Cosmology

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Early cosmic inflation

The Λ-CDM standard cosmological model is successful in many ways.

However, there are several problems – some of them at a conceptual level.

Three problems have been noted as particularly difficult –

- The horizon problem
- The relict particle problem
- The flatness problem

– and the “early inflationary scenario” provides a solution to these, which constitutes the basis for introducing it.

We will continue by discussing these problems, and then outline the mechanism of early cosmic inflation, and how it solves the problems.

The horizon problem

The horizon problem starts with the observation that – depending on the the behaviour of $a(\tau)$ – there are regions in the sky from which we receive CMB such that when the CMB was released, the points of origin of the CMB have had no intersecting past light cones.

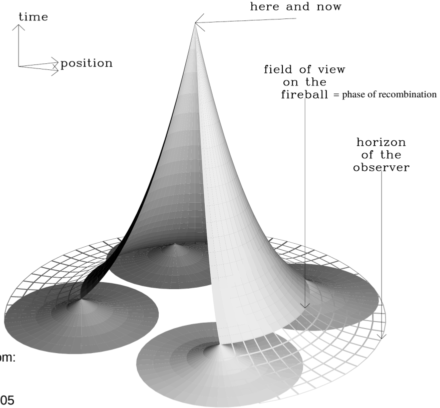


Illustration taken from:
D.E. Liebscher,
Cosmology,
Springer-Verlag, 2005

Nevertheless, the temperature of the CMB is almost exactly the same no matter in which direction in the sky we look. The angular temperature fluctuations around the average temperature are very small and appear randomly distributed.

This indicates that the plasma from which the CMB was released at recombination has had a very uniform temperature, apart from very small fluctuations. This may be traced back to either of the following:

(1) The "initial state" (e.g. before hadronization, i.e. the state of the quark gluon-plasma) has been in a uniform (homogeneous) thermal equilibrium "everywhere in space", or

(2) The plasma has reached a very-near-to equilibrium state through interaction.

Possibility (1) is usally not very much favoured because there doesn't seem to be a natural explanation for it. The state space for the material in the early Universe is vast, so why has the material in the Universe started from an equilibrium state?

[On the other hand, one may wonder how strong an objection to (1) can be made in the light of having readily accepted that the state of the material in the Universe is homogeneous and isotropic – it is also a highly idealized assumption.]

The favoured argument is (2) – everything is driven into thermal (and kinetic) equilibrium by interaction.

That is certainly consistent with the way thermal equilibrium has been treated and used to gain estimates on various processes like neutrino decoupling, nucleosythesis, recombination etc. It has some conceptual implications. On one hand, one need not worry about what “initial state” there has been since, given sufficiently strong interaction (and long enough time), any “initial state” would have resulted in a thermal equilibrium state of the plasma just before recombination. On the other hand, if that is the case, one may wonder how it would be possible to “retrodict” what the initial state had been.

However, the horizon problem now lies in the observation that there are points, or regions, from which CMB emanated at recombination, which have had non-overlapping past light cones, and therefore could not have interacted by any “causal” interaction (where the interaction proceeds with no more than the velocity of light). Therefore, these points cannot have equilibriated by interaction during their previous history. (And then, why has the CMB almost exactly the same temperature at any direction in the sky?)

[While this argument is plausible, again it is not entirely clear how compelling it really is, given that it is not viewed as a problem to assume spatial homogeneity and isotropy of energy density and pressure.]

Conformal time and cosmological horizons

The point of non-overlapping past lightcones is best illustrated by introducing a new time-coordinate, called “conformal time”.

Assume an FLRW spacetime with $k = 0$: $M = (0, \infty) \times \mathbb{R}^3$,

$$ds^2 = d\tau^2 - a(\tau)^2(dx^2 + dy^2 + dz^2)$$

Then one can introduce a new time-coordinate

$$\eta = \eta(\tau) = \int_{\tau_1}^{\tau} \frac{d\tau'}{a(\tau')}$$

where $\tau_1 \in (0, \infty)$ can be chosen arbitrary (but fixed). The time-coordinate η is called **conformal time** and depends on $a(\tau)$. One nice effect is that using the conformal time coordinate, the FLRW metric is **conformally equivalent** to the Minkowski metric, i.e. we have

$$ds^2 = \tilde{a}(\eta)^2(d\eta^2 + dx^2 + dy^2 + dz^2)$$

with $\tilde{a}(\eta) = a(\tau(\eta))$.

It is now of interest to determine the range $(\eta_0, \eta_\infty) \subset \mathbb{R}$ of η which clearly depends of the asymptotic behaviour of $a(\tau)$ for $\tau \to 0$ and $\tau \to \infty$. Assume $0 < \alpha < 1$:

Let $a(\tau) \approx \tau^\alpha$ for $\tau \in (0, \tau_1)$. Then $\eta_0 = \int_{\tau_1}^0 a(\tau)^{-1}\, d\tau > -\infty$ exists.

If $a(\tau) \approx \tau^\alpha$ for $\tau \in (\tau_1, \infty)$, then $\eta_\infty = \int_{\tau_1}^\infty a(\tau)^{-1}\, d\tau = \infty$.

However, if $a(\tau) \approx \tau^\alpha$ with $\alpha > 1$ for $\tau \in (\tau_1, \infty)$, then $\eta_\infty = \int_{\tau_1}^\infty a(\tau)^{-1}\, d\tau < \infty$.

Therefore, an FLRW spacetime with $k = 0$ for a Λ-CDM cosmological model is conformally equivalent to a “timelike strip”

$$(\eta_0, \eta_\infty) \times \mathbb{R}^3\,, \quad ds^2 = \tilde{a}(\eta)^2 (d\eta^2 + dx^2 + dy^2 + dz^2)$$

of Minkowski spacetime, where $\eta_0 < \eta_\infty$ are finite real numbers, since for Λ-CDM, $a(\tau) \approx \tau^{1/2}$ as $\tau \to 0$ (radiation dominated Universe at early cosmic time) and $a(\tau) \approx \sinh(\tau/\tau_\Lambda)^{2/3}$ for $\tau \to \infty$) (Λ-dominated Universe at late cosmic time).

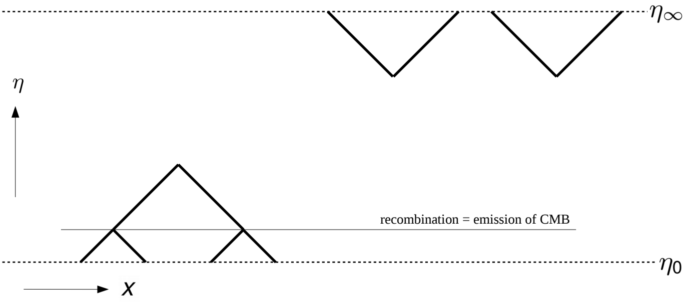
Now it is easy to verify that if $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are two Lorentzian metrics on a 4-dimensional manifold M which are **conformally equivalent**, meaning that

$$\tilde{g}_{\mu\nu}|_x = \lambda(x) \cdot g_{\mu\nu}|_x \quad \text{for all } \ x \in M$$

with a smooth function $\lambda : M \to (0, \infty)$, then a curve (worldline) on M is lightlike w.r.t. to $\tilde{g}_{\mu\nu}$ if and only if it is lightlike w.r.t. $g_{\mu\nu}$.

Therefore, the future and past light cones w.r.t. $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$ are exactly the same for conformally equivalent metrics.

So, for a Λ-CDM cosmological model with $k = 0$ we can, using the conformal time-coordinate η, depict the forward- and backward light cones as the light cones in a finite "timelike strip" of Minkowski spacetime.

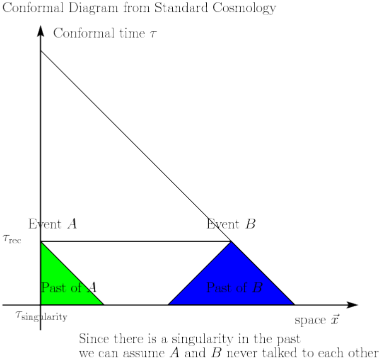


In the lower left part of the picture, the situation of the illustration on slide 3 is shown in 2 spacetime dimensions. The past light cones of two emission points of CMB at recombination are disjoint if the CMB is observed far enough in the future.
In the right upper part, two sufficiently distant observers have disjoint future light cones (cf. Problem 8.4)

This suggests that the horizon problem (if perceived as a problem) is mildened if the behaviour of $a(\tau)$ can be altered – while preserving its main asymptotic characteristics for $\tau \to 0$ (radiation) and for $\tau \to \infty$ (Λ-dominated) – so that to the past of recombination, or before, the past light cones of emission points of CMB aquire some overlap. That is actually possible.

The idea is that around 10^{-33}...10^{-32} seconds after $\tau = 0$ (the "initial singularity"), just before the electroweak phase transition, the scale factor $a(\tau)$ has undergone a rapid accelerated expansion, driven by an energy release from the – hypothetical – phase transition of a GUT into its parts dominated by strong and electroweak interactions. We will come back to that. Depending on the assumptions, within the timespan $\tau = 10^{-33}$...10^{-32} seconds the scale factor grows enormously, by a factor of 10^{26} or more.

Such a behaviour of $a(\tau)$ leads to a larger overlap (or overlap at all) of past lightcones prior to recombination, as shown in picture on the following slide, using actually conformal time on the time axis:



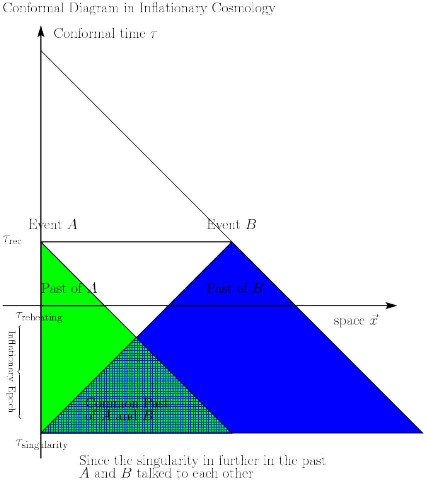


Illustration how a short, early accelerated expansion of $a(\tau)$ leads to larger overlap of past lightcones. The time coordinate that is denoted by τ in the diagrams is actually our conformal time η.

This early inflationary scenario also solves two other problems:

The relict particle problem: There are (mostly theoretical) indications for a GUT (grand unified theory) in which the strong interaction and the electroweak interaction are described as one interaction; this should manifest itself at suffcently high energies, similarly as the electroweak interaction unifies weak and electromagnetic interaction at sufficiently high energies. However, in the unified theories, there appear new types of particles (in the electroweak theory, the $W^{\pm}$ and Z bosons, which are however not stable). In the phase transition of the GUT to strong and electroweak interaction, there should appear a high abundance of heavy (and likely, stable) *magnetic monopoles* which so far have not been observed – they are called “relict” particles from the GUT phase transition, and their absence from observation poses a problem. That problem is softened if the GUT phase transition is accompanied by an inflationary growth of the scale factor because that dilutes the relict particle density to a negligible amount, explaining why these particles haven’t been discovered.

The flatness problem: Observations indicate that the Universe is flat, i.e. $k = 0$, to a high degree of accuracy, so high that it amounts to a “scale mismatch”. Also here, an early inflationary phase offers an explanation: Even if the Universe was not flat, the deviation from flatness would have been “flattened out” by an early inflationary phase, very much as a large increase of the radius of a ball would make its surface appear flat over a fixed solid angle.