# **Entropy production and entanglement forces?**

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merci à:

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# Outline

 Quantum Field Theory for lossy systems history system+bath paradigm Langevin vs. Rytov

### • current issues

"thermodynamic cut" energy conservation entropic/entanglement forces

### **Quantum Field Theory for lossy systems**

### Quantum Dissipative Systems

Ulrich Weiss Modern Condensed Matter Physics vol. 13 (3rd ed., World Scientific 2008)

bath: Feynman-Vernon influence functional spin-boson physics: NIBA & co

**The Theory of Open Quantum Systems** Heinz-Peter Breuer and Francesco Petruccione (Oxford University Press 2002)

(non)Lindblad master equation damped oscillator, Brownian motion, Born-Markov & co

### A Note on the Quantization of Dissipative Systems

Wesley E. Brittin *Phys. Rev.* **77** (1950) 396–97

 $\Delta x \Delta \dot{x} \ge \hbar e^{-\alpha t}/m$ 

(4)

violates the uncertainty principle for an oscillator,

$$\Delta x \Delta \dot{x} \ge \hbar/m. \tag{5}$$

Equation (5) is valid for the oscillator even when damping is taken into account, but to get the correct result one must treat the coupled system—oscillator plus radiation field.<sup>2</sup> Kanai conjectures that it is prob-

<sup>2</sup> Bauer & Jensen [*Z Phys* 1948]

$$H = \frac{1}{2}p^2 e^{-\alpha t} + \frac{1}{2}\omega^2 x^2 e^{+\alpha t}$$
$$p = e^{\alpha t} \dot{x}$$

Irreversibility and generalized noise

Herbert B. Callen and Theodore A. Welton *Phys. Rev.* **83** (1951) 34

quantum Johnson-Nyquist voltage noise

$$\langle V^2 \rangle = (2/\pi) \int_0^\infty R(\omega) E(\omega, T) d\omega,$$
 (4.8)

where

$$E(\omega, T) = \frac{1}{2}\hbar\omega + \hbar\omega [\exp(\hbar\omega/kT) - 1]^{-1}. \quad (4.9)$$

It may be recognized that  $E(\omega, T)$  is, formally, the expression for the mean energy at the temperature T of an oscillator of natural frequency  $\omega$ .

impedance  $\delta V = Z \delta \dot{Q}$ ,  $R = \operatorname{Re} Z$ 

dissipation: system "acts on" bath loss of coherence: bath fluctuations acting on system

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#### "remarkable formula"

Ford, Lewis & O'Connell [*Phys Rev Lett* 1985] van Kampen, Nijboer & Schram [*Phys Lett A* 1968] Davies [*Chem Phys Lett* 1972] Obcemea [*Int J Q Chem* 1987]

Einige Probleme aus der Theorie der elektrischen Schwankungserscheinungen

W. L. Ginsburg Fortschr. Phys. **1** (1953) 51

zero-point potential energy of damped circuit

$$\langle U_0 \rangle = \frac{\hbar}{2\pi \sqrt{LC}} \cdot \frac{1}{\sqrt{4-\alpha^2}} \left[ \frac{\pi}{2} - \arctan \frac{\alpha^2 - 2}{\alpha \sqrt{4-\alpha^2}} \right]$$

damping  $\alpha = RC\omega$ , resonance  $\omega = (LC)^{-1/2}$ 

dissipation: system "acts on" bath loss of coherence: bath fluctuations acting on system



Theory of Electrical Fluctuations and Thermal Radiation

S. M. Rytov

(Academy of Sciences USSR 1953)

$$\begin{split} &\mathrm{i}\omega\varepsilon(\omega)\mathbf{E}+\nabla\times\mathbf{H} &= \mathbf{j}_{\mathrm{th}}+\mathbf{j}_{\mathrm{ext}}\,,\\ &-\mathrm{i}\omega\mu(\omega)\mathbf{H}+\nabla\times\mathbf{E} &= \mathbf{0}\,, \end{split}$$

Maxwell–Langevin equations

$$\begin{array}{lll} \langle j_{\rm th} \rangle &=& \mathbf{0} \\ \langle j_{\rm th}(x) j_{\rm th}(x') \rangle &=& \kappa T \operatorname{Re} \sigma \, \delta(x - x') \\ \langle j_{\rm th}^*(\mathbf{x}, \omega) j_{\rm th}(\mathbf{x}', \omega') \rangle &=& \frac{2\hbar\omega^2}{\mathrm{e}^{\hbar\omega/\kappa T} - 1} \operatorname{Im} \varepsilon_{kl}(\omega) 2\pi \delta(\omega - \omega') + \mathsf{magn.} \end{array}$$

fluctuation-dissipation theorem ("2nd": W. Eckhardt 1982) consistent quantization for linear, causal media (Scheel, Knöll & Welsch 1998)

PHYSICAL REVIEW

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JULY 15, 1960

#### Dissipation in Quantum Mechanics. The Harmonic Oscillator

I. R. Senitzky

U. S. Army Signal Research and Development Laboratory, Fort Monmouth, New Jersey (Received February 29, 1960) Phys Rev 119 (1960) 670

### equation of motion for system energy

$$\langle H_{\text{osc}} \rangle = \langle H_{\text{osc}}^{(0)} \rangle e^{-\beta t} + \hbar \omega \{ \frac{1}{2} + [\exp(\hbar \omega / kT) - 1]^{-1} \} (1 - e^{-\beta t}).$$
 (60)

• dissipative losses balanced by fluctuating forces

#### Energy balance for a dissipative system

X. L. Li, G. W. Ford, and R. F. O'Connell, *Phys Rev E* **48** (1993) 1547 comment by Senitzky: *Phys Rev E* **51** (1995) 5166

#### approximations:

- bath near thermal equilibrium (M. Lax)
- weak damping  $\beta \ll \omega$
- small renormalization

commutators preserved zero point energy preserved

#### **Coupled surface polaritons and the Casimir force**

Carsten Henkel, Karl Joulain, Jean-Philippe Mulet, and Jean-Jacques Greffet *Phys. Rev. A* **69** (2004) 023808

surface plasmon contribution to pressure

$$F = \frac{\hbar\Omega}{4\pi d^3} \left( \alpha(z) - \frac{\gamma \text{Li}_3(z^2)}{4\pi\Omega} \right), \qquad (12)$$

short distance d, Drude damping  $\gamma$ , surface plasmon frequency  $\Omega$ , metal: z = 1,  $\alpha(1) \approx 0.1388$ ,  $\text{Li}_3(1) = \zeta(3)$ 

summation over surface plasmons: Intravaia and Lambrecht [*Phys Rev Lett* 2005] [*Phys Rev A* 2007]

# **Energy and Entropy**



Energy flux balance

$$\delta \dot{Q}_{\rm in} = \delta \dot{Q}_{\rm out}$$

... otherwise climate heating

Net entropy flux

$$\mathrm{d}\dot{S}_{\mathrm{in}} - \mathrm{d}\dot{S}_{\mathrm{out}} \simeq \frac{\delta\dot{Q}_{\mathrm{in}}}{T(\odot)} - \frac{\delta\dot{Q}_{\mathrm{out}}}{T(\eth)} < 0$$

"solar information source" [L. Boltzmann/E. Schrödinger]

entropy source/sink: material bodies, complex many-body systems

## "Entanglement forces"

#### example: "quantum friction force"



heuristics:

lateral delay of image charge, dipole ...

Barton [New J Phys  $\geq$  2010], Behunin & Hu [Phys Rev A 2010], Pieplow & Henkel [New J Phys 2013], Intravaia & al [J. Phys. Condens. Matt. 2015], ...



pair production ("unstable vacuum")
plate-plate entangled state
"afresh" zero temperature baths
non-equilibrium stationary state

Pendry [*J mod Opt* 1998], Philbin & Leonhardt [*New J Phys* 2009]

## Thermodynamic cut – toy model





#### spectrum of heat current

competition: interaction vs bath coupling

standard language: 'continuous variables' (covariance matrix  $\langle q_i q_j \rangle$ , partial transpose ...) • goal here: extend the entanglement criterion to spectral correlations  $S_{ij}(\omega) \leftarrow \langle q_i(t)q_j(t') \rangle$ Dorofeyev [*Can J Phys* 2013], Biehs & Agarwal [*JOSA B* 2013], Barton [*J Stat Phys* 2015] ... **Two-Temperature van der Waals Potentials**, J. P. Rosenkrans, B. Linder, and R. A. Kromhout, *J Chem Phys* 49 (1968) 2927

#### Mystery of vacuum fluctuations?

... challenge to cosmology

... down to earth: regularized by matter (de)coupling

#### System+bath physics

... roots of thermodynamics (fluctuation–dissipation) ... poor men's master equations vs NEGF

#### Challenges

... thermal anomaly (low Matsubara frequencies) ... heat transport: take materials/interfaces seriously