Cosmology

Summer Term 2020, Lecture 24

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01 July 2020

Recombination – continued

A more accurate estimate on the recombination process can be obtained by the condition of thermal-kinetic equilibrium that appeared in Lecture 22. One of its rate equations for recombination is

$$\begin{aligned}\frac{1}{a^3}\frac{d}{d\tau}(n_e a^3) &= n_e^{(0)} n_{\mathrm{p}}^{(0)} \langle \sigma v \rangle \left(\frac{n_{\mathrm{H}}}{n_{\mathrm{H}}^{(0)}} - \frac{n_e^2}{n_e^{(0)} n_{\mathrm{p}}^{(0)}} \right) \\ &= n_b \langle \sigma v \rangle \left[(1 - X_e) \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-\epsilon_0 / k_B T} - X_e^2 n_b \right]\end{aligned}$$

where for the last equation we have used

$$\frac{n_e^{(0)} n_{\mathrm{p}}^{(0)}}{n_{\mathrm{H}}^{(0)}} = \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-\epsilon_0 / k_B T} , \quad n_e = n_b X_e$$

Using also $n_b a^3 \approx \mathrm{const}$, the rate equation turns into

$$\frac{d}{d\tau} X_e = (1 - X_e)\beta - X_e^2 n_b \alpha^{(2)}$$

with

$$\beta = \langle \sigma v \rangle \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-\epsilon_0/k_B T} \quad \text{“ionization rate”}$$

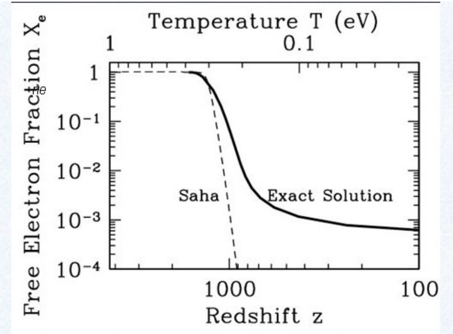
$$\alpha^{(2)} = \langle \sigma v \rangle \qquad \text{“recombination rate”}$$

Either from the theory of quantum electrodynamics, or from laboratory experiments, one obtains

$$\alpha^{(2)} \approx 9.78 \cdot \frac{\alpha^2}{m_e^2} \left(\frac{\epsilon_0}{k_B T} \right)^{1/2} \ln \left(\frac{\epsilon_0}{k_B T} \right)$$

where $\alpha \approx 1/137$ is the fine structure constant.

One can then numerically solve the rate equation in terms of a function $X_e = X_e(T)$. The result looks like in the graph below which has already appeared, where the numerical solution $X_e(T)$ is called "exact solution". It shows that the recombination rate drops as the cosmic expansion goes on and recombination takes place over a much longer timespan than predicted by the Saha equation.



WIMPs as CDM

The methods used for estimates on neutrino decoupling, nucleosynthesis etc. can also be used for some considerations about cold dark matter (CDM). Among the mainstream opinion is held that the effects associated with “dark matter” are due to a form of matter which so far has not been identified in laboratory experiments, or at any rate experiments below galaxy scale.

One of the suggestions is that dark matter consists of WIMPs, “weakly interacting massive particles”. The idea is that WIMPs behave like very massive neutrinos: They interact very weakly with other forms of matter, but they are very heavy. A favoured mass scale for WIMPs is $\approx 100\,\mathrm{GeV}$. While WIMPs are patterned after neutrinos, it is not necessarily assumed that they are part of the weak interaction in the current standard model of elementary particle physics; they might interact, e.g. with the known forms of matter, by some other, yet unknown force.

Assuming that there are WIMPs $\mathcal{W}$ together with anti-WIMPs $\overline{\mathcal{W}}$, a first guess is that they interact with associated “leptons” ℓ and $\overline{\ell}$ like neutrinos would, so that there are transformation processes like

$$\mathcal{W} + \overline{\mathcal{W}} \longleftrightarrow \ell + \overline{\ell}$$

The idea here is that the ℓ-leptons are electrically charged and interact strongly with the quark-gluon plasma, other leptons and photons and thereby keep the WIMPs in equilibrium.

Then the WIMPs should decouple already at an energy scale $k_B T$ which is only little below 100 GeV, and since they interact very little and are heavy, they should from a very early cosmic time on behave like "dust".

In fact, the corresponding equation of state, $n_{\mathcal{W}} a^3 = \mathrm{const}$, qualifies "cold" dark matter. The motivation is that thereby, the energy density of WIMPs will be high at later cosmic time in comparison to baryons and this is the primary reason for introducing CDM.

A favoured scenario is that just prior to the decoupling process, the ℓ-leptons and their anti-leptons annihilate almost completely, and the WIMPs and their antiparticles annihilate up to a (significant) excess of $\mathcal{W}$s.

One can then run the previous arguments backwards: Since Ω_{CDM} is known, one can, on the basis of the assumptions made so far, gain an estimate on the cross-section of the process $\mathcal{W} + \overline{\mathcal{W}} \longleftrightarrow \ell + \overline{\ell}$. This is an important benchmark when trying to develop an extension of the standard model of elementary particles which includes WIMPs.

By assumption, the ℓ-leptons are in equilibrium, so we have

$$n_{\ell,\overline{\ell}} = n^{(0)}_{\ell,\overline{\ell}}$$

The condition of kinetic-thermal equilibrium then assumes the form

$$\frac{1}{a^3}\frac{d}{d\tau}\left(n_{\mathcal{W}}a^3\right) = \langle \sigma v \rangle \left((n^{(0)}_{\mathcal{W}})^2 - n^2_{\mathcal{W}}\right)$$

Using $a \cdot T \approx \mathrm{const}$ and introducing $Y = n_{\mathcal{W}}/(k_B T)^3$, this turns into

$$\frac{dY}{d\tau} = T^3 \langle \sigma v \rangle \left((Y^{(0)})^2 - Y^2\right)$$

Note that T here is not an equilibrium temperature of the WIMP dust (after decoupling) but $k_B T$ is an expression for its mean energy.

With

$$\tau_{dec} = \text{time (before) decoupling}$$
$$\tau_0 = \text{present cosmic time}$$

one then obtains

$$\varrho_{\mathcal{W}}\big|_{\tau_0} = m_{\mathcal{W}} Y(\tau_0)(k_B T(\tau_{dec}))^3 \left(\frac{a(\tau_0) T(\tau_0)}{a(\tau_{dec}) T(\tau_{dec})} \right)^3 \approx m_{\mathcal{W}} Y(\tau_0) \frac{(k_B T(\tau_{dec}))^3}{30}$$

One can then solve the differential equation for $Y(\tau)$ according to the conditions

$$k_B T(\tau_{dec}) = 100\,\text{GeV}\,, \quad m_{\mathcal{W}} = 100\,\text{GeV}\,, \quad \Omega_{\mathcal{W}} = \Omega_{CDM} \approx 0.3$$

This gives

$$\langle \sigma v \rangle \approx 10^{-40}\,\text{cm}^2$$

There are supersymmetric extensions of the standard model of elementary particle physics with particles of a rest energy of around 100 GeV and a cross-section for the leptonic annihilation process of $10^{-40}\,\text{cm}^2$.

However:

- (⊖) Supersymmetric extensions of the standard model predict a large number of new types of particles also in energy ranges accessible to present-day particle accelerators. None of them have been found so far – so it is doubtful if supersymmetry is realized in nature.
- (⊖) The gravitational accretion induced by WIMPs may even be too strong, so that it would have led to a much larger rate of black hole formation, even when the first galaxies had formed (or prior to that) than is apparently observed today. (That might change with future observational methods – gravitational wave detectors have only recently confirmed stellar size black holes which previously had not been observed.)

To summarize:

The Λ-CDM cosmological model provides qualitative and quantitative statements on the evolution of the Universe which are in agreement with the observations.

The primordial nucleosynthesis can be well explained at quantitative level, and is in good agreement with observations, particularly concerning the observed ratio of $^4\mathrm{He}/\mathrm{H}$

The recombination time-scale fits with the redshift of the CMB

In these introductory lectures, we have only presented very basic calculations on nucleosynthesis and recombination. These topics are matters of current research and there are much more elaborate and accurate considerations in the literature, even at textbook level. Everyone is invited to get deeper into these matters and to explore the current state of research and methods in these areas on their own initiative.

The nature of CDM remains elusive (and to a lesser extent, the nature of Λ, or of “dark energy”). This poses a challenge to cosmology – and to the connection between general relativity and elementary particle physics.