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Cosmology Problem Sheet 7

Problem 7.1 [12 points]

Derive the Friedmann equations for the scale factor $a(\tau)$, energy density $\varrho(\tau)$ and pressure $p(\tau)$ for a FLRW cosmological spacetime filled with a homogeneous, isotropic ideal fluid, fulfilling Einstein's equations.

Hint: There is some overlap with Problem 5.4.

Problem 7.2 [6 points]

Consider the stress-energy-tensor $T_{\mu\nu}$ of a perfect fluid on a FLRW spacetime. Show that the vanishing of the divergence of $T_{\mu\nu}$, $\nabla_\sigma(g^{\sigma\lambda}T_{\lambda\nu}) = 0$, implies the equation

$$\frac{d}{d\tau}(\varrho a^3) + p\frac{d}{d\tau}a^3 = 0\,.$$

Problem 7.3 [6 points]

Consider a FLRW spacetime with $k = 0$, and assume that the scale factor takes the form

$$a(\tau) = \gamma\tau^{4/5} \quad (\tau > 0)$$

where γ is some positive constant. Sketch the form of the backward/forward lightcone of a point $(\tau_0, x_0, 0, 0)$ in a diagram with respect to (τ, x, y, z) coordinates. Explain, using the diagram, why temporal change of the scale factor can be interpreted as a change of the velocity of light.

Problem 7.4 $*$ [extra problem, 8 extra points]

Derive the equation of state $p = \varrho/3$ for electromagnetic radiation in spatially homogeneous and isotropic equilibrium.

Hint: If you have no clue here — take a look at the books. The derivation can be found also in several books on thermodynamics.

The solutions to the problems are to be handed in by Fri, 12 June 2020, 12 am, using the moodle-tool for the Cosmology course, see

https://moodle2.uni-leipzig.de/course/index.php?categoryid=2765

You should upload your solution as pdf file. It is perfectly ok if you work out the solutions in hand-written form and scan/photograph them and convert the files

into a single pdf file. However, please make sure that the result is very well readable, and that the pdf files aren't excessively large. Please leave some margin space for marking. The marked solutions will be made available in the moodle-tool.