Cosmology

Summer Term 2020, Lecture 21

Rainer Verch

Inst. f. Theoretische Physik
Universität Leipzig
22 June 2020

Decoupling of neutrinos and the neutrino background temperature

Prior to decoupling, the electron neutrinos and anti-neutrinos interact with the electrons and positrons through the **weak interaction** through the following processes:

$$\begin{aligned} e^- + e^+ &\longleftrightarrow \nu_e + \overline{\nu}_e \\ e^\pm + \nu_e &\longleftrightarrow \nu_e^\pm + \nu_e \\ e^\pm + \overline{\nu}_e &\longleftrightarrow e^\pm + \overline{\nu}_e \end{aligned}$$

There are similar processes for the other leptons ($\mu^\pm, \tau^\pm$) and their corresponding anti-/neutrinos ($\nu_\mu, \overline{\nu}_\mu; \nu_\tau, \overline{\nu}_\tau$).

Furthermore: The electrons and positrons (and other lepton) are electrically charged and therefore they interact with the photons. The electrons and positrons (and other leptons) as well as the photons interact also with other electrically charges particles, like hadrons (e.g. protons.) Through the mutual interactions, the neutrinos are kept in equilibrium with the rest of the “plasma”, and therefore, prior to decoupling, their temperature is the same as the rest of the plasma.

Now we attempt to obtain an estimate for the decoupling temperature τ_{dec} of the neutrinos.

Here, one exploits that there is quite a bit known about the weak interaction describing the interaction processes between electrons/positron and anti-/neutrinos as above, both from laboratory experiments (in particle accelerators) or the theory of weak interactions in elementary particle physics. Using this, one as for the processes under consideration:

- Interaction cross section: $\sigma = (\hbar G_{\rm weak} k_B T)^2$
  where $G_{\rm weak} \approx 1.16 \cdot 10^{-5} (GeV)^{-2}$ is the weak interaction coupling constant
- $t_{\rm weak} = \hbar / (G_{\rm weak}^2 (k_B T)^5)$ is the interaction time scale for the processes

Moreover, the particle density of the electrons at very high temperatures (relativistic limit) is $n_e \approx (k_B T/\hbar)^3$; and with $a(\tau) \sim \tau^{1/2}$, one gets $T(\tau) \sim 1/\tau^{1/2}$, or more precisely,

$$H(\tau) \approx \sqrt{\frac{\kappa (k_B T(\tau))^4}{8\pi\hbar^3}}$$

Thereby one obtains the estimate

$$t_{\rm weak} \gtrsim t_{exp} \qquad \text{at} \;\; T \approx 10^{10}\, K$$

Thus, at τ_{dec} with $T(\tau_{dec}) \approx 10^{10}\,K$ the anti-/neutrinos decouple from the electrons and positrons and therefore, they also decouple from the photons and the rest of the “plasma”

At some late time τ_{an}, the “plasma” temperature reaches $T = 0.5\,MeV = 5 \cdot 10^9\,K$ corresponding to the rest mass $0.5\,MeV$ of $e^{\pm}$. From this time on, the electron-positron annihilation $e^- + e^+ \longrightarrow 2\gamma$ overwhelms the reverse process. Making τ_{an} later if necessary, one can assume – approximately for the current consideration – that the particle densities n_{e^-} and n_{e^+} are approximately zero after τ_{an}.

Then one uses the principle of conservation of the total entropy:

$$s_T a(\tau)^3\Big|_{\tau=\tau_{dec}} = s_T a(\tau)^3\Big|_{\tau=\tau_{an}} \;,$$

with

$$s_T = \frac{2\pi^2}{45}\left(\frac{k_B T}{\hbar}\right)^3 \hat{g}_*$$

Now insert the values at the relevant times:

$$\hat{g}_*|_{\tau_{dec}} = g_\gamma \left(\frac{T_\gamma}{T}\right)^3 + \frac{7}{8}\left(g_{e^\pm}\left(\frac{T_{e^\pm}}{T}\right)^3 + g_\nu\left(\frac{T_\nu}{T}\right)^3\right)$$

where $T(\tau_{dec}) = T_\gamma(\tau_{dec}) = T_{e^\pm}(\tau_{dec}) = T_\nu(\tau_{dec})$.

$$\hat{g}_*|_{\tau_{an}} = g_\gamma \left(\frac{T_\gamma}{T}\right)^3 + \frac{7}{8} g_\nu \left(\frac{T_\nu}{T}\right)^3 , \quad T = T(\tau_{an})$$

On the other hand: Neutrinos have a very small rest mass, so one may treat them as radiation. In the relativistic limit, they obey $\varrho \sim T^4$ and $\varrho \sim 1/a^4$ once they have decoupled from other matter (and exchange no energy with other degrees of freedom). Therefore, we have

$$a(\tau_{dec}) T_\nu(\tau_{dec}) = a(\tau_{an}) T_\nu(\tau_{an})$$

Using the abbreviations $T_{\nu,dec} = T_\nu(\tau_{dec})$, $T_{\nu,an} = T_\nu(\tau_{an})$, $a_{dec} = a(\tau_{dec})$ etc, inserting the relations of this sheet into the conservation of the total entropy yields:

$$a_{dec}^3 \left((g_\gamma + \frac{7}{8} g_{e\pm}) T_{\gamma,dec}^3 + \frac{7}{8} T_{\nu,dec} \right) = a_{an}^3 \left(g_\gamma T_{\gamma,an}^3 + \frac{7}{8} T_{\nu,an} \right)$$

Observing $T_{\gamma,dec} = T_{\nu,dec}$ and $a_{dec} T_{\nu,dec} = a_{an} T_{\nu,an}$ then gives

$$\frac{T_{\gamma,an}}{T_{\nu,an}} = \left(\frac{g_\gamma + \frac{7}{8} g_{e\pm}}{g_\gamma} \right)^{1/3} = \left(\frac{11}{4} \right)^{1/3}$$

where the last equation is is obtained by setting

- $g_\gamma = 2$ (2 helicity states of the photon)
- $g_{e\pm} = 4$ (2 helicity states for both e^- and e^+ in the relativistic limit)

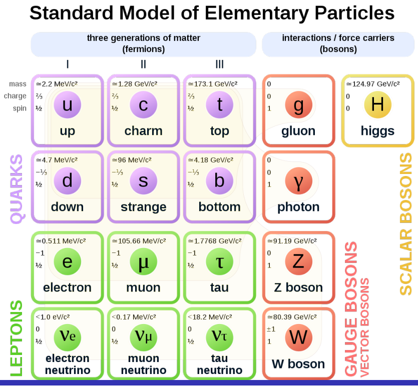
If one sets $\tau_{an} = \tau_0$, i.e. "today", the conclusion is that there should be a background radiation of electron-neutrinos in thermal equilibrium (with itself) at the temperature

$$T_\nu|_{\tau_0} = \left(\frac{4}{11} \right)^{1/3} T_\gamma|_{\tau_0} = \left(\frac{4}{11} \right)^{1/3} \cdot 2.73\, K = 1.96\, K$$

on noting that $T_\gamma|_{\tau_0}$ is the temperature of the CMB.

So far, this theoretically predicted neutrino background temperature cannot be observed – we have indicated earlier that the observation of cosmic neutrino radiation is a problem due to the weak interaction of neutrinos with any form of matter. Of course, the observation of a cosmic neutrino background at the predicted temperature would greatly corroborate the picture of the FLRW-Λ-CDM-“big bang” standard cosmological scenario, on top of other milestones that it passes.

In our example calculation of the neutrino background temperature, we have pretended that there was only one type (generation) of lepton, but there are 3 of them in the currently accepted standard model of elementary particles (see picture right for graphical illustration of the “building blocks”, taken from wikipedia: “standard model of elementary particle physics”, see enlarged or on wikipedia for better resolution). Taking 3 lepton generations into account, the neutrino background temperature changes. It would moreover change if there were more than 3 lepton generations.



In the following couple of slides, a table of the the thermal history of the Universe is presented, based on

- the FLRW-Λ-CDM homogeneous/isotropic standard cosmological model
- the standard model of elementary particle physics

The table is taken from wikipedia: “chronology of the universe” where it can be viewed in one go. Similar tables (sometimes with more detailed explanations) are given in the cosmology textbooks.

The table starts “on top” at the earliest time that can theoretically be represented in the FLRW model, and goes down to ever tater times.

Timescales of processes are governed by the interaction strengths of the known interactions: Strong force, weak force, electromagnetic force. The weak and electromagnetic force are known to be parts of a more general, “unified” electroweak interaction above a sufficiently high energy scale. It is conjectured, but presently not confirmed, that the electroweak interaction and the strong force are parts of a “grand unified theory” (GUT) at a still higher energy scale. The transition form a GUT to separate strong and electroweak forces and its relation to an early “cosmic inflation” in the table is speculative (discussed later). Processes of around 100 MeV and below are accessible in present day accelerator experiments and can be regarded as confirmed.

Epoch	Time	Redshift	Radiation temperature (Energy)	Description
Planck epoch	$<10^{-43}$ s		$>10^{32}$ K ($>10^{19}$ GeV)	The Planck scale is the physical scale beyond which current physical theories may not apply, and cannot be used to calculate what happened. During the Planck epoch, cosmology and physics are assumed to have been dominated by the quantum effects of gravity.
Grand unification epoch	$<10^{-36}$ s		$>10^{29}$ K ($>10^{16}$ GeV)	The three forces of the Standard Model are unified (assuming that nature is described by a Grand Unified Theory).
Inflationary epoch, Electroweak epoch	$<10^{-32}$ s		10^{28} K ~ 10^{22} K (10^{15} ~ 10^{9} GeV)	Cosmic inflation expands space by a factor of the order of 10^{26} over a time of the order of 10^{-33} to 10^{-32} seconds. The universe is supercooled from about 10^{27} down to 10^{22} kelvins.[8] The strong interaction becomes distinct from the electroweak interaction.

Epoch	Time	Redshift	Radiation temperature (Energy)	Description
Quark epoch	10^{-12} s ~ 10^{-6} s		$>10^{12}$ K (>100 MeV)	The forces of the Standard Model have separated, but energies are too high for quarks to coalesce into hadrons, instead forming a quark–gluon plasma. These are the highest energies directly observable in the Large Hadron Collider.
Hadron epoch	10^{-6} s ~ 1 s		$>10^{10}$ K (>1 MeV)	Quarks are bound into hadrons. A slight matter-antimatter-asymmetry from the earlier phases (baryon asymmetry) results in an elimination of anti-hadrons.
Neutrino decoupling	1 s		10^{10} K (1 MeV)	Neutrinos cease interacting with baryonic matter. The sphere of space that will become the observable universe is approximately 10 light-years in radius at this time.

Epoch	Time	Redshift	Radiation temperature (Energy)	Description
Lepton epoch	1 s ~ 10 s		10^{10} K ~ 10^9 K (1 MeV ~ 100 keV)	Leptons and antileptons remain in thermal equilibrium.
Big Bang nucleosynthesis	10 s ~ 10^3 s		10^9 K ~ 10^7 K (100 keV ~ 1 keV)	Protons and neutrons are bound into primordial atomic nuclei, hydrogen and helium-4. Small amounts of deuterium, helium-3, and lithium-7 are also synthesized. At the end of this epoch, the spherical volume of space which will become the observable universe is about 300 light-years in radius, baryonic matter density is on the order of 4 grams per m^3 (about 0.3% of sea level air density)—however, most energy at this time is in electromagnetic radiation.
Photon epoch	10 s ~ $1.168 \cdot 10^{13}$ s (370 ka)		10^9 K ~ 4000 K (100 keV ~ 0.4 eV)	The universe consists of a plasma of nuclei, electrons and photons; temperatures remain too high for the binding of electrons to nuclei.

Recombination	370 ka	1100	4000 K (0.4 eV)	Electrons and atomic nuclei first become bound to form neutral atoms. Photons are no longer in thermal equilibrium with matter and the universe first becomes transparent. Recombination lasts for about 100 ka, during which universe is becoming more and more transparent to photons. The photons of the cosmic microwave background radiation originate at this time. The spherical volume of space which will become the observable universe is 42 million light-years in radius at this time. The baryonic matter density at this time is about 500 million hydrogen and helium atoms per m^3, approximately a billion times higher than today. This density corresponds to pressure on the order of 10^{-17} atm.
Dark Ages	370 ka ~? 150 Ma (Only fully ends by about 1 Ga)	1100 ~ 20	4000 K ~ 60 K	The time between recombination and the formation of the first stars. During this time, the only source of photons was hydrogen emitting radio waves at hydrogen line. Freely propagating CMB photons quickly (within about 3 million years) red-shifted to infrared, and universe was devoid of visible light.
Star and galaxy formation and evolution	Earliest galaxies: from about ?300-400 Ma (first stars: similar or earlier) Modern galaxies: 1 Ga ~ 10 Ga (Exact timings being researched)	From about 20	From about 60 K	The earliest known galaxies existed by about 380 Ma. Galaxies coalesce into "proto-clusters" from about 1 Ga (redshift z = 6) and into galaxy clusters beginning at 3 Ga (z = 2.1), and into superclusters from about 5 Ga (z = 1.2). See: list of galaxy groups and clusters, list of superclusters.

Reionization	Onset 250 Ma ~ 500 Ma Complete: 700 Ma ~ 900 Ma Ends: 1 Ga (All timings approximate)	20 ~ 6	60 K ~ 19 K	The most distant astronomical objects observable with telescopes date to this period; as of 2016, the most remote galaxy observed is GN-z11, at a redshift of 11.09. The earliest "modern" Population III stars are formed in this period.
Present time	13.8 Ga	0	2.7 K	Farthest observable photons at this moment are CMB photons. They arrive from a sphere with the radius of 46 billion light-years. The spherical volume inside it is commonly referred to as the observable universe.