UNIVERSITÄT LEIPZIG

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Cosmology Problem Sheet 9

Problem 9.1 [12 points]

Consider a FLRW cosmological spacetime with $k = 0$.

(a) Suppose that the density parameters are $\Omega_M = \Omega_{\text{bar}} + \Omega_{CDM} > 0$, $\Omega_\Lambda > 0$ and $\Omega_{\text{rad}} = 0$ at present cosmic time $\tau_0 = 1.3 \cdot 10^{10}$ yrs. $H_0 = H(\tau_0)$. Verify that

$$a(\tau) = \left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3} \sinh(\tau/\tau_\Lambda)^{2/3}, \quad \tau_\Lambda = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}$$

is a solution to the Friedmann equations.

(b) Using the solution from **(a)** with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, check if H_0 matches the currently observed value.

(c) Assume $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7 - 10^{-4}$, $\Omega_{\text{rad}} = 10^{-4}$. In this case, the scale factor given in **(a)** is no longer a solution to the Friedmann equations. Determine the (smallest) redshift $z = z(\tau_r)$ so that $\varrho_{\text{rad}}(\tau) > \varrho_M(\tau)$ for all $\tau < \tau_r$ (τ_r then marks the cosmic time prior to which the Universe was "radiation dominated").

Problem 9.2 [12 points]

Verify the asymptotic high-energy / low energy asymptotic expressions (given in the Lecture) for n_T, ϱ_T and p_T for a Bose/Fermi gas in thermal equilibrium (described by the Boltzmann distribution function) at temperature T.

The solutions to the problems are to be handed in by Thu, 25 June 2020, 8 pm, using the moodle-tool for the Cosmology course, see

https://moodle2.uni-leipzig.de/course/index.php?categoryid=2765

You should upload your solution as pdf file. It is perfectly ok if you work out the solutions in hand-written form and scan/photograph them and convert the files into a single pdf file. However, please make sure that the result is very well readable, and that the pdf files aren't excessively large. Please leave some margin space for marking. The marked solutions will be made available in the moodle-tool.