Cosmology

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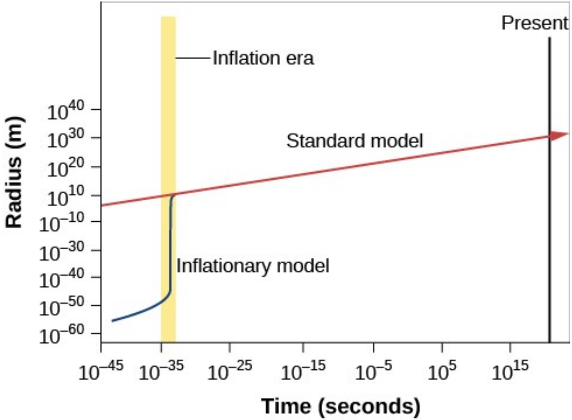
The early cosmic inflationary scenario hypothesizes that there has been a very short period of very rapid accelerated growth of the scale factor $a(\tau)$, as sketched in the following picture (taken from https://courses.lumenlearning.com/astronomy/chapter/the-inflationary-universe/) [There are very many similar illustrations on the internet, but none of them is, in my view, really ideal. Take it mainly as a qualitative picture.]



As mentioned before, the energy which drives this expansion is believed to originate from the spontaneous symmetry breaking of a GUT as the Universe begins to expand and the ambient temperature falls below the point where strong and electroweak interactions are unified. Such a phase transition where spontaneous symmetry breaking occurs is usually accompanied by release of latent energy, as the following analogy should illustrate:

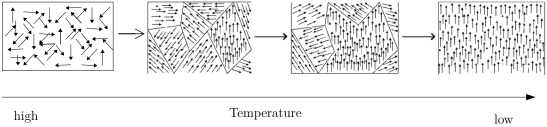


Illustration of a phase transition of some magnetic material.
At high temperature, the elementary magnets take random directions around their sites, averaging over small domains gives zero total magnetization.
As the temperature is lowered, magnetized domains start to form (Weiss domains).
On further lowering the temperature, the magnetized domains grow.
Below a critical temperature, there is a spontaneous magnetization of the material.
This is a phase transition from a ``symmetric phase'' at high temperature – symmetric because there is no preferred direction of magnetization (so for an external test magnet, no matter how it is directed, there is no response to the material) to a phase with ``broken symmetry'' (an external test magnet reacts to the magnetized material with a preferred alignment).
The phase transition is accompanied with a release of latent heat which previously was stored in the domain walls, and prior to that, in the kinetic energy of the elementary magnets. The latent heat release transfers energy to the ambient degrees of freedom outside of the material.

The symmetry breaking effect of GUT $\rightarrow$ strong & electroweak would be described in terms of a quantized gauge field theory. No rigorous such theory is presently at hand, so one resorts to something very much simpler.

Spontaneous symmetry breaking is an effect that occurs in field theories with gauge symmetry, both for classical and quantum fields (although it is hard to establish that for quantum fields). Actually, the effect should be described in terms of quantum fields, but in some circumstances, the cumulative effects, especially for high energy densities and pressure, can be approximately described by an "effective", classical gauge field theory. An example is the usual treatment of atoms in quantum mechanics: The electromagnetic fields interacting with the electrons (described by a quantum theory) should be described by quantum electrodynamics, but in most circumstances, they are described as classical gauge fields. That is typically a good approxiation if the field strengths are high enough (so that the electromagnetic field contains many photons to "average over" and produce effectively – at suitable scales – the effects of a classical field).

A similar approach is taken to describe early cosmic inflation. The effects of the degrees of freedom releasing the energy from a (hypothetical) GUT $\to$ strong & electroweak symmetry breakdown are described by an **inflaton field**, a complex scalar field with a $U(1)$ gauge symmetry in an **effective potential**, on the FLRW spacetime at very early cosmic times. It should be emphasized that the inflaton field is a *phenomenological field*; it is not a fundamental field of degrees of freedom of a GUT, but supposed to be a model for the collective behaviour of the degrees of freedom during the GUT breakdown at certain scales. It can be compared to a macroscopically averaged magnetization field in the analogy of spontaneous magnetization.

(Some authors however seem to take the view that the inflaton field is fundamental and term it “quintessence”. In the early development of the inflationary scenario, it was also thought that the inflaton is the Higgs field; this point of view is now mostly abandoned.)

The inflaton field

We present the simplest version of the inflaton field, as a complex scalar field in an effective potential.

Assume that we have an FLRW spacetime with $k = 0$ and scale factor $a(\tau)$; therefore, the spacetime manifold and metric are given by

$$M = (0, \infty) \times \mathbb{R}^3 , \qquad ds^2 = d\tau^2 + a(\tau)^2(dx^2 + dy^2 + dz^2)$$

The **inflaton field** φ is a smooth function $\varphi : M \to \mathbb{C}$, such that:

- φ is a solution to the field equation

$$\Box\varphi(\tau, \boldsymbol{x}) + V'(|\varphi(\tau, \boldsymbol{x})|) = 0$$

  where $\Box$ is the d'Alembert operator (wave operator) of the FLRW spacetime, and $V : \mathbb{R}_+ \to \mathbb{C}$ is called the **effective potential**, and $V'(r) = dV(r)/dr$. The effective potential also depends on other parameters like the ambient temperature (or energy scale) T, and thereby, it has actually a τ-dependence.

  On an FLRW spacetime with $k = 0$, the field equation takes the form (cf. Problem sheet 11)

$$\partial_\tau^2\varphi + 3H\partial_\tau\varphi - \frac{1}{a^2}\Delta_{\mathbf{x}}\varphi + V'(|\varphi|) = 0 \quad (\varphi = \varphi(\tau, \boldsymbol{x}))$$

- Moreover, the FLRW spacetime should fulfill the Friedmann equations (at a timescale of around $10^{-36}\,s < \tau < 10^{-32}\,s$) with $\varrho = \varrho_{\mathrm{rad}} + \varrho_\varphi$, $p = p_{\mathrm{rad}} + p_\varphi$ where (cf. Problem sheet 11)

$$\varrho_\varphi = \frac{1}{2}\left(|\partial_\tau \varphi|^2 + \frac{1}{a^2}|\vec{\nabla}_{\boldsymbol{x}}\varphi|^2\right) + V(|\varphi|)$$

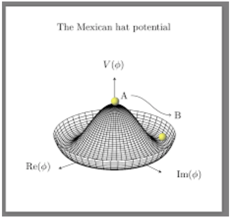
$$p_\varphi = \frac{1}{2}|\partial_\tau \varphi|^2 - \frac{1}{6a^2}|\vec{\nabla}_{\boldsymbol{x}}\varphi|^2 - V(|\varphi|)$$

Note that here all matter density is modelled as radiation (we are in an epoch where the baryons are in a quark-gluon plasma). The assumptions of FLRW spacetime should still be valid, so for consistency, ϱ_φ and p_φ cannot depend on the space coordinates $\boldsymbol{x}$ which motivates that actually, φ should not depend on $\boldsymbol{x}$. Therefore, $|\vec{\nabla}_{\boldsymbol{x}}\varphi|^2 = 0$. Furthermore, in the GUT phase transition, the inflaton field should carry the dominant part of energy so we have $\varrho_\varphi \gg \varrho_{\mathrm{rad}}$. Thus neglecting ϱ_{rad} and p_{rad}, the Friedmann equations & eqns. of motion for φ during the inflationary phase of GUT symmetry breaking reduce to

$$H(\tau)^2 - \frac{\kappa}{3}\left(\frac{1}{2}|\partial_\tau \varphi(\tau)|^2 + V(|\varphi(\tau)|)\right) = 0$$

$$\partial_\tau^2 \varphi(\tau) + 3H(\tau)\partial_\tau \varphi(\tau) + V'(|\varphi(\tau)|) = 0$$

Now we wish to discuss briefly the connection to symmetry breaking. If that occurs depends on the form of the potential. Let us consider here a potential of the form $V(r) = \gamma(\epsilon_0 - r)^2$, where ϵ_0 and γ are positive constants.
Then the function $v(z) = V(|z|)$, $z = x + iy \in \mathbb{C}$, takes the form of the famous "Mexican hat potential":



The function $v(z)$ (corresponding to $V(\phi)$ in the picture) has the following extremal values:

$$\text{local maximum}: \ z = 0, \ v(0) = \gamma\epsilon_0^2 \,, \quad \text{minima for } |z| = \epsilon_0, \ v|_{|z|=\epsilon_0} = 0$$

Under the transformations $z \mapsto \mathrm{e}^{i\beta} z$ ($\beta \in \mathbb{R}$), one has that the local maximum of the potential, $z = 0$, is left invariant, since $\mathrm{e}^{i\beta} 0 = 0$. Actually, under these transformations, $z = 0$ is the only invariant point in $\mathbb{C}$. On the other hand, if z is a minimizer for v, i.e. $|z| = \epsilon_0$, then $z' = \mathrm{e}^{i\beta} z$ is again a minimizer for v, but not the same: The transformations $z \mapsto \mathrm{e}^{i\beta} z$ ($\beta \in \mathbb{R}$) move a potential-minimizing configuration to another one. This behaviour is associated with symmetry braking in the sense that a configuration of higher symmetry (under $U(1)$ gauge transformations in this case) is not the one which carries the lowest energy.

To see this at the level of the inflaton field φ: Admitting for consistency with the Friedmann equations only solutions to the eqn. of motion which are homogeneous (i.e. depend only on τ), the eqn of motion is

$$\partial_\tau^2 \varphi(\tau) + 3H(\tau)\partial_\tau \varphi(\tau) + V'(|\varphi(\tau)|) = 0$$

Thus, there are the following constant solutions:

$$\varphi_0(\tau) = 0\,, \quad \varphi_\alpha(\tau) = \mathrm{e}^{i\alpha}\epsilon_0 \quad (\alpha \in [0, 2\pi))$$

Under the gauge transformation $\varphi(\tau) \mapsto \mathrm{e}^{i\beta}\varphi(\tau)$, we see that φ_0 is unchanged (invariant), while $\varphi_\alpha \mapsto \varphi_{\alpha+\beta}$ is mapped to another solution.

For these constant solutions, we find the energy densities

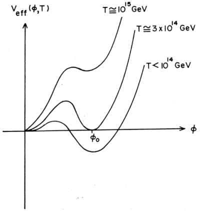
$$\varrho_{\varphi_0}(\tau) = \gamma\epsilon_0^2 > \varrho_{\varphi_\alpha}(\tau) = 0$$

so we have indeed the situation that is usually associated with symmetry breaking: The more symmetric configuration, or state (solution to the field equations), is not the ground state, i.e. there are other, less symmetric configurations which carry lower energy.

We remark that it is adequate to consider the transformations $\varphi(\tau, \boldsymbol{x}) \mapsto \mathrm{e}^{i\beta}\varphi(\tau, \boldsymbol{x})$ as gauge transformations of the inflaton field: As long as β is independent of τ and $\boldsymbol{x}$, the transformations don't change the field equation (so they map solutions to the field equations again to solutions) and they don't change energy density and pressure (since the effective potential only depends on $|\varphi|$). This is an example of **rigid gauge transformations** which don't depend on space and time. If the gauge transformations are allowed to depend on space and time, one would have to introduce spacetime dependent gauge potentials in the field equation, like in $U(1)$-electrodynamics. However, this is not considered here.

Therefore, the mechanism of a symmetry breakdown is also present in the (phenomenological) description of the inflaton field, if at some initial time $\tau_{\rm ini} \approx 10^{-36}$ s, i.e. at the onset of inflation, the inflaton field is (approximately) in its most symmetric configuaration $\varphi_0 = 0$, being a local maximum of the energy density, and at the end of inflation, at a final time $\tau_{\rm fin} \approx 10^{-32}$ s, the inflaton field is in (one of) its less symmetric, energy minimizing configuration(s) $\varphi_\alpha = {\rm e}^{i\alpha}\varepsilon_0$. The energy difference would have been used to drive the expansion of the scale factor.

There is a problem with this: A constant solution is an exact solution to the eqn. of motion for φ for any scale factor, so if at $\tau_{\rm ini}$, φ is a constant solution, then it cannot transit to another value from $\tau_{\rm ini}$ to $\tau_{\rm fin}$ with a fixed, Mexican hat like effective potential. This forces the effective potential to depend on further parameters which change with time. The original suggestion of A. Guth was of the following from:



With such a temperature dependent potential, inflation works – i.e. the inflaton field fulfills its designed purpose – but there are problems. We will discuss this.