Cosmology

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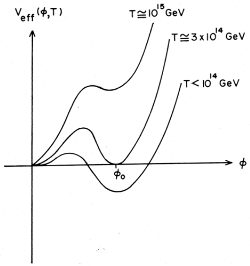
As mentioned, originally the idea in the first papers on cosmic inflation was that the effective potential of the inflaton field is temperature dependent, looking as follows:



This temperature dependent behaviour of the effective potential was patterned after the behaviour of the effective potential of the Higgs field – as mentioned, initially the Higgs field was viewed as a candidate for the inflaton field. That Higgs field like behaviour of the effective potential is nowadays known as **old inflationary model**.

The old inflationary model describes the onset phase of inflation with an exponential growth of the phase factor. But it was soon noticed that it had problems to describe the end phase of inflation in a satisfactory manner, or to correctly model an end phase of inflation at all, or at the right time scales. Basically, the gradient of the effective potential of the the old inflationary model around $\varphi = 0$ is too steep even as the temperature drops.

One can qualitatively distinguish three main phases of early cosmic inflation that the effective potential should describe dynamically:

(1) onset: $\tau = \tau_{\rm ini}$

The ambient temperature T reaches the critical temperature $T_{\rm crit}$ for the GUT $\to$ electroweak and strong phase transition.
The effective potential $V = V_T$ has a local minimum which is not gauge invariant.
At $\tau_{\rm ini}$, $\varphi \approx \varphi_0$ and $a(\tau) \approx {\rm e}^{\sqrt{h_0}\tau}$ where $h_0 = (\kappa/3)\gamma\epsilon_0^2$, but this is not an exact solution to the Friedmann equations because the effective potential varies in time.
If the “potential well” of the effective potential is flat enough around $\varphi = 0$, one can also imagine that one has a solution which shows a form of “tunneling” (analogous to the quantum mechanical tunnel effect) from an (approximately) constant solution $\varphi = \varphi_0$ to an energetically lower solution $\varphi = \varphi_\beta$ between $\tau_{\rm ini}$ and $\tau_{\rm fin}$.

Now one should bear in mind that at the present level of description, the inflaton field is to be seen as a "macroscopic average" of a more complicated behavior at a smaller scale, like a macroscopically averaged magnetization field in the phase transition of a magnet from the symmetric (unordered, unmagnetized) to an unsymmetric (ordered, magnetized) phase. However, in the analogy with cosmic inflation, one would have to take into account the cosmic expansion as a growth of the distance between the magnetized domains in the material. That must happen at the appropriate time scale so that the magnetized domains can still coalesce and form larger magnetized domains, and allow the latent heat to be released to the ambient degrees of freedom (including, of course, the scale factor) and not remain trapped in the domain walls. This is modelled in the second phase.

(2) Slow-roll inflation: $\tau_{\rm ini} < \tau < \tau_{\rm fin}$

The field equation

$$\partial_\tau^2 \varphi + 3H\partial_\tau \varphi + V'(\phi) = 0$$

is analogous to the equation of motion of a massive particle in 1 space dimension (identifying $\varphi(t) = x(t)$ = particle position) under the influence of a potential and a friction term. If the potential is not "too steep", the particle "moves (rolls) slowly", and that can be used to regulate the scale at which $a(t)$ expands, so that it expands with acceleration > 0 over a long enough time to produce a sufficient growth of the scale factor at $\tau_{\rm fin}$, but not with a too high acceleration that would prevent coalescence of the transitioned domains.

For a sufficient expansion, one needs still an almost exponential growth of $a(\tau)$. To this end, inspired by the analogy of the "slowly rolling particle", one assumes

$$|\dot{H}| \ll H^2 \quad \text{and} \quad |\partial_\tau^2 \varphi|^2 \ll H^2 |\dot{\varphi}|^2$$

This implies

$$p_\varphi \approx -\varrho_\varphi$$

This amounts to neglecting $\dot{\varphi}$ compared to $V(\varphi)$ and $\partial_\tau^2 \varphi$ compared to $V'(\varphi)$ [mind hidden factors of c], which in turn yields

$$\frac{\dot{H}}{H^2} \approx \frac{1}{2\kappa} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2$$

Therefore, $a(\tau)$ has almost exponential growth for a “long” time (in units of $1/H$) if the right hand side of that approximate equation is very small, i.e.

$$\varepsilon(\varphi) = \frac{1}{2\kappa} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \ll 1$$

One the other hand, the condition $|\partial_\tau^2 \varphi|^2 \ll H^2 |\dot{\varphi}|^2$ amounts to

$$\eta(\varphi) = \frac{1}{3\kappa} \left| \frac{V''(\varphi)}{V(\varphi)} \right|^2 \ll 1$$

The parameters $\varepsilon(\varphi)$ and $\eta(\varphi)$ are called the **slow-roll parameters** and the condition that they are very small (compared to 1) is called **slow-roll condition**.

(3) Re-heating: $\tau \gtrsim \tau_{\rm fin}$

At the end of the slow-roll phase, the scale factor has drastically expanded which leads to a corresponding drastic drop of the ambient temperature scale.

However, a too low temperature would prevent the usual big bang scenario to come to pass; the decoupling processes/phase transitions that we have looked at previously would happen too soon.

The idea is that towards the end of the slow-roll phase, the interaction between the inflaton field and other forms of matter (quarks, gluons, leptons, neutrinos, photons) becomes increasingly effective, and that the "inflatons" start to decay into these other forms of matter and thereby heat them as the process releases energy. This is also suggested as the mechanism to stop inflation, so that $a(\tau) \sim \tau^{1/2}$ after $\tau_{\rm fin}$.

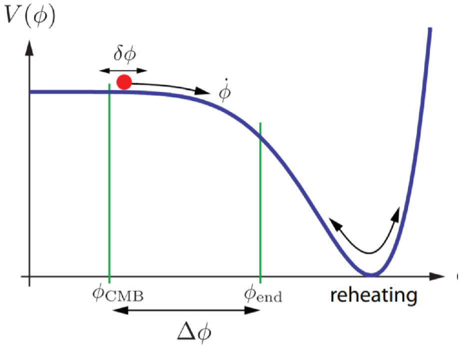
The picture is: On approaching $\tau_{\rm fin}$, the effective potential now has its minima lying in a steep potential well, and φ is oscillating around a potential minimizing configuration. The energy density fulfills the equation

$$\dot{\varrho}_\varphi + 3H(\varrho_\varphi + p_\varphi) = \Gamma \varrho_\varphi = -(1 + 4H)\varrho_{\rm rad}$$

where $\varrho_{\rm rad}$ is the energy density of all other forms of matter (as mentioned), described in the high-energy limit as radiation. Note that $\varrho_\varphi \approx p_\varphi$ since $V(\varrho) \approx 0$.

If $\Gamma \gg H$ at the end of the slow-roll phase, this means that a large fraction of the inflaton energy is transferred to all other matter, which thereby heats up ("re-heating"). The re-heating becomes ineffective once most of the "inflatons" have decayed.

An illustration of the qualitative shape of the effective potential for the "slow-roll" scenario of early inflation appears in the picture below. The slow-roll phase is (approximately) within the green bars.
(Again, there are very many similar pictures on the internet, and in the case of this picture I am not even sure about the approriate attribution.)



The slow-roll scenario is nowadays the “mainstream” scenario presented in textbooks on cosmology. However, it has already been superseeded by more refined versions which come under names like “chaotic inflation”, “hybrid inflation” and several more. Here, for simplicity only one scalar inflaton field has been considered, but there is also “multifield inflation” which uses several scalar fields (or fields of tensor type, or even more complicated types of fields).

Thereby, estimates on the inflationary phase are model-dependent. What is common to the models is that the inflationary phase is very short, generally within a range of $\tau_{\rm ini} \approx 10^{-35}$ s and $\tau_{\rm fin} \approx 10^{-34\ldots-32}$ s. Usually, $T \approx 10^{16}$ GeV at $\tau_{\rm ini}$. Then, depending on the model of inflation used, one can obtain a scale factor growth within the range $a(\tau_{\rm fin})/a(\tau_{\rm ini}) \approx 10^5 \ldots 10^{260}$. This is quite a bit of a range!

Remarks

The early cosmic inflationary model solves some problems which are connected with the standard cosmological model. One may say that in some way the solution of the horizon problem is one of its features – also because it does not, like the relict particle problem, solve a problem that has its roots in a hypothetical GUT (even if GUT is used to some degree for the solution of the problem) but a problem of a more fundamental nature (if perceived as such). (Personally, I am not very convinced by the flatness problem, I leave it to you to find out more about it and form your opinion.)

In view of the zoology of inflationary models, one may have doubts if early cosmic inflation is a mechanism of principle. In fact, it is a straightforward exercise that you can pick your favourite scale factor $a(\tau)$ with $\partial_\tau^2 a > 0$; then the Friedmann equations with an inflaton field yield

$$\dot{H} = -\frac{1}{2}\kappa|\dot{\varphi}|^2$$

If φ is taken to be real-valued, one can parametrize H by φ instead of τ; in other words, setting $\tilde{H}(y) = H \circ \varphi^{-1}(y)$ one gets

$$\partial_y \tilde{H}(y) - \frac{3}{2}\kappa \tilde{H}(y)^2 = -\frac{1}{2}\kappa^2 V(y)$$

This means that basically, given a and hence H, one can choose an effective potential which produces the given a, with a suitable φ, as solution to the Friedmann equations.

This may help illustrating that more is needed to make the inflationary model not completely arbitrary and just a hidden way of putting in what you want to get out. In fact, in the more recent years there has been some discussion on whether the early inflationary scenario is specific enough to qualify as a scientific – in the sense of falsifyable by observations – theory. I am not sure if a conclusion has been reached.

To add to the benefits of the inflationary scenario not yet mentioned: The current view is that it helps to explain the temperature fluctuations in the CMB and thereby, the "seeds of pattern formation" in the Universe which, some time after recombination, results in the formation of stars and galaxies etc. I am afraid that I won't get very far in the last lecture on that.

At the level of a purely phenomenological description of the inflaton field, there is not really much of a relation to a GUT – and this is really what is needed in order to make the inflaton field picture more convincing and to actually constrain it more. While I am no expert in this field and most books on cosmology don't have much to say about that, it is worth pointing out that this is possible in principle – so that the parameters of a GUT (types of fields, masses, couplings) can be related to the inflaton field parameters ($\tau_{\rm ini}$, $\tau_{\rm fin}$, ε, η, Γ, ...). Some discussion concerning that context appears in Mukhanov's book. A very rewarding read in this respect is a (no longer really up-to-date, but) beautifully written early review article presenting a lot of motivational and conceptual background on the cosmic inflationary scenario by Robert Brandenberger, "Quantum field theory methods and inflationary Universe models", Rev. Mod. Phys. **57** (1985) 1-60. For an overview on more modern developments, see the "TASI Lectures on primordial cosmology" by Daniel Baumann, https://arxiv.org/abs/1807.03098 .