Lecture Notes by Jürgen Vollmer

Theoretical Mechanics

— Working Copy –

— 2022-02-16 04:54:23+01:00—

Fakultät für Physik und Geowissenschaften, Universität Leipzig

Index

D-tuples, 28
N body problem
    EOM, 115
\ set difference, **18**
⋂ set intersection, **18**
⋃ set union, **18**
Kronecker δ_{ij}, 37
∅ empty set, 15
∃, there is, **18**
$\mathbb{R}^D$, 28
⊆ subset, **16**
Levi-Civita tensor ε_{ijk}, 43
[, 63
2 body
    spaceship lost, 210

∀, for all, **18**

acceleration, **61**
action, 166
algorithm
    Euler Lagrange EOM, 171
    phase space plot, 176
angular momentum, **78**
    conservation, 78
    particle spin, 148
    torque, 151
angular momentum conservation
    central force, 117
axiom
    force balance, 32
    principle of least action, 166
    torque balance, 46
axis of inertia, 151

BAC-CAB rule, 44
basis
    orthonormal basis in $\mathbb{R}^3$, 43
bifurcation
    pitchfork, **180**
    saddle node
        soap film, 173
    saddle-node, **185**
Buckingham-Pi Theorem, 6

cardinality, **15**
carousel
    freely rotating, 208
Cartesian product, *see* product, Cartesian
celestial mechanics, 115
center of mass
    1D collision, 128
    2 particle collision, 138
    extended body, 147
central force
    angular momentum conservation, 117
centrifugal governor, **180**
collision
    2 particles
        1D with center-of-mass frame, 128
        general case, 138
    inelastic, with rotor, 78
    one dimensional, 77
complex number
    2D rotation, 26
    complex conjugate, 26
    field, 25
    rotation in plane, 192
cone
    particle on cone, 197
conservation laws
    translation invariance, 131

conservative force, 73
energy conservation, **75**
work, **74**
constant of motion, **4**, **70**
angular momentum, 78
kinetic energy, 71
momentum, 76
total energy, 75
contour line, **82**, *87*
coordinate system
integration volume, 142
coordinates
basis change, 39
Cartesian, 14, **36**
integration volume, 144
polar, 14
and complex numbers, 26
Kepler problem, 122
mathematical pendulum, 174
spherical
spherical pendulum, 193
Coriolis
on turntable, 213
cross product
geometric interpretation, 45
Jacobi identity, 45
cyclic variables, **187**
cycloid, 52
cylindrical coordinates
integration volume, 144
unit vectors, **182**

d'Alembert's principle, 170
degrees of freedom, **2**
determinant, 143
Jacobi, 144
recursion, 143
diagonalization
two masses over pulley, 207
dimensionless dynamics, 6
Dirac notation, 39
DOF, *see* degrees of freedom
driven pendulum, 169
dynamics, **5**
dimensionless, 6

Effective potential, **191**
effective potential
Mexican hat, 197
Particle on Cone, 197
Einstein summation convention, 38
element, **15**
energy
of impact in accidents, 76
EOM, *see* equation of motion
initial condition, 65
equation of motion, **64**
autonomous, 92
centrifugal governor, 180
free flight, 189
harmonic oscillator, 189
mathematical pendulum, 174
pearl on rotating ring, 177
phase-space plot, 93
spherical pendulum, 193
uniform motion, 187
equations of motion
free flight, 94
non-dimensionalization, 7
with Stokes friction, 98
with turbulent friction, 105
Hookian spring, 108
Kepler orbit, 120
pendulum
non-dimensionalization, 7
Stokes friction
sperms, 104

falling through Earth, 145
Fermat's principle, 172
field, **25**
complex numbers, 25
rational numbers, 25
real numbers, 25
smallest, 25
for all ∀, **18**

force
balance, 32
conservative, 73
Hookian spring, 64
force balance
mobile, 152
free flight
cat and men, 75
EOM
Lagrange, 189
equations of motion, 94
non-dimensionalization, 7
with spin, 152
with Stokes friction
ball in air, 103
friction
static, 32
Stokes, 98
Stokes vs. turbulent, 107
turbulent, 105

generalized coordinates, **162**
groove length in venyl records, 88
group, **21**
90°-rotations with respect to two vertical axis, 22
3 elements, 23
4 elements, 53
Abelian, **21**
commutative, **21**
dihedral of order 6, 24
edit text field, 22
rotations of regular polygons, 53
smallest, 21
symmetries of rectangle, 23

harmonic force
Earth interior, 145
harmonic oscillator, 108
Lagrange EOM, 189
linear EOM, 109
solution for damped, 111
heterocline, **175**
homocline, **175**
Hookian spring, 108
hypotrochoids, 58

IC, *see* initial conditions
inertia
axis of, 151
ball, 150
tensor of, 150
inertial system, **63**
initial condition, 65
initial conditions, *see also* equations of motion, *see also* ordinary differential equation, **3**
inner product, 34
geometric interpretation, 35
integral
line, **72**
volume, **141**
integration
of an ODE, 95
integration volume, 144
interval, **20**
isosurface, **82**, *87*

Jacobi determinant, 144
Jacobi identity, 43, 45
junctors
overview, 17
precedence, 17

Kepler problem
Kepler's 1st law, 123
Kepler's 2nd law, 121
Kepler's 3rd law, 122
Lagrange approach, 198
textbf, 115, 120
kinetic energy, **71**
conservation, 71
Kronecker δ, 37

Lagrange
cyclic variable, **187**
Mexican hat, 197
spaceship lost, 210
two masses over pulley, 207
Lagrange formalism
and dissipation, 180
Lagrangian, 166

Cartesian coordinates, **166**
in generalized coordinates, 171
Levi-Civita tensor ε_{ijk}, 43
line integral, **72**
venyl record, 88
logical junctors
overview, 17
precedence, 17

mass density, **140**
mathematical pendulum
Lagrange coordinates, 162
Lagrange EOM, 174
matrix, **28**
determinant, **143**
Jacobi, 144
multiplication, 29
vector space, 29
mechanical similarity
textbf, 121
momentum, **76**
conservation, 76
momentum conservation
and translation invariance, 188

Newton
2nd law, **64**
3rd law, **66**
vs. Aristotle, 67
non-dimensionalization
free flight, 7
pendulum, 7
numbers
complex, **19**
field property, 25
even, **19**
integer, **19**
group property, 24
natural, **19**
negative, **19**
odd, **19**
rational, **19**
real, **19**

ODE, *see* ordinary differential equation
ordinary differential equation, **92**
autonomous, 92
linear, **109**
solution as implicit equation, 100
solution strategy
integration, 95
linear ODEs with const coefficients, 109
linear ODEs with const coefficients and degeneracy, 112
separation of variables, 99
system
decoupling, 95
oscillator
harmonic, 108

parameter, **4**
partial differential equations (PDE), 92
particle spin, **148**, 148
partition, **140**
pearl on rotating ring, 185
Lagrange EOM, 177
pendulum
driven, 169, 208
horizontal driven double, 211
non-dimensionalization, 7
period
order-of-magnitude guess, 9
steel-can, 216
pendulum on rails, 203
Phase Diagram, **184**
phase space
Mexican hat, 197
phase vector, **3**
phase-space plot, **93**
harmonic oscillator, 94
physical quantity, **5**
pitchfork bifurcation, **180**
point particle, **115**
polar coordinates, 14

and complex numbers, 26
integration volume, 144
unit vectors, **163**
polynomials
vector space, 29, 31
potential, **73**
principle
d'Alembert's, 170
principle of least action, 166
product
Cartesian, **18**

reference frame, **63**
rest frame, 63
rest frame, 63
rollercoaster trail, 169
rotation
in 2D by complex number, 26
Rotation barrier, **192**
rotation barrier, 196
rotation symmetry
Mexican hat, 197
Particle on Cone, 197
roulettes, 58

saddle-node bifurcation, **185**
separation of variables, 99
set, **15**
cardinality, **15**
complement, **18**
definition by property, 16
difference (\), **18**
element, **15**
empty ∅, 15
equivalence, 17
examples, 15
intersection (⋂), 18
logical quantors, 18
set operations, 18
sets of sets
examples, 16
subset (⊆), **16**
superset (⊇), **16**
union (⋃), **18**
set operations, **18**
sign function, **106**
slackline, 32
Snellius' law, 172
Soap film, 173
solid of revolution, 146
spaceship lost, 210
spherical coordinates
integration volume, 144
unit vectors, **177**, 185
spherical pendulum
Lagrange EOM, 193
spirograph, 58
spring on rails, 200
state vector, **2**
steel-can pendulum, 216
Stokes friction, **98**
system, **2**

Taylor approximation, **101**
of exponential function, 102
of sine function, 102
Taylor expansion, **101**
multi dimensional, **154**
tensor of inertia, 150
there is ∃, **18**
torque, **46**, **151**
balance, 46
lever, 42
on particle, **151**
torque balance
mobile, 48, 152
total energy, **75**
conservation, 75
trajectory, **3**
translation invariance
and conservation laws, 131
momentum conservation, **188**
trochoid, 52
Tsunami
order-of-magnitude of wave period, 10
order-of-magnitude of wave speed, 9
turbulent friction, **105**
two masses
over pulley, 207
two-body problem, 120

uniform motion
EOM

Lagrange, 187

variation of a function, **165**
variational calculus
Fermat's principle, 172
shortest path
in plane, 167
on catenoid, 168
on cylinder, 168
on sphere, 214
soap film, 173
variational principles
potential and steady state, 152
vector
cross product, **43**
gradient, 71
orthogonal, 35
time derivative, 61
vector product, **43**
Jacobi identity, 43
vector space, **27**
$\mathbb{R}^D$, 28
basis, **36**
coordinate transformation, 39
coordinates, **36**
displacements, 27
inner product, 34
invariance of the inner product, 39
matrix, 29
polynomials, 29, 31
scalar product, 38
volume integral, **141**

water rocket, 66, **80**, 97
work, **72**
conservative force, **74**
energy of impact, 76

© Jürgen Vollmer — 2022-02-16 04:54:23+01:00