Lecture Notes by Jürgen Vollmer

Theoretical Mechanics

— Working Copy, Chapter 1 –

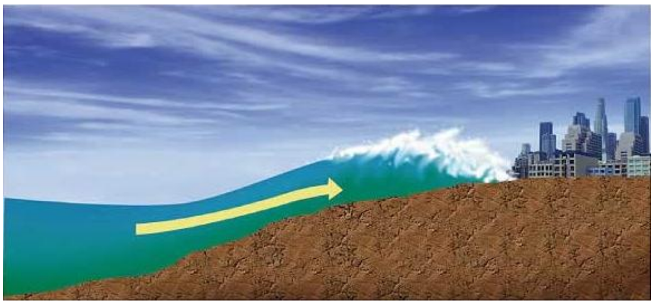
— 2021-11-01 02:07:58+01:00—

Fakultät für Physik und Geowissenschaften, Universität Leipzig

LECTURES DELIVERED AT FAKULTÄT FÜR PHYSIK UND GEOWISSENSCHAFTEN, UNIVERSITÄT LEIPZIG
http://www.uni-leipzig.de/physik/~vollmer

Contents

1
Basic Principles



-Ilhador-/wikimedia, Public domain

At the end of this chapter we will be able to estimate the speed of a Tsunami wave.

1.1 Basic notions of mechanics

Definition 1.1: System

A mechanical *system* is comprised of particles labeled by an index $i \in \mathbb{I}$, that have masses m_i, reside at the positions $\boldsymbol{x}_i$, and move with velocities $\boldsymbol{v}_i$.

Remark 1.1. We say that the system has N particles when $\mathbb{I} = \{1, \dots, N\}$.

Remark 1.2. Bold-face symbols indicate here that $\boldsymbol{x}_i$ describes a position in space. For a D-dimensional space one needs D numbers[1] to specify the position, and $\boldsymbol{x}_i$ may be thought of as a vector in $\mathbb{R}^D$. We say that $\boldsymbol{x}_i$ is a D-vector. In Chapter 2 we will take a closer look at vectors and their properties.

[1] Strictly speaking we do not only need numbers, but must also indicate the adopted units.

Remark 1.3. In order to emphasize the close connection between positions and velocities, the latter will also be denoted as $\dot{\boldsymbol{x}}$.

Remark 1.4. In hand writing vectors are commonly denoted by an arrow, i. e. $\vec{x}$ rather than $\boldsymbol{x}$.

Example 1.1: A piece of chalk

We wish to follow the trajectory of a piece of chalk through the lecture hall. In order to follow its position and orientation in space, we *decide* to model it as a set of two masses that are localized at the tip and at the tail of the chalk. The positions of these two masses $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ will both be vectors in $\mathbb{R}^3$. For instance we can indicate the shortest distance to three walls that meet in one corner of the lecture hall. In this model we have $N = 2$ and $D = 3$.

Definition 1.2: Degrees of Freedom (DOF)

A system with N particles whose positions are described by D-vectors has $D\,N$ *degrees of freedom (DOF)*.

Remark 1.5. Note that according to this definition the number of DOF is a property of the model. For instance, the model for the piece of chalk has $D\,N = 6$ DOF. However, the length of the piece of chalk does not change. Therefore, one can find an alternative description that will only evolve 5 DOF. (We will come back to this in due time.)

Definition 1.3: State Vector

The position of all particles can be written in a single *state vector*, $\boldsymbol{q}$, that specifies the positions of all particles. Its components are called coordinates.

Remark 1.6. For a system with N particles whose positions are specified by D-dimensional vectors, $\boldsymbol{x}_i = (x_{i,1}, \dots, x_{i,D})$, the vector $\boldsymbol{q}$ takes the form $\boldsymbol{q} = (x_{1,1}, \dots, x_{1,D}, x_{2,1}, \dots, x_{2,D}, \dots, x_{N,1}, \dots, x_{N,D})$, which comprises the coordinates $x_{1,1}, \dots, x_{N,D})$. For conciseness we will also write $\boldsymbol{q} = (\boldsymbol{x}_1, \dots, \boldsymbol{x}_N)$. The vector $\boldsymbol{q}$ has DOF number of entries, and hence $\boldsymbol{q} \in \mathbb{R}^{DN}$.

Remark 1.7. The velocity associated to $\boldsymbol{q}$ will be denoted as $\dot{\boldsymbol{q}} = (\dot{\boldsymbol{x}}_1, \dots, \dot{\boldsymbol{x}}_N)$.

Definition 1.4: Phase Vector

The position and velocities of all particles form the *phase vector*, $\boldsymbol{\Gamma} = (\boldsymbol{q}, \dot{\boldsymbol{q}})$.

Definition 1.5: Trajectory

The *trajectory* of a system is described by specifying the time dependent functions

$$\begin{aligned} & \boldsymbol{x}_i(t), \boldsymbol{v}_i(t), \quad i = 1, \dots, N \\ \text{or} \quad & \boldsymbol{q}(t), \dot{\boldsymbol{q}}(t) \\ \text{or} \quad & \boldsymbol{\Gamma}(t) \end{aligned}$$

Definition 1.6: Initial Conditions (IC)

For $t \in [t_0, \infty)$ the trajectory is uniquely determined by its *initial conditions (IC)* for the positions $\boldsymbol{x}_i(t_0)$ and velocities $\boldsymbol{v}_i(t_0)$, i.e. the point $\boldsymbol{\Gamma}(t_0)$ in phase space.

Remark 1.8. This definition expresses that the future evolution of a system is *uniquely* determined by its ICs. Such a system is called deterministic. Mechanics addresses the evolution of deterministic systems. At some point in your studies you might encounter stochastic dynamics where different rules apply.

Example 1.2: Throwing a javelin

The ICs for the flight of a javelin specify where it is released, $\boldsymbol{x}_0$, when it is thrown, the velocity $\boldsymbol{v}_0$ at that point of time, and the orientation of the javelin. In a good trial the initial orientation of the javelin is parallel to its initial velocity $\boldsymbol{v}_0$, as shown in Figure 1.1

based on Atalanta, creativecommons, CC BY-SA 3.0

Figure 1.1: Initial conditions for throwing a javelin, cf. Example 1.2.

Remark 1.9. In repeated experiments the ICs will be (slightly) different, and one observes different trajectories.

1. A seasoned soccer player will hit the goal in repeated kicks. However, even a professional may miss occasionally.

2. A bicycle involves a lot of mechanical pieces that work together to provide a predictable riding experience.

3. A lottery machine involves a smaller set of pieces than a bike, but it is constructed such that unnoticeably small differences of

initial conditions give rise to noticeably different outcomes. The outcome of the lottery can not be predicted, in spite of best efforts to select identical initial conditions.

Definition 1.7: Constant of Motion

A function of the positions x_i and velocities v_i is called a *constant of motion*, when it does not evolve in time.

Remark 1.10. For a given initial conditions a constant of motion takes the same value for the full trajectory. However, it may take different values for different trajectories, i.e. different choices of initial conditions.

Example 1.3: Length of a piece of chalk

During the flight the positions x_1 and x_2 of the piece of chalk will change. However, the length L of the piece of chalk will not, and at any given time it can be determined from x_1 and x_2. Hence, L is a constant of motion that takes the same value for all trajectories of the piece of chalk.

Example 1.4: Energy conservation for the piece of chalk

We will see that the sum of the potential and the kinetic energy is conserved during the flight of the piece of chalk. This sum, the total energy E, is a constant of motion. The potential energy depends on the position and the kinetic energy is a function of the velocity. Trajectories that start at the same position with different speed will therefore have different total energy. Hence, E is a constant of motion that can take different value for different trajectories of the piece of chalk.

Definition 1.8: Parameter

In addition to the ICs the trajectories will depend on *parameters* of the system. Their values are fixed for a given system.

Example 1.5: A piece of chalk

For the piece of chalk the trajectory will depend on whether the hall is the Theory Lecture Hall in Leipzig, a briefing room in a ship during a heavy storm, or the experimental hall of the ISS space station. To the very least one must specify how the gravitational acceleration acts on the piece of chalk, and how the room moves in space.

Remark 1.11. The set of parameters that appear in a model depends on the *choices* that one makes upon setting up the experiment. For instance

Beckham's banana kicks can only be understood when one accounts for the impact of air friction on the soccer ball.

Air friction will not impact the trajectory of a small piece of talk that I throw into the dust bin.

By adopting a clever choice of the parameterization the trajectory of the piece of chalk can be described in a setting with 5 DOF. The length of the piece of chalk will appear as a parameter in that description.

Definition 1.9: Physical Quantities

Positions, velocities and parameters are *physical quantities* that are characterized by at least one number and a unit.

in 3. length or duration?

Example 1.6: Physical Quantities

1. The mass, M, of a soccer ball can be fully characterized by a number and the unit kilogram (kg), e.g. $M \approx 0.4\,\mathrm{kg}$.
2. The length, L, of a piece of chalk can be fully characterized by a number and the unit meter (m), e.g. $L \approx 7 \times 10^{-2}\,\mathrm{m}$.
3. The duration, T, of a year can be characterized by a number and the unit second, e.g. $T \approx \pi \times 10^{7}\,\mathrm{s}$.
4. The speed, v, of a car can be fully characterized by a number and the unit, e.g. $v \approx 42\,\mathrm{km\,h^{-1}}$.
5. A position in a D-dimensional space can fully be characterized by D numbers and the unit meter.
6. The velocity of a piece of chalk flying through the lecture hall can be characterized by three numbers and the unit m/s. However, one is missing information in that case about its rotation.

Remark 1.12. Analyzing the units of the parameters of a system provides a fast way to explore and write down functional dependencies. When doing so, the units of a physical quantity Q are denoted by $[Q]$. For instance for the length L of the piece of chalk, we have $[L] = \mathrm{m}$. For a dimensionless quantity d we write $[d] = 1$.

Example 1.7: Changing units

Suppose we wish to change units from km/h to m/s. A transparent way to do this for the speed of the car in the example above is by multiplications with one

$$v = 72 \frac{\mathrm{km}}{\mathrm{h}} \, \frac{1\,\mathrm{h}}{3.6 \times 10^{3}\,\mathrm{s}} \, \frac{1 \times 10^{3}\,\mathrm{m}}{1\,\mathrm{km}} = \frac{72}{3.6} \mathrm{m\,s^{-1}} = 20\,\mathrm{m\,s^{-1}}$$

Definition 1.10: Dynamics

The characterization of all possible trajectories for all admissible ICs is called *dynamics* of a system.

1.1.1 Self Test

Problem 1.1. **The degrees of freedom of a frisbee**

a) How would you describe the position of a frisbee in space?

b) How many degrees of freedom does your parameterization involve?

c) Are there constants of motion in your description?

d) Specify at least three parameters required for the description.

Problem 1.2. **Useful numbers and unit conversions**

a) Verify that
   - one nano-century amounts to π seconds,
   - a colloquium talk at our Physics Department must not run take longer than a micro-century,
   - a generous thumb-width amounts to one atto-parsec.

b) The Physics Handbook of Nordling and Österman (2006) defines a beard-second, i. e. the length an average beard grows in one second, as 10 nm. In contrast, Google Calculator uses a value of only 5 nm. I prefer the one where the synodic period of the moon amounts to a beard-inch. Which one will that be?

c) In the furlong–firkin–fortnight (FFF) unit system one furlong per fortnight amounts to the speed of a tardy snail (1 centimeter per minute to a very good approximation), and one micro-fortnight was used as a delay for user input by some old-fashioned computers (it is equal to 1.2096 s). Use this information to determine the length of one furlong.

1.2 Dimensional analysis

Mathematics does not know units. Experimental physicists hate large sets of parameters because the sampling of high-dimensional parameter space is tiresome. A remedy to both issues is offered by the Buckingham-Pi-Theorem. We state it here in a form accessible with our present level of mathematical refinement. The discussion of a more advanced formulation may appear as a homework problem later on on this course.

Theorem 1.1: Buckingham-Pi-Theorem

A dynamics with n parameters, where the positions $\boldsymbol{q}$ and the parameters involve the three units meter, seconds and kilogram, can be rewritten in terms of a *dimensionless dynamics* with $n-3$ parameters, where the positions $\boldsymbol{\xi}$, velocities $\boldsymbol{\zeta}$, and parameters π_j with $j \in \{1, \ldots, n-3\}$ are given solely by numbers.

Example 1.8: Non-dimensionalization for a pendulum

Let $\boldsymbol{x}$ denote the position of a pendulum of mass M that is attached to a chord of length L and swinging in a gravitational field $\boldsymbol{g}$ of strength g (see Figure 1.2).
The units of these quantities are $[\boldsymbol{x}] = \mathrm{m}$, $[M] = \mathrm{kg}$, $[L] = \mathrm{m}$, and $[g] = \mathrm{m/s^2}$, respectively. The position $\boldsymbol{x}$ describes the position of the system. Its evolution will depend (potentially) on the three parameters, M, L, and g, plus the direction of $\boldsymbol{g}$.
In this problem we choose L as length scale and $\sqrt{L/g}$ as time scale. Then the dimensionless positions will be $\boldsymbol{\xi} = \boldsymbol{x}/L$, the dimensionless velocities will be $\boldsymbol{\zeta} = \dot{\boldsymbol{x}}/\sqrt{g\,L}$.
There is no way to turn M into a dimensionless parameter. Therefore, the evolution of (ξ, ζ) can not depend on M. The only dimensionless parameter that remains in the model is the direction of $\boldsymbol{g}$.

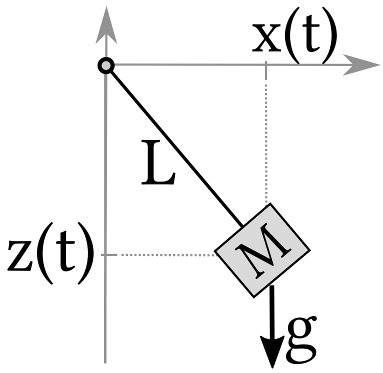


Figure 1.2: Pendulum discussed in Example 1.8

Example 1.9: Non-dimensionalization for the flight of a piece of chalk

Let $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ denote the position of the tip and the tail of a model for a piece of chalk, where tip and tail are associated to masses m_1 and m_2. The piece of chalk has a length L. It performs a free flight in a gravitational field with acceleration $\boldsymbol{g}$ of strength g.
The units of these quantities are $[\boldsymbol{x}_i] = \mathrm{m}$, $[m_i] = \mathrm{kg}$, $L = \mathrm{m}$, and $[g] = \mathrm{m/s^2}$, respectively. There are four parameters, $n = 4$, plus the direction of $\boldsymbol{g}$.
In this problem we choose L as length scale and $\sqrt{L/g}$ as time scale. Then the dimensionless positions will be $\boldsymbol{\xi}_i = \boldsymbol{x}_i/L$, the dimensionless velocities will be $\boldsymbol{\zeta} = \dot{\boldsymbol{x}}_i/\sqrt{g\,L}$.
The two masses m_1 and m_2 give rise to the dimensionless parameter $\pi_1 = m_1/m_2$, and in three dimensions the direction of $\boldsymbol{g}$ must be characterized by another two dimensionless parameters.

Proof of the Buckingham-Pi-Theorem. We first look for combinations of the parameters with the following units

$$\mathrm{m} = \left[p_1^{\alpha_1}\right]\,\left[p_2^{\alpha_2}\right]\,\ldots\,\left[p_n^{\alpha_n}\right]$$

$$\mathrm{s} = \left[p_1^{\beta_1}\right]\,\left[p_2^{\beta_2}\right]\,\ldots\,\left[p_n^{\beta_n}\right]$$

$$\mathrm{kg} = \left[p_1^{\gamma_1}\right]\,\left[p_2^{\gamma_2}\right]\,\ldots\,\left[p_n^{\gamma_n}\right]$$

Each of these equations involves constraints on the exponents in order to match the exponents of the three units that can be expressed as a system of linear equations. The solvability conditions for such systems imply that they conditions can always be met by an appropriately chosen set of three parameters. Without loss of generality we denote them as p_1, p_2 and p_3, and we have

$$\begin{aligned} \mathrm{m} &= \left[p_1^{\alpha_1}\right] \left[p_2^{\alpha_2}\right] \left[p_3^{\alpha_n}\right] \\ \mathrm{s} &= \left[p_1^{\beta_1}\right] \left[p_2^{\beta_2}\right] \left[p_3^{\beta_n}\right] \\ \mathrm{kg} &= \left[p_1^{\gamma_1}\right] \left[p_2^{\gamma_2}\right] \left[p_3^{\gamma_n}\right] \end{aligned} \tag{1.2.1}$$

Thus we use the parameters $p_1, \dots, p_3$ to remove the units from our description. In its dimensionless form it will involve the positions and velocities

$$\begin{aligned} \xi &= q\, p_1^{-\alpha_1}\, p_2^{-\alpha_2}\, p_3^{-\alpha_n} \\ \zeta &= \dot{q}\, p_1^{\beta_1-\alpha_1}\, p_2^{\beta_2-\alpha_2}\, p_3^{\beta_n-\alpha_n} \end{aligned}$$

Similarly, the dimensionless form of the parameters p_i of the dynamics are obtained by multiplying the original parameters with appropriate powers of the expressions (1.2.1) of the units. For p_1 to p_3 this gives rise to one. Additional parameters will turn into dimensionless groups of parameters that provide π_1 to π_{n-3}. □

expand and provide more examples

1.2.1 *Self Test*

Problem 1.3. **Oscillation period of a particle attached to a spring**

In a gravitational field with acceleration $g_{\text{Moon}} = 1.6\,\text{m}/\text{s}^2$ a particle of mass $M = 100\,\text{g}$ is hanging at a spring with spring constant $k = 1.6\,\text{kg}/\text{s}^2$. It oscillates with period T when it is slightly pulled downwards and released. We describe the oscillation by the distance $x(t)$ from its rest position.

a) Determine the dimensionless distance $\xi(t)$, and the associated dimensionless velocity $\zeta(t)$.

b) Provide an order-of-estimate guess of the oscillation period T.

Problem 1.4. **Earth orbit around the sun**

a) Light travels with a speed of $c \approx 3 \times 10^8\,\text{m}\,\text{s}^{-1}$, and it takes 500 s to travel from Sun to Earth. What is the Earth-Sun distance D, i. e. one Astronomical Unit (AU) in meters?

b) The period of the trajectory of the Earth around the Sun depends on D, on the mass $M = 2 \times 10^{30}\,\text{kg}$ of the sun, and on the gravitational constant $G = 6.7 \times 10^{-11}\,\text{m}^3/\text{kg}\,\text{s}^2$. Estimate, based on this information, how long it takes for the Earth to travel once around the sun.

c) Express your estimate in terms of years. The estimate of (b) is of order one, but still off by a considerable factor. Do you recognize the numerical value of this factor?

d) Upon discussing the trajectory $\boldsymbol{x}(t)$ of planets around the sun later on in this course, we will introduce dimensionless positions of the planets $\boldsymbol{\xi}(t) = \boldsymbol{x}(t)/L = (x_1(t)/L, x_2(t)/L, x_3(t)/L)$. How would you define the associated dimensionless velocities?

1.3 *Order-of-magnitude guesses*

Many physical quantities take a value close to one when they are expressed in their "natural" dimensionless units. When the choice is unique, then clearly it is also natural. Otherwise, the appropriate choice is a matter of experience.

We will come back to this when we employ non-dimensionalization in the forthcoming discussion. We demonstrate this based on a discussion of

Example 1.10: The period of a pendulum

We consider a pendulum of mass M attached at a stiff bar of negligible mass. With this bar it is fixed to a pivot at a distance L from the mass such that it can swing in a gravitational field inducing an acceleration g. In this example we make use of the fact that the bar has fixed length L, and describe the position of the mass by the angle $\theta(t)$ (see Figure 1.3).

As discussed in Example 1.8 the time unit for this problem is $\sqrt{L/g}$. Hence we estimate that the period T of the pendulum is of the order of $T \simeq \sqrt{L/g}$. Explicit calculations to be performed later on will reveal that this estimate is off by a factor 2π when the amplitude is small, $|\theta(t)| \ll 1$. For large oscillation amplitudes θ_0 the period will increase further, tending to infinity when θ_0 approaches π. Hence, we conclude that

$$T = f(\theta_0)\sqrt{L/g} \qquad \text{with } \ f(\theta_0) \simeq 2\pi \ \text{ for } \ \theta_0 \ll 1\,.$$

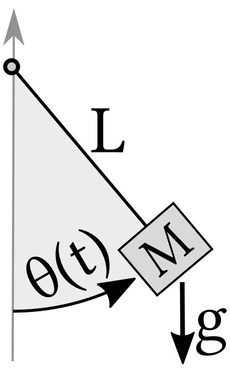


Figure 1.3: Pendulum discussed in Example 1.10

Example 1.11: The speed of Tsunami waves

A Tsunami wave is a water wave that is generated by an earth quake or an underwater land slide. Typical wave lengths are of an order of magnitude $\lambda = 100\,\text{km}$. They travel through the ocean that has an average depth of about $D = 4\,\text{km}$, much smaller than λ. Therefore, we expect that the wave speed v_{Tsunami} is predominantly set by the ocean depth and the gravitational acceleration $g \approx 10\,\text{m/s}^2$, i.e.

$$v_{\text{Tsunami}} \approx \sqrt{gD} = 2 \times 10^2\,\text{m/s} \approx 700\,\text{km/h}$$

This estimate suggests that the 2004 Indian Ocean Tsunami traversed the distance from Indonesia to the East African coast, $L \approx 10\,000\,\mathrm{km}$, in about

$$\frac{L}{v_{\mathrm{Tsunami}}} \approx \frac{1 \times 10^4\,\mathrm{km}}{700\,\mathrm{km/h}} = \frac{100}{7}\mathrm{h} \approx 15\,\mathrm{h}$$

This is very close to the value of 16 h reported in Wikipedia.

Example 1.12: The period of Tsunami waves

In spite of their speed and devastating power, Tsunamis are very hard to detect on the open sea because their period T is very long. It can be estimated as the time that the wave needs to run once through its wavelength[2]

$$T \approx \frac{\lambda}{v_{\mathrm{Tsunami}}} = \frac{\lambda}{\sqrt{gD}} = \frac{100\,\mathrm{km}}{700\,\mathrm{km/h}} = \frac{1}{7\,\mathrm{h}} \approx 10\,\mathrm{min}$$

Here, our estimate is too small by about a factor of three.

[2] Observe that this physical argument goes beyond the blind use of dimensional analysis. The equation for T involves the length scales λ and D in a non-trivial combination that is set by a physical argument.

We conclude that estimates based on dimensional analysis provide valuable insight in time scales of physical processes, even in situations where a detailed mathematical treatment is very delicate.

1.3.1 *Self Test*

Problem 1.5. **Printing the output of Phantom cameras**

With a set of three phantom cameras one can simultaneously follow the motion of 100 particles in a violent 3d turbulent flow. Data analysis of the images provides particle positions with a resolution of 25,000 frames per second. You follow the evolution for 20 minute, print it double paged with 8 coordinates per line and 70 lines per page. A bookbinder makes 12 cm thick books from every 1000 pages. You put these books into bookshelves with seven boards in each shelf. How many meters of bookshelves will you need to store your data on paper?

1.4 *Problems*

Problem 1.6. **Dimensional Analysis of Flight Trajectories**

a) How does the initial velocity v_0 impact the distance W of a thrown object (stone, ball, or shot) or a jump?

b) How does the initial velocity v_0 depend on
the force F acting by the responsible muscle
the accelerated mass M, and
the distance L of the path where the acceleration is performed?

c) Estimate the maximum distance
of throwing a stone of mass $m = 200\,\text{g}$,
of a standing jump for a human and a grass hopper.

d) Make an explicit analysis of standing jumps by exploring how their distance scales with the ratio of characteristic sizes (i.e., body length) of the jumper.

Problem 1.7. **Water waves**

The speed of waves on the ocean depends only on their wave length L and the gravitational acceleration $g \simeq 10\,\text{m/s}^2$.

a) How does the speed of the waves depend on L and g?

b) Unless it is surfing, the speed of a yacht is limited by its hull speed, i.e. the speed of a wave with wave length identical to the length of the yacht. Estimate the top speed of a 30 ft yacht.

c) Close to the beach the water depth H become a more important parameter than the wave length. How does the speed of the crest and the trough of the wave differ? What does this imply about the form of the wave?

Problem 1.8. **Golf on Moon and Earth**

snapshot retrieved from a NASA movie

In the end of the Apollo14 mission, on February 6, 1971, astronauts Alan Shepard and Ed Mitchell modified discarded equipment to perform sport on Moon: Mitchell threw a scoop handle as if it were a javelin (see the Apollo14 Lunar surface journal of the NASA). Shepard attached a golf club head to a handle of a sample tool, and hit two golf balls that still reside on Moon. Rumours tell that the golf balls went "miles and miles and miles".

a) According to Newton's laws of gravity the gravitational acceleration amounts to $G\,M/R^2$, where G is a constant, M the mass of the planet or moon, and R its radius. The Earth radius is four times larger than the one of Moon. Estimate the gravitational acceleration g_M on Moon.
Hint: The acceleration on the Earth surface is $g = 10\,\text{m}\,\text{s}^{-1}$.

b) In contrast to what you have found in a) the gravitational acceleration on Moon is about one sixth of the value on Earth. Use this difference to estimate the difference of the average density of the Moon and of Earth.

c) On Earth a long-distance golf shot can go a few hundred meters. By which factor does this distance increase on Moon?

d) Assume that the shot on Earth can go for 500 m, when one neglects friction due to the Earth atmosphere. Estimate the release velocity of the shot and its time of flight.

e) How long will the golf ball go on Moon, and how long will it fly?

1.4.1 Proofs

1.5 Further reading

The first chapter of Großmann (2012) provides a clear and concise introduction to basic calculus with an emphasis on applications to physics problems.

The introductory chapters of Morin (2014, 2007) provide an excellent introductions to problem solving strategies in physics and dimensional analysis.

Mahajan (2010) and Zee (2020) provided full-grown text books addressing the art of narrowing down solutions of a broad range of physics and mathematics problems.

Harte (1988) is a classic text that introduces and illustrates modeling strategies for problems derived from environmental science.

Bibliography

Archimedes, 1878, in *Pappi Alexandrini Collectionis, Book VIII, c. AD 340*, edited by F. O. Hultsch (Apud Weidmannos, Berlin), p. 1060, cited following https://commons.wikimedia.org/wiki/File:Archimedes_lever,_vector_format.svg.

Arnol'd, V. I., 1992, *Ordinary Differential Equations* (Springer, Berlin).

Epstein, L. C., 2009, *Thinking Physics, Understandable Practical Reality* (Insight Press, San Francisco).

Finney, G. A., 2000, American Journal of Physics **68**, 223–227.

Gale, D. S., 1970, American Journal of Physics **38**, 1475.

Gommes, C. J., 2010, Am. J. Phys. **78**, 236.

Großmann, S., 2012, *Mathematischer Einführungskurs für die Physik* (Springer), very clear introduction of the mathematical concepts for physics students., URL https://doi.org/10.1007/978-3-8348-8347-6.

Harte, J., 1988, *Consider a spherical cow. A course in environmental problem solving* (University Science Books, Sausalito, CA).

Kagan, D., L. Buchholtz, and L. Klein, 1995, Phys.Teach. **33**, 150.

Lueger, O., 1926–1931, *Luegers Lexikon der gesamten Technik und ihrer Hilfswissenschaften* (Dt. Verl.-Anst., Stuttgart), URL https://digitalesammlungen.uni-weimar.de/viewer/resolver?urn=urn:nbn:de:gbv:wim2-g-3163268.

Mahajan, S., 2010, *Street-fighting Mathematics. The Art of Educated Guessing and Opportunistic Problem Solving* (MIT Press, Cambridge, MA), ISBN 9780262514293 152, some parts are available from the author, URL https://mitpress.mit.edu/books/street-fighting-mathematics.

Morin, D., 2007, *Introduction to Classical Mechanics* (Cambridge), comprehensive introduction with a lot of exercises—many of them with worked solutions. The present lectures cover the Chapters 1–8 of the book. I warmly recomment to study Chapters 1 and 6., URL https://scholar.harvard.edu/david-morin/classical-mechanics.

Morin, D., 2014, *Problems and Solutions in Introductory Mechanics* (CreateSpace), a more elementary introduction with a lot of solved exercises self-published at Amazon. Some Chapters can also be downloaded from the autor's home page, URL `https://scholar.harvard.edu/david-morin/mechanics-problem-book`.

Murray, J., 2002, *Mathematical Biology* (Springer).

Nordling, C., and J. Österman, 2006, *Physics Handbook for Science and Engineering* (Studentlitteratur, Lund), 8 edition, ISBN 91-44-04453-4, quoted after Wikipedia's List of humorous units of measurement, accessed on 5 May 2020.

Purcell, E. M., 1977, American Journal of Physics **45**(3).

Seifert, H. S., M. W. Mills, and M. Summerfield, 1947, American Journal of Physics **15**(3), 255.

Sommerfeld, A., 1994, *Mechanik*, volume 1 of *Vorlesungen über theoretische Physik* (Harri Deutsch, Thun, Frankfurt/M.).

Zee, A., 2020, *Fly by Night Physics: How Physicists Use the Backs of Envelopes* (Princeton UP), ISBN 9780691182544.

Index