These exercises are intended to be solved emulating a real-time exam situation. Take the time it takes for you to solve each exercise and compare it to the suggested time in parenthesis.

1. **Forces, work, and line integrals**

   (a) A force $\mathbf{A}$ performs work along a path starting at point a and ending at point b. What can be said about $\nabla \times \mathbf{A}$, and about a and b, if the work performed by $\mathbf{A}$ is zero for every path between a and b? (*5 min*)

   (b) The Lorentz force acting on a point charge of charge q moving with velocity $\mathbf{v}$ on an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ is given by

   $$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

   Show that the magnetic component of this force $\mathbf{F_B} = q(\mathbf{v} \times \mathbf{B})$ does not ever perform any work. (*5-10 min*)

   (c) Consider the force

   $$\mathbf{F} = (2x + yz)\hat{i} + (xz)\hat{j} + (yx)\hat{k}$$

   Calculate the work performed by $\mathbf{F}$ along the closed path joining the curves ϵ, γ, and ω.
   ϵ is the top half of a counterclockwise ellipse on the xy- plane given by the parameterization

   $$\epsilon = \{\mathbf{q}(t) = (x, y, z) = (a\cos t, b\sin t, 0) \mid 0 \leq t \leq \pi\}$$

   γ is the straight line joining the points $(-a, 0, 0)$ and $(0, 0, c)$, given by the parameterization

   $$\gamma = \{\mathbf{q}(t) = (x, y, z) = (-a + at, 0, ct) \mid 0 \leq t \leq 1\}$$

   and ω is the straight line joining the points $(0, 0, c)$ and $(a, 0, 0)$, given by the parameterization

   $$\omega = \{\mathbf{q}(t) = (x, y, z) = (at, 0, c - ct) \mid 0 \leq t \leq 1\}$$

   Sketch a picture of the path to help you visualize the problem. (*15-20 min*)

2. **Initial value problem**

   (a) Consider the EOM
   $$10\ddot{x} + 5x = 0$$
   with general solution
   $$x(t) = c_1 \cos\omega(t - t_0) + c_2 \sin\omega(t - t_0)$$
   Find the value of ω and the solution given the initial conditions
   $$x(t_0) = 9;\ \dot{x}(t_0) = 5$$
   (*10 min*)

   (b) Solve the following differential equation
   $$\dot{y} = xe^{-y}$$
   subject to the initial condition
   $$y(x = 0) = 1$$
   (*10 min*)

3. **Bonus problems**

   (a) Consider the force
   $$\mathbf{\Gamma}(\mathbf{x}) = \frac{1}{c_1(\mathbf{x})}\Big(c_2(\mathbf{x})\mathbf{\Sigma}(\mathbf{x}) + \mathbf{\Omega}(\mathbf{x}) \times \mathbf{\Delta}(\mathbf{x})\Big)$$
   where c_1 and c_2 are scalar functions, and $\mathbf{\Omega}(\mathbf{x}) = \mathbf{\Sigma}(\mathbf{x}) \times \mathbf{\Gamma}(\mathbf{x})$. Find the (non-zero) functional form of c_1 and c_2 such that $\mathbf{\Gamma}(\mathbf{x})$ can be rewritten as
   $$\mathbf{\Gamma}(\mathbf{x}) = -\frac{1}{c_1(\mathbf{x})}\Big(\mathbf{\Delta}(\mathbf{x}) \times \mathbf{\Omega}(\mathbf{x})\Big)$$
   if $\mathbf{\Delta}(\mathbf{x})$ and $\mathbf{\Gamma}(\mathbf{x})$ are orthogonal (with $\mathbf{\Gamma}(\mathbf{x})$, $\mathbf{\Sigma}(\mathbf{x})$, and $\mathbf{\Omega}(\mathbf{x})$ non-zero). (*15-20 min*)

   (b) For a force $\mathbf{F}$, the conditions of being irrotational $\nabla \times \mathbf{F} = 0$ and performing a net work of zero along a closed path are often used to define if the force is conservative. These two conditions are interchangeable. Show explicitly that these two conditions are equivalent. *Hint:* Revise Stoke's theorem. (*5-10 min*)