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Cosmology Problem Sheet 3

Problem 3.1 (Hubble distance and redshift in Minkowski spacetime) [12 points]

Let $d = d(t)$ be the luminosity distance of a stellar object from the terrestial observer who is assumed to be approximately at rest in an inertial system which has time-coordinate t, assuming validity of Minkowski geometry.

Assume also validity of Hubble's relation

$$H_0 d = cz = c \left(\frac{\sqrt{1 + \dot{d}/c}}{\sqrt{1 - \dot{d}/c}} - 1 \right)$$

for all $t > 0$, where c is the vacuum velocity of light, z is the redshift factor expressed as a function of $\dot{d}$ (the recession velocity) and H_0 is the Hubble-parameter (which here is assumed constant). Then $d = d(t)$ is a solution to the differential equation following from this relation ($\dot{d}(t)$ is derivative of $d(t)$ with respect to t).

Suppose that a solution $d(t)$ to that differential equation has the following properties:

1. $d(t)$ is continuously differentiable, with $d(t) > 0$ for all $t > 0$.
2. $0 < \frac{1}{c}\dot{d}(t_0) < 1$ for some $t_0 > 0$.
3. $\dot{d}(t)$ has a (unique) limit for $t \to \infty$.

Show that:

It holds that $0 < \frac{1}{c}\dot{d}(t) < 1$ for all $t > 0$, and

$$\lim_{t\to\infty} \frac{1}{c}\dot{d}(t) = 1 \quad \text{and} \quad \lim_{t\to\infty} d(t) = \infty \,.$$

(*Remark*: This can also be shown without making assumption 3.)

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Problem 3.2 [7 points]

Assume that the linearized Hubble expansion law

$$H_0 d = v$$

holds at all times and distances, where d is the average distance between galaxies and v is the average recession velocity between galaxies. Assume that H_0 is constant in time, with value $H_0 = 70\text{km}/(\text{sec}\cdot Mpc)$. The average distance between the galaxies is approx. $10\,Mpc$ today, with an average diameter of the dense bulge region of about $10\,\text{k}pc$. Calculate the time in the past when the bulge regions of the galaxies would have had, on average, 50% overlap.

Problem 3.3 - Age of an old star [5 points]

For some old stars, one can measure the abundance ratio Th/Eu of the chemical elements Thorium and Europium spectroscopically. In one case, which will be denoted as $\star$, the abundance ratio was measured to be

$$\left(\frac{\text{Th}}{\text{Eu}}\right)_\star(t_{\star,\text{today}}) = 0.219\,.$$

For the solar system, the findings indicate an abundance ratio of

$$\left(\frac{\text{Th}}{\text{Eu}}\right)_\odot(t_{\odot,\text{today}}) = 0.369\,.$$

Thorium has a half-life (decay) time of $\tau_{\text{Th}} = 14.05$ Gyr, while Europium is considered as stable (the most abundant isotopes have $\tau_{\text{Eu}} > 10^{18}$ yr).

Assuming that the solar system came into being at $t_{\odot,\text{ini}}$ around 4.6 Gyr ago ($\Delta t_\odot = t_{\odot,\text{today}} - t_{\odot,\text{ini}} = 4.6$ Gyr), and that

$$\left(\frac{\text{Th}}{\text{Eu}}\right)_\star(t_{\star,\text{ini}}) \quad\approx\quad \left(\frac{\text{Th}}{\text{Eu}}\right)_\odot(t_{\odot,\text{ini}})$$

at the time $t_{\star,\text{ini}}$ when the $\star$ was formed, give an estimate for the age $\Delta t_\star = t_{\star,\text{today}} - t_{\star,\text{ini}}$ of the star.

Hint: The $\star$ is sufficiently close so that one can assume $t_{\star,\text{today}} \approx t_{\odot,\text{today}}$. You will need the formula

$$\left(\frac{\text{Th}}{\text{Eu}}\right)(t_{\text{today}}) = \left(\frac{\text{Th}}{\text{Eu}}\right)(t_{\text{ini}}) \cdot 2^{-\Delta t/\tau_{\text{Th}}}\,.$$

The solutions to the problems are to be handed in by Thu, 13 May 2020, 4 pm, using the moodle-tool for the Cosmology course, see

https://moodle2.uni-leipzig.de/course/index.php?categoryid=2765

You should upload your solution as pdf file. It is perfectly ok if you work out the solutions in hand-written form and scan/photograph them and convert the files into a single pdf file. However, please make sure that the result is very well readable, and that the pdf files aren't excessively large. Please leave some margin space for marking. The marked solutions will be made available in the moodle-tool.