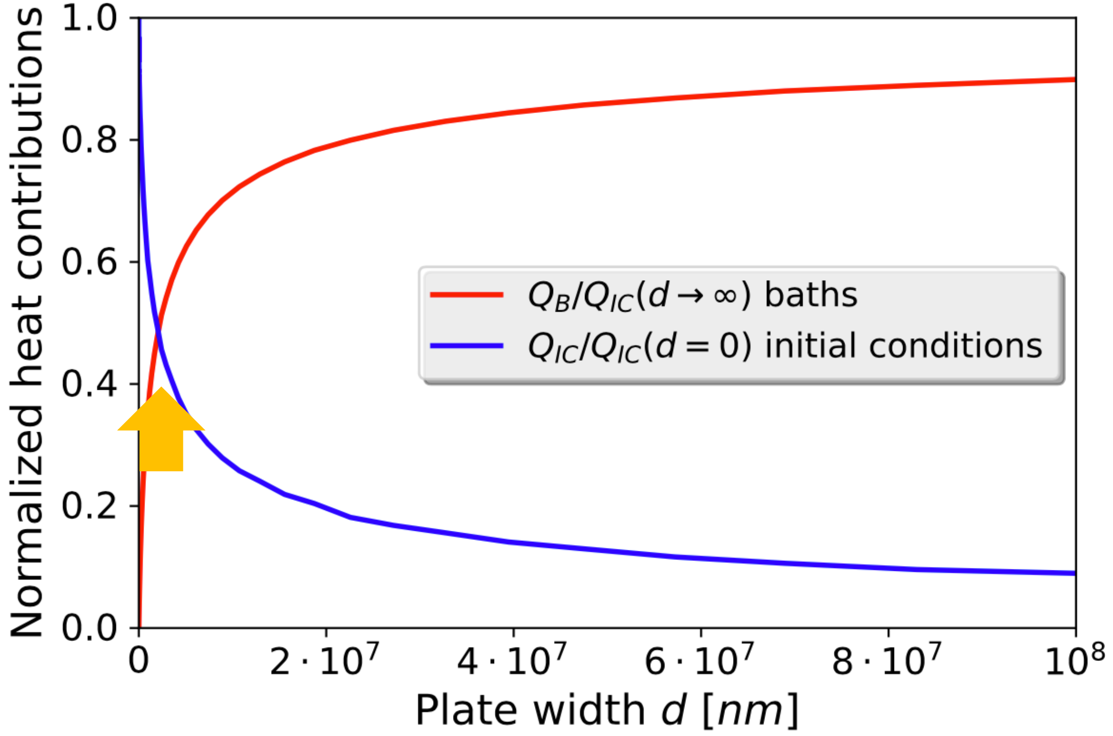
Heat flux: Numerics

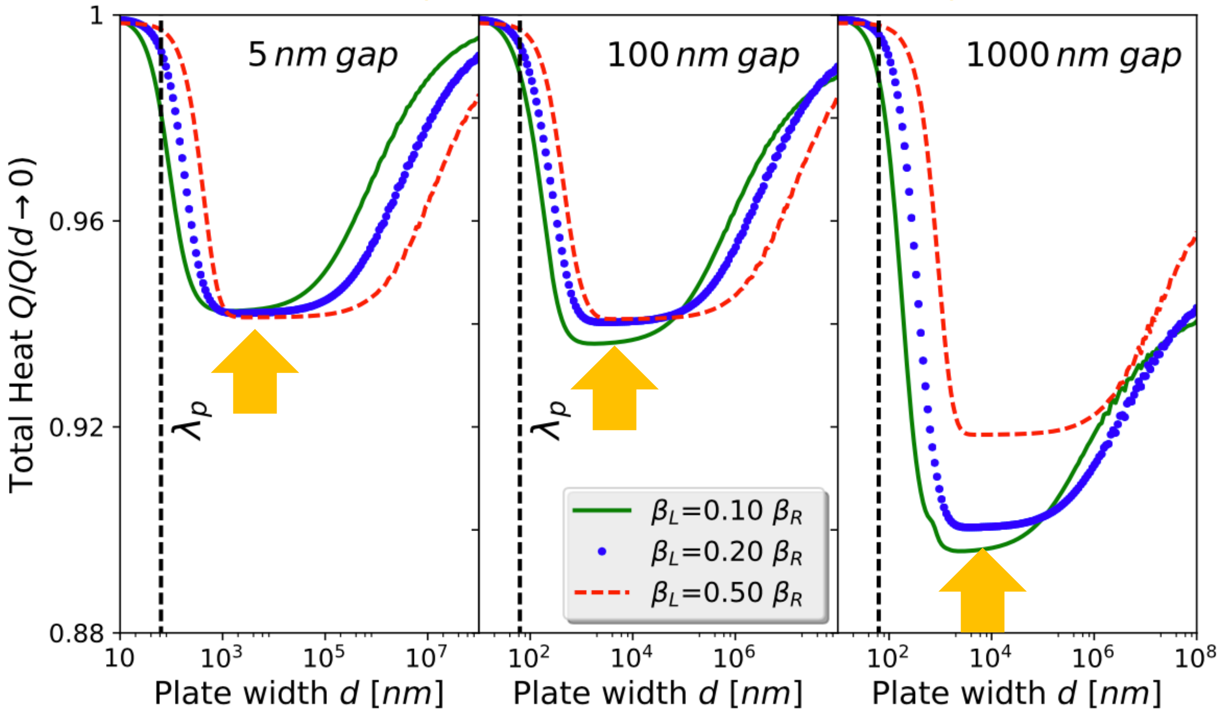
Identical plates:

- Same (dissipative) material
- Same thickness *d*
- 2 “THERMAL SIDES”
- MINIMUM



Competition of contributions (SHIELDING vs. RADIATION)

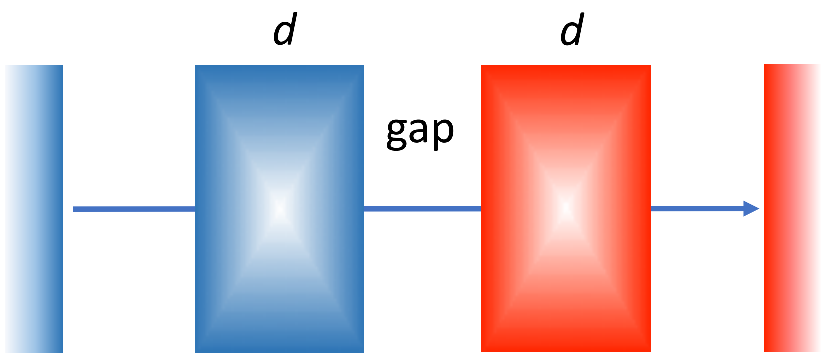




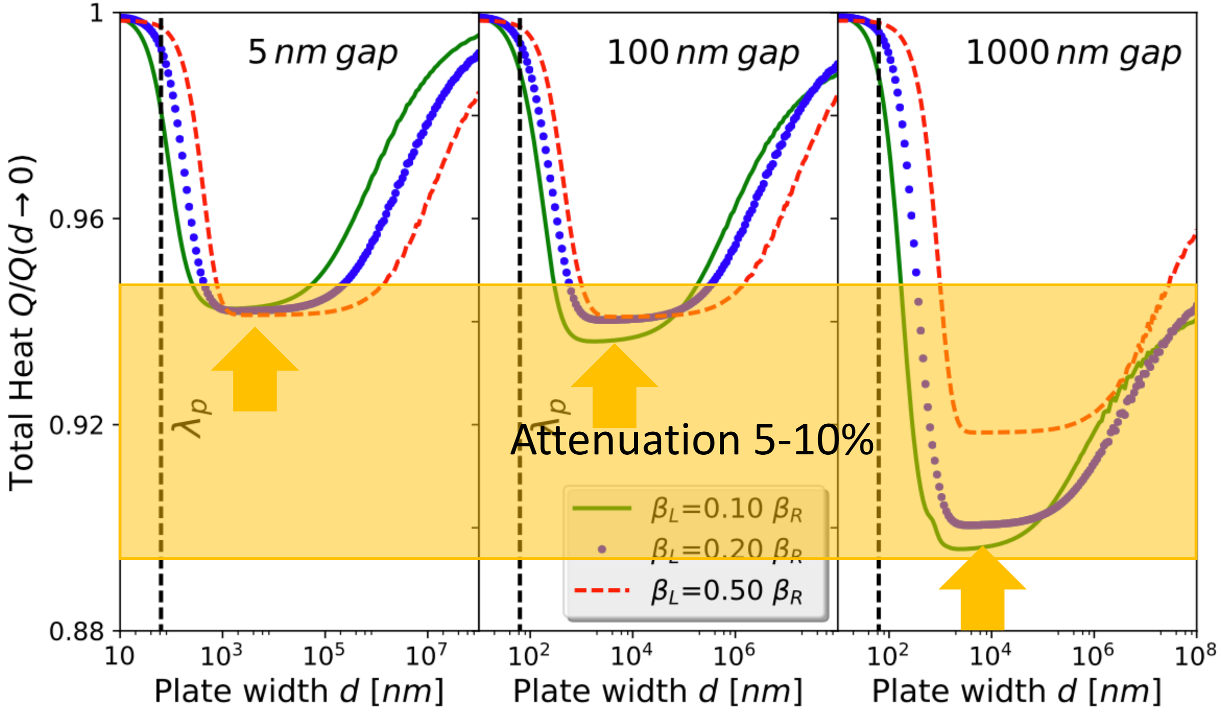
Heat flux: Numerics

Identical plates:

- Same (dissipative) material
- Same thickness *d*
- 2 “THERMAL SIDES”
- MINIMUM (5-10 %)



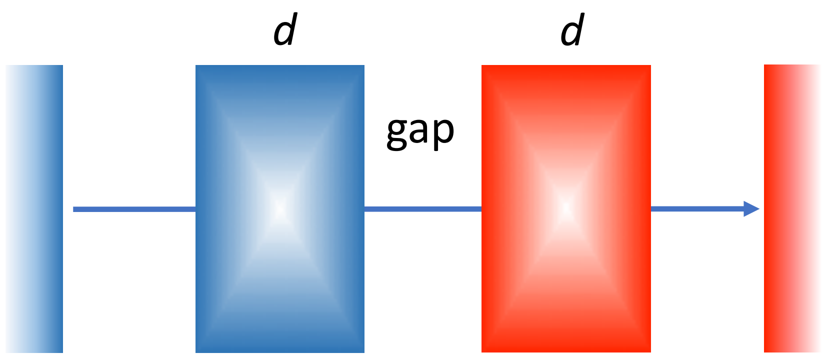
Competition of contributions
(SHIELDING vs. RADIATION)



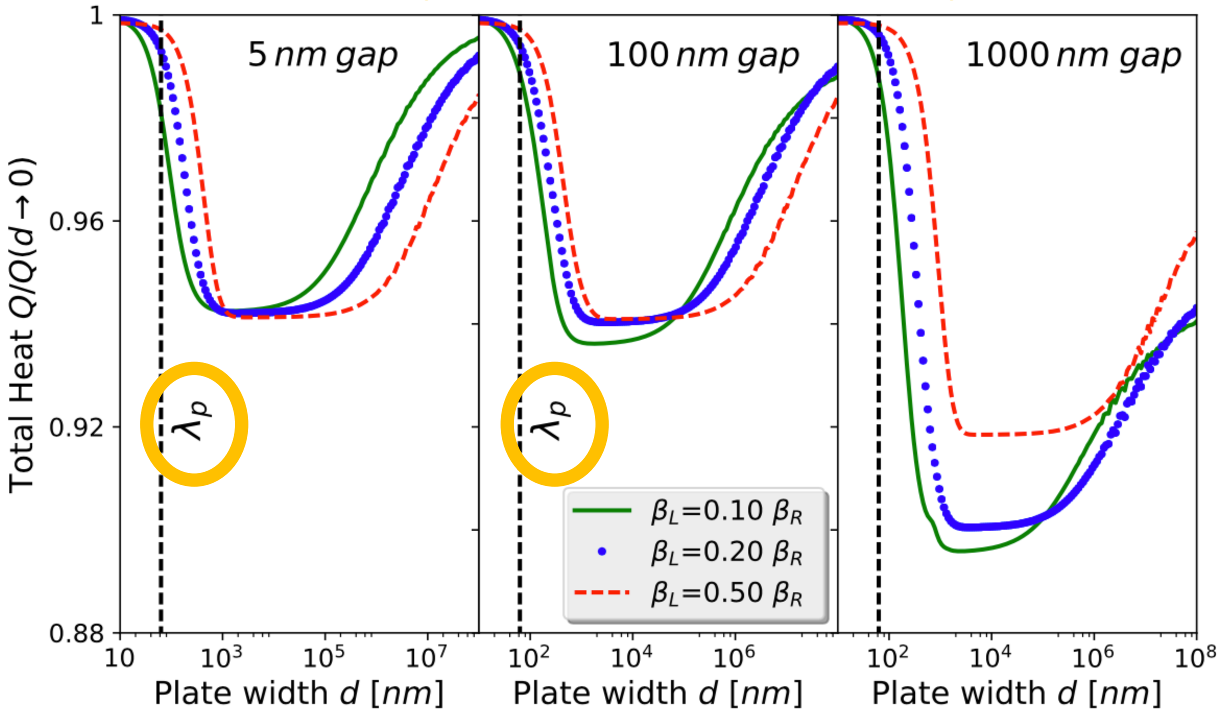
Can this attenuation be stronger???

Heat flux: Numerics

Identical plates:
- Same (dissipative) material
- Same thickness *d*
- 2 “THERMAL SIDES”
- MINIMUM



Competition of contributions (SHIELDING vs. RADIATION)

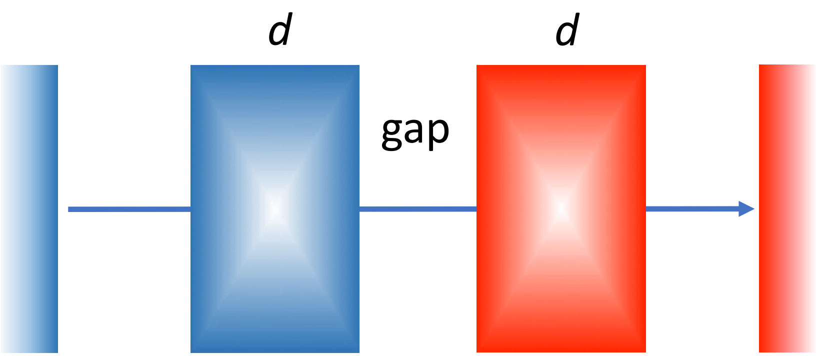


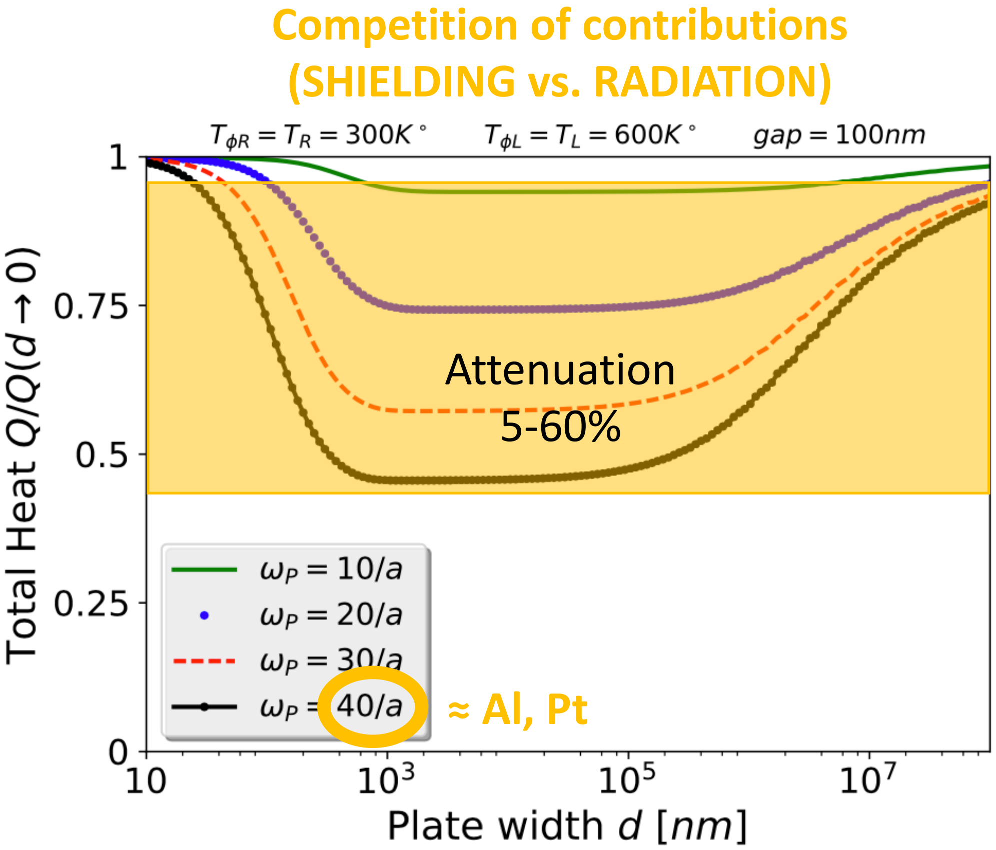
Can this attenuation be stronger???
YES.
Increasing the reflectivity.
(enhance shielding)

Heat flux: Numerics

Identical plates:

- Same (dissipative) material
- Same thickness *d*
- 2 "THERMAL SIDES"
- MINIMUM (5-60 %)





Can this attenuation be stronger???
YES.
Increasing the reflectivity.
(enhance shielding)

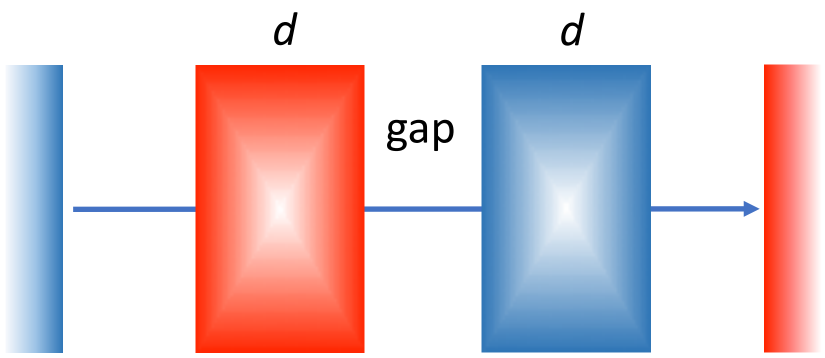
Heat flux: Numerics

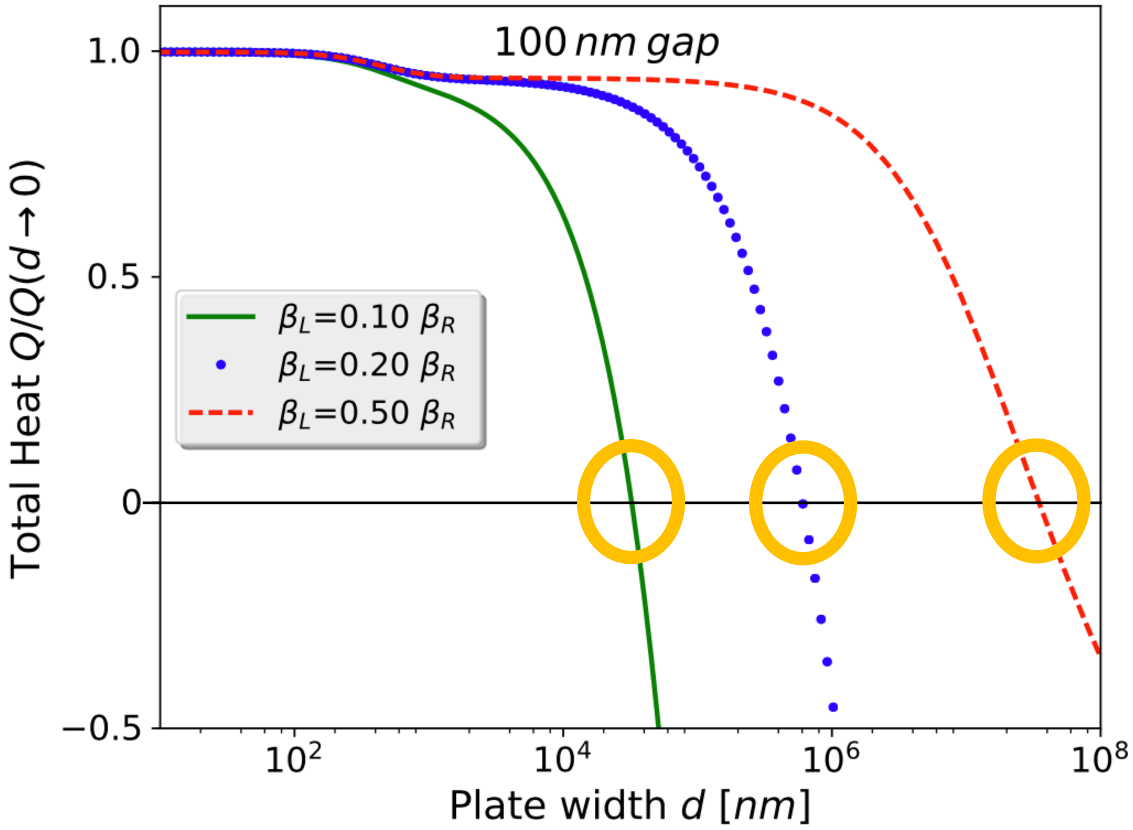
Identical plates:

- Same (dissipative) material
- Same thickness d
- Crossed fluxes

$Q_{IC} < 0 < Q_B$

$Q_{IC} > 0 > Q_B$





Competition of contributions
(SHIELDING vs. RADIATION)

0 flux between the plates in NON-Eq. scenario

Conclusions

- We developed a full first-principles quantum approach based on a canonical quantization and open quantum systems frameworks.
- We deduced the steady situation and describe the physics of each contribution to the field operator.
- We applied for the two finite width plates and studied the Casimir forces and heat fluxes in different non-equilibrium scenarios.
- For the force, we reproduced all the known-results, giving also a consistent way to reobtain the non-equilibrium half-spaces result as a limiting case of the non-equilibrium finite width case.
- For the heat flux, we showed that Landauer formula is obtained in different situations but it is not valid in every situation in the two plates configuration.
- We showed that the scales of convergence of the force and the heat flux as functions of the thickness are different.
- For the "two thermal sides" scenario, we showed that a minimum in the heat flux exists, which could result in an attenuation of 60% for some materials.
- We showed that 0 heat flux between the plates can be obtained also in non-equilibrium scenarios.

Refs: 1 – AERL, Phys. Rev. D 95, 025009 (2017).
2 – AERL, Poggi P. M., Lombardo F. C., Giannini V., arXiv:1709.09277 (Oct2017). Submitted to PRA.

Thank you!

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2 – AERL, Poggi P. M., Lombardo F. C., Giannini V., arXiv:1709.09277 (Oct2017). Submitted to PRA.