The Casimir effect for thin films and the models of dissipation

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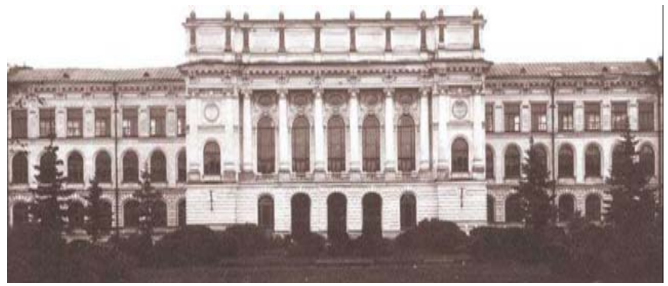


CONTENT

1. INTRODUCTION

In the past, it was believed that differences in theoretical predictions of the Drude and plasma models at small distances are of about a few percent.

In the difference force measurements by Bimonte and Decca, the predictions of both models can differ by a factor of 1000.

We show that the Casimir free energy and pressure for thin metal films calculated using the Drude and plasma models are also quite different, both quantitatively and qualitatively.

This is not of theoretical importance only, but should be taken into account in the determination of stability of metallic coatings.

2. CASIMIR FREE ENERGY AND PRESSURE OF THIN FILMS

$$\mathcal{F}(a,T) = \frac{k_B T}{2\pi} \sum_{l=0}^{\infty}{}' \int_0^\infty k_\perp dk_\perp \sum_\alpha \ln\left[1 - r_\alpha^{(2,1)} r_\alpha^{(2,3)} e^{-2ak_l^{(2)}}\right]$$

$$k_l^{(2)} \equiv k_l^{(2)}(k_\perp) = \sqrt{k_\perp^2 + \varepsilon^{(2)}(i\xi_l)\frac{\xi_l^2}{c^2}}$$

Reflection coefficients for TM and TE polarizations

$$r_{\rm TM}^{(2,n)} = \frac{\varepsilon_l^{(n)} k_l^{(2)} - \varepsilon_l^{(2)} k_l^{(n)}}{\varepsilon_l^{(n)} k_l^{(2)} + \varepsilon_l^{(2)} k_l^{(n)}}$$

$$r_{\rm TE}^{(2,n)} = \frac{k_l^{(2)} - k_l^{(n)}}{k_l^{(2)} + k_l^{(n)}}$$

$$\varepsilon_l^{(n)} \equiv \varepsilon^{(n)}(i\xi_l)$$

3. METALLIC FILM BETWEEN DIELECTRIC PLATES

The dielectric permittivity of metal at low frequencies is described:

by the plasma model

$$\varepsilon_{l,p}^{(2)} = 1 + \frac{\omega_{p,2}^2}{\xi_l^2}$$

$$k_{0,p}^{(2)} = \sqrt{k_\perp^2 + \frac{\omega_{p,2}^2}{c^2}}$$

by the Drude model

$$\varepsilon_{l,D}^{(2)} = 1 + \frac{\omega_{p,2}^2}{\xi_l(\xi_l + \gamma_2)}$$

$$k_{0,D}^{(2)} = k_\perp$$

Reflection coefficients at zero Matsubara frequency for nonmagnetic metals

$$r_{\mathrm{TM},D}^{(2,n)}(0,k_\perp) = -1, \quad r_{\mathrm{TE},D}^{(2,n)}(0,k_\perp) = 0$$

$$r_{\mathrm{TM},p}^{(2,n)}(0,k_\perp) = -1,$$

$$r_{\mathrm{TE},p}^{(2,n)}(0,k_\perp) = \frac{\sqrt{c^2k_\perp^2 + \omega_{p,2}^2} - ck_\perp}{\sqrt{c^2k_\perp^2 + \omega_{p,2}^2} + ck_\perp}$$

Reflection coefficients at zero Matsubara frequency for magnetic metals

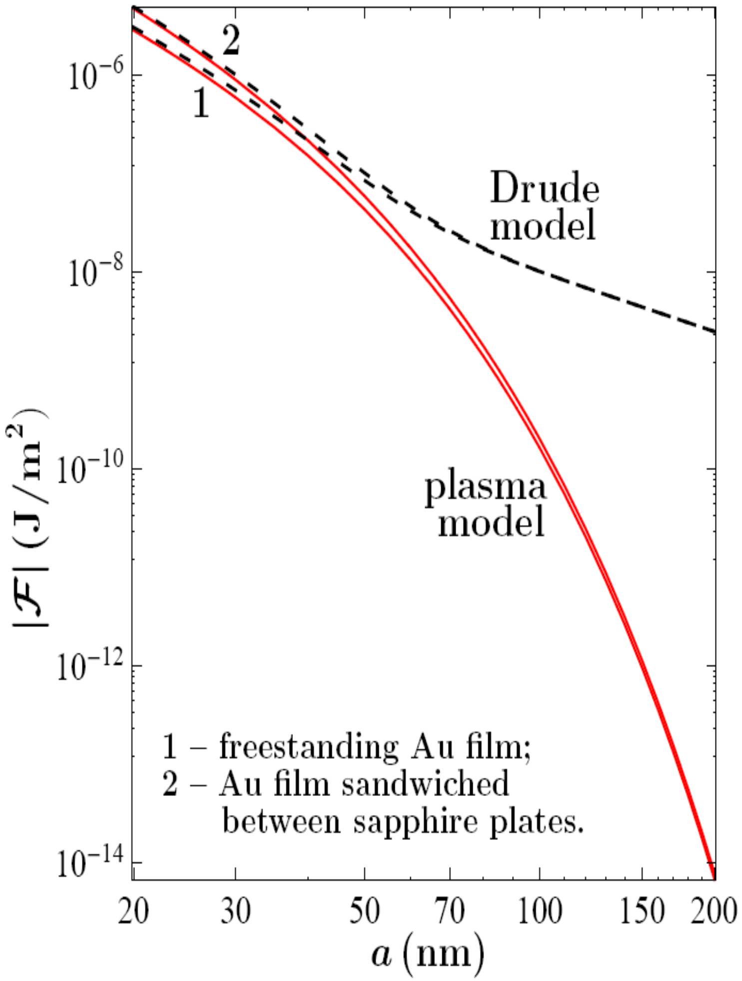
$$r_{\mathrm{TM},D}^{(2,n)}(0,k_\perp) = -1,$$

$$r_{\mathrm{TE},D}^{(2,n)}(0,k_\perp) = \frac{1-\mu_{\mathrm{Ni}}}{1+\mu_{\mathrm{Ni}}}$$

$$r_{\mathrm{TM},p}^{(2,n)}(0,k_\perp) = -1,$$

$$r_{\mathrm{TE},p}^{(2,n)}(0,k_\perp) = \frac{\sqrt{c^2k_\perp^2+\mu_{\mathrm{Ni}}\omega_{p,\mathrm{Ni}}^2}-\mu_{\mathrm{Ni}}ck_\perp}{\sqrt{c^2k_\perp^2+\mu_{\mathrm{Ni}}\omega_{p,\mathrm{Ni}}^2}+\mu_{\mathrm{Ni}}ck_\perp}$$

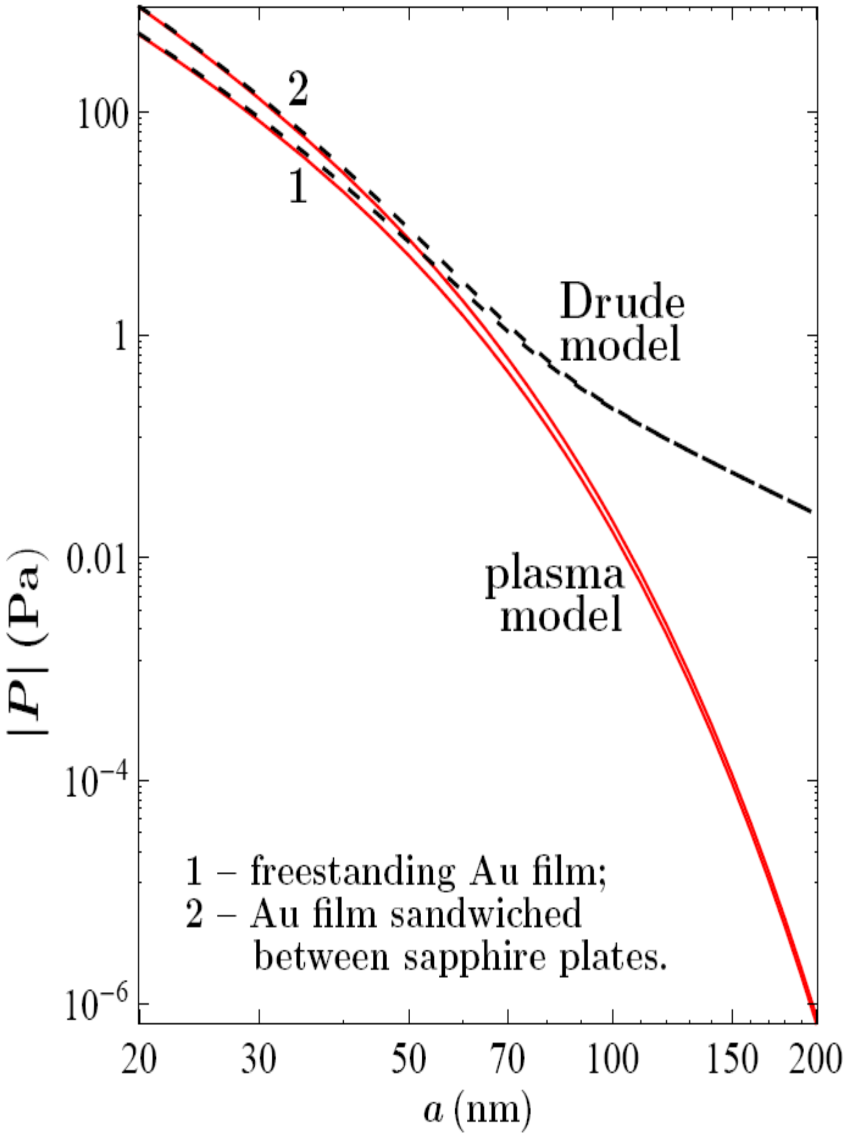
3.1. Results for Au film



For a free-standing Au film

a (nm)	$\frac{\mathcal{F}_D(a,300\text{K})}{\mathcal{F}_p(a,300\text{K})}$
50	1.7
100	50
200	3.2×10^5

Klimchitskaya, Mostepanenko,
PRA, v.92, 042109 (2015)



For a free-standing Au film

a (nm)	$\frac{P_D(a,300\text{K})}{P_p(a,300\text{K})}$
50	1.2
100	10
200	3.2×10^4

3.2. Classical limit and ideal metal limit

For the Drude model, the classical limit starts from a thickness of about 100nm

$$\mathcal{F}_D(a,T) \approx -\frac{k_B T}{16\pi a^2}\zeta(3)$$

For the plasma model, there is no classical limit

$$\mathcal{F}_p(a,T) \approx -\frac{k_B T}{4\pi a}\frac{\omega_{p,2}}{c}e^{-\frac{2a\omega_{p,2}}{c}}\left(1+\sum_{l=1}^{\infty} e^{-\frac{a\xi_l^2}{c\omega_{p,2}}}\right)$$

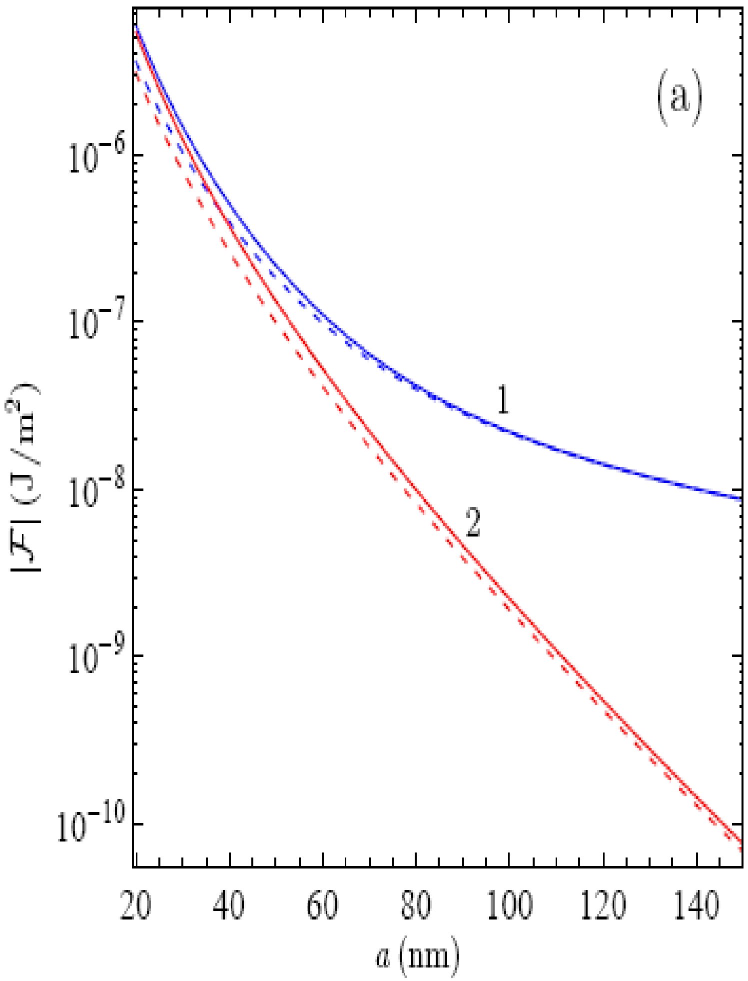
$$\lim_{\omega_{p,2}\to\infty} \mathcal{F}_D(a,T) = -\frac{k_B T}{16\pi a^2}\zeta(3)$$

This result is in contradiction with the fact that electromagnetic fluctuations cannot penetrate in the interior of an ideal metal and, thus, its Casimir free energy must be equal to zero.

If real metal is described by the plasma model, the Casimir free energy, caused by quantum fluctuations of the electromagnetic field, vanishes when the metal becomes an ideal one, as it should be.

$$\lim_{\omega_{p,2}\to\infty} \mathcal{F}_p(a,T) = 0$$

3.3. Results for magnetic Ni film



The magnitudes of the free energy per unit area for a free-standing (solid lines) or sandwiched between sapphire plates (dashed lines) Ni film computed at 300K using the Drude (lines 1) or plasma (lines 2) models of dielectric permittivity.

Klimchitskaya, Mostepanenko, PRB, v.94, 045404 (2016)

4. LOW-TEMPERATURE BEHAVIOR OF THE CASIMIR FREE ENERGY OF METALLIC FILMS

Metals described by the plasma model

$$\mathcal{F}_p(a,T) = E_p(a,T) + \Delta_T\mathcal{F}_p(a,T) \qquad \Delta_T\mathcal{F}_p(a,T) = -\frac{2\pi^2(k_BT)^4}{15\hbar^3c^2\omega_p(e^{2a\omega_p/c}-1)}$$

$$S_p(a,T) = -\frac{\partial\mathcal{F}_p(a,T)}{\partial T} = \frac{8\pi^2k_B(k_BT)^3}{15\hbar^3c^2\omega_p(e^{2a\omega_p/c}-1)}$$

$$S_p(a,T) \to 0$$

The Casimir entropy of a metallic film described by the plasma model satisfies the Nernst heat theorem.

Klimchitskaya, Mostepanenko, PRA, v.95, 012130 (2017)

Metals described by the Drude model

$$\mathcal{F}_D(a,T) = \mathcal{F}_p(a,T) + \mathcal{F}_D^{(0)}(a,T) - \mathcal{F}_p^{(0)}(a,T) + \mathcal{F}^{(\gamma)}(a,T)$$

$$\mathcal{F}_p^{(0)}(a,T) = \frac{k_B T}{16\pi a^2} \int_0^\infty y\,dy \left\{ \ln\left(1 - e^{-\sqrt{y^2+\tilde{\omega}_p^2}}\right) + \ln\left[1 - r_{\mathrm{TE},p}^2(y) e^{-\sqrt{y^2+\tilde{\omega}_p^2}}\right]\right\},$$

$$\mathcal{F}_D^{(0)}(a,T) = \frac{k_B T}{16\pi a^2} \int_0^\infty y\,dy \ln\left(1 - e^{-y}\right) = -\frac{k_B T}{16\pi a^2}\zeta(3),$$

$$
\begin{aligned}
\mathcal{F}_D(a,T) &= \mathcal{F}_p(a,T) + \mathcal{F}^{(\gamma)}(a,T) \\
&- \frac{k_B T}{16\pi a^2}\left[\zeta(3) + I_1\left(\frac{2a\omega_p}{c}\right) + I_2\left(\frac{2a\omega_p}{c}\right)\right],
\end{aligned}
$$

$$
I_1(\tilde{\omega}_p) = -\left[\mathrm{Li}_3(e^{-\tilde{\omega}_p}) + \tilde{\omega}_p \mathrm{Li}_2(e^{-\tilde{\omega}_p})\right],
$$

$$
\begin{aligned}
I_2(\tilde{\omega}_p) \approx &-\left(\tilde{\omega}_p + 17 + \frac{112}{\tilde{\omega}_p} + \frac{432}{\tilde{\omega}_p^2} + \frac{960}{\tilde{\omega}_p^3} + \frac{960}{\tilde{\omega}_p^4}\right) e^{-\tilde{\omega}_p} \\
&+ 4\left[\tilde{\omega}_p K_1(\tilde{\omega}_p) + 9K_2(\tilde{\omega}_p) + \frac{30}{\tilde{\omega}_p} K_3(\tilde{\omega}_p)\right].
\end{aligned}
$$

Klimchitskaya, Mostepanenko, PRA, v.95, 012130 (2017)

$$\lim_{T\to 0} \mathcal{F}^{(\gamma)}(a,T) = 0, \quad \lim_{T\to 0} \frac{\partial \mathcal{F}^{(\gamma)}(a,T)}{\partial T} = 0.$$

$$S_D(a,0) = \frac{k_B}{16\pi a^2}\left[\zeta(3) + I_1\left(\frac{2a\omega_p}{c}\right) + I_2\left(\frac{2a\omega_p}{c}\right)\right]$$

The Casimir entropy of a metallic film with perfect

crystal lattice described by the Drude model

DOES NOT SATISFY the Nernst heat theorem.

Klimchitskaya, Mostepanenko, PRA, v.95, 012130 (2017)

5. CASIMIR FREE ENERGY FOR DIELECTRIC FILMS

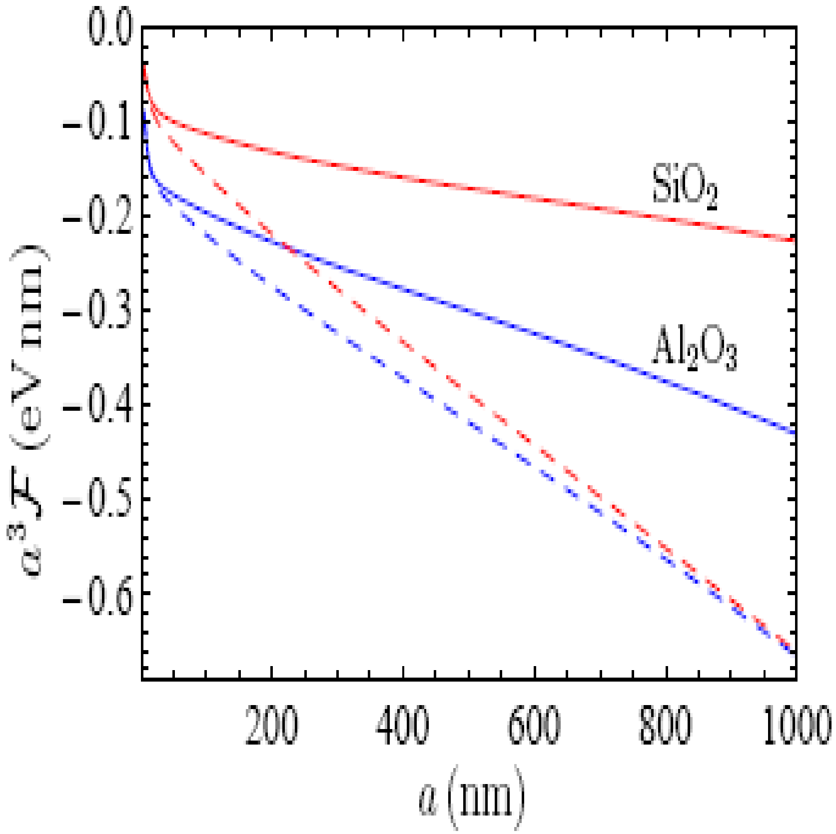
Dielectric permittivity of perfect dielectrics

$$\varepsilon_f(\mathrm{i}\zeta) = 1 + \sum_{j=1}^{N} \frac{g_j}{\omega_j^2 + \omega_c^2\zeta^2 + \gamma_j\omega_c\zeta},$$

Dielectric permittivity with account of free charge carriers which are present in any dielectric material at nonzero temperature

$$\tilde{\varepsilon}_f(\omega) = \varepsilon_f(\omega) + \mathrm{i}\frac{4\pi\sigma_0(T)}{\omega}, \qquad \sigma_0 \sim \exp(-b/T),$$

5.1. Results for free-standing dielectric films



The Casimir free energy per unit area for films made of silica (the top pair of lines) and sapphire (the bottom pair of lines) multiplied by the third power of film thickness computed at 300K with neglected (the solid lines) and included (the dashed lines) of dc conductivity of respective material.

Klimchitskaya, Mostepanenko, J. Phys.: Condens. Matt., v.29, 275701 (2017)

5.2. Classical limit for dielectric films

Perfect dielectrics

$$r_{\mathrm{TM}}^{(f,v)}(0,y) \equiv r_{f,0} = \frac{1-\varepsilon_{f,0}}{1+\varepsilon_{f,0}},$$

$$r_{\mathrm{TE}}^{(f,v)}(0,y) = 0.$$

$$\mathcal{F}^{(l=0)}(a,T) = -\frac{k_B T}{16\pi a^2}\sum_{n=1}^{\infty}\frac{r_{f,0}^{2n}}{n}\int_0^{\infty} y\,dy\,e^{-ny}$$

$$= -\frac{k_B T}{16\pi a^2}\sum_{n=1}^{\infty}\frac{r_{f,0}^{2n}}{n^3} = -\frac{k_B T}{16\pi a^2}\mathrm{Li}_3(r_{f,0}^2),$$

Classical limit for dielectric materials when the presence of free charge carriers at nonzero temperature is taken into account

$$\mathcal{F}^{(l=0)}(a,T) = \frac{k_B T}{16\pi a^2}\int_0^\infty y\,dy\,\ln(1-e^{-y})$$
$$= -\frac{k_B T}{16\pi a^2}\zeta(3),$$

This is the same result as for metals described by the Drude model.

5.3. Low-temperature behavior of the Casimir free energy

Perfect dielectrics:

$$\mathcal{F}(a,T) = E(a) + \Delta_T\mathcal{F}(a,T) \qquad \varepsilon_f(\mathrm{i}\zeta) = 1 + \sum_{j=1}^{N} \frac{g_j}{\omega_j^2 + \omega_c^2\zeta^2 + \gamma_j\omega_c\zeta}$$

$$\Delta_T\mathcal{F}(a,T) = -\frac{(k_BT)^2}{\hbar a^2}\,\frac{\mathrm{Li}_2(r_{f,0}^2)}{12(\varepsilon_{f,0}^2-1)}\sum_{j=1}^{N}\frac{g_j\gamma_j}{\omega_j^4} - \frac{(k_BT)^3}{(\hbar c)^2}\,\frac{\zeta(3)r_{f,0}^2(\varepsilon_{f,0}+1)}{4\pi}.$$

$$S(a,T) = -\frac{\partial\Delta_T\mathcal{F}(a,T)}{\partial T} = \frac{k_B^2T}{\hbar a^2}\,\frac{\mathrm{Li}_2(r_{f,0}^2)}{6(\varepsilon_{f,0}^2-1)}\sum_{j=1}^{N}\frac{g_j\gamma_j}{\omega_j^4} + k_B\left(\frac{k_BT}{\hbar c}\right)^2\frac{3\zeta(3)r_{f,0}^2(\varepsilon_{f,0}+1)}{4\pi}.$$

The Casimir entropy of a film made of perfect dielectric satisfies the Nernst heat theorem.

Dielectrics with account of free charge carriers at nonzero temperature:

$$\tilde{\mathcal{F}}(a,T) = \mathcal{F}(a,T) - \frac{k_B T}{16\pi a^2}\left[\zeta(3) - \mathrm{Li}_3(r_{f,0}^2)\right]$$

Here the terms decreasing with temperature exponentially fast are omitted.

$$\tilde{S}(a,0) = \frac{k_B}{16\pi a^2}\left[\zeta(3) - \mathrm{Li}_3(r_{f,0}^2)\right] > 0$$

The Casimir entropy of a dielectric film with taken into account free charge carriers at nonzero temperature DOES NOT SATISFY the Nernst heat theorem.

6. CONCLUSIONS

1. For thin metallic films at nonzero temperature the use of the Drude and plasma models at low frequencies leads to significantly different results which can be easily discriminated. This is quite different from the case of two metallic plates interacting through a vacuum gap, where the difference by a factor of two is reached only at large separations of about 6 micrometers.

2. There is no classical limit for the Casimir free energy of thin metallic films described by the plasma model. Even for thick metallic films the Casimir free energy preserves its essentially quantum character. The Casimir entropy of thin metallic films described by the plasma model satisfies the Nernst heat theorem.

3. If the film metal is described by the Drude model, the classical limit for the Casimir free energy is reached for about 100nm film thickness. This results in a nonzero Casimir free energy in an ideal-metal limit which contradicts to the fact that electromagnetic fluctuations cannot penetrate in the interior of an ideal metal. The Casimir entropy of thin metallic films described by the Drude model violates the Nernst heat theorem.

4. The Casimir free energy of dielectric films depends significantly on weather we include or omit the conductivity at nonzero temperature in the dielectric permittivity.
For perfect dielectrics with omitted conductivity the Nernst heat theorem is satisfied. The inclusion of conductivity at nonzero temperature leads to violation of the Nernst heat theorem.