Cosmology

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The age of the Universe

There are several methods of determining the age of the Universe, where in many cases it is easier to set lower limits than upper limits.

Distance determination of stellar objects at the largest observable distances together with the finite velocity of light sets a lower limit for the age of the visible Universe between 13 and 14 G years.

Other indications come from the relative abundances of isotopes with different decay times. This is described on the pages by E. Wright, see the link on the web-page for the course. The results indicate an age of the Universe of around 13 G years.

A very crude lower limit can be derived from the fact that heavier elements which are bred in stars by nuclear fusion don't leave the star atmosphere. Therefore, they are only released (and for some part, formed) and form asteroids, planets etc when a star undergoes a supernova explosion at the end of its life cycle. For main sequence stars, that means that several G years must have passed before heavier elements can have formed. That is consistent with nucleosynthesis results in the hot big bang scenario: The most abundant elements at the end of the hot big bang phase are Hydrogen and Helium, heavier elements appear only in tiny fractions.

Problems with Special Relativity and Newtonian Gravity at large scales

The observations show:

- At large scales ($\approx 10 G\ell y$) the Universe appears homogeneous and isotropic.
- Combining this with Olbers' paradoxon: The Universe cannot have been like this forever (with an almost constant density of luminous matter averaged over sufficiently large scales of distance and time) because we see a "dark" night sky.
- The redhift of galaxies and the Hubble relation support the view that the Universe is not static. The galaxies recede from each other with increasing time. If the Hubble law holds for all times and distances, then –

  ($\star$) With incrasing time, the recession velocity increases, approaching asymptotically the velocity of light.

  ($\star\star$) With decreasing time, i.e. going back to the past, the galaxies must have been much closer together; further in the past, the Universe must have been very dense and hot.

Problems with Special Relativity and Newtonian Gravity at large scales

Item ($\star$) and the observed large-scale homogeneity and isotropy of the (luminous) mass distribution in the Universe pose a problem if the dominant “force” between mass distributions at lage scales is Newtonian gravity.

- Newtonian gravity is always attractive — so why does the recession velocity grow with distance (and grow in time)? The recession velocity should rather slow down than increase.
- Since the large-scale mass distribution is homogeneous and isotropic, an “outer” mass distribution cannot excert a “gravitational pull” on an “inner” mass distribution.

Therefore, the Hubble law, if valid on all time- and distance scales, appears paradoxical in the setting of Special Relativity and Newtonian gravity.

Problems with Special Relativity and Newtonian Gravity at large scales

There may be ways of overcoming this paradox in cosmology by introducing other kinds of forces, new rules of light propagation at large scales, etc. (For some discussion of such approaches, see e.g. Weinberg's book.) However, Newtonian gravity and Special Relativity are inconsistent on any scale (not just cosmological scales) as had been noticed by Einstein, and that observation provided motivation for him to develop General Relativity (GR). In the setting of GR, the cosmological spacetime models of Friedmann, Lemaître, Robertson and Walker (FLRW) consistently describe cosmological expansion for large-scale homogeneous and isotropic mass distributions. While initially met with some scepticism, these spacetime models now underlie the mainstream discussion in cosmology; in particular, the present standard cosmological model. Therefore, we will next introduce some basics of GR, and look at the FLRW spacetimes in more detail.

Some basics of GR, Part 1: Motivations

- Newton's law of gravity is not compatible with Special Relativity: It is not Lorentz covariant, and it implies "action-at-a-distance": Any local change of a mass distribution changes the gravitational field it creates instantaneously everywhere in space, i.e. gravitational effects propagate, according to Newton's law, faster than the speed of light.
- The mass-energy equivalence of Special Relativity implies that in the presence of inhomogeneous gravitational fields, there cannot be global inertial systems (only "local approximations").
- The equivalence of inertial mass and gravitational mass implies that light is deflected by gravitational fields.

The way in which Einstein modified Special Relativity addressing all these points — and removing the difficulties — has some formal similarities to the generalization of electro- and magnetostatics into electrodynamics. One of the key points is to replace the "rigid" Minkowski metric, which in all inertial coordinate systems takes the form $(\eta_{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1)$ by a "dynamical" generalization, changing in space and time.

To see an analogy: In electrodynamics, the electrostatic potential φ and magnetostatic vector potential $\vec{A}$ are made dynamical by observing that charge densities ϱ and current densities $\vec{j}$ not only react (as "test objects") to the force fields generated by the potentials, but simultaneously they act as sources for the potentials.

Maxwell's equations are the dynamical equations of that mutual interaction.

In Special Relativity, the Minkowski metric $(\eta_{\mu\nu})$ determines, in particular, the propagation of light. As $(\eta_{\mu\nu})$ is constant in inertial coordinates, the light propagation is along straight lightlike lines in inertial coordinates.

Light is deflected by gravitational fields. Therefore, as an ansatz, one may, in the presence of gravitational fields, replace the constant $(\eta_{\mu\nu})$ by a metric $(g_{\mu\nu}(x))$ $(x = (x^0, x^1, x^2, x^3))$ which depends on time- and space-coordinates.

Then one can set up an analogy

$$\begin{aligned} \text{electrodynamic potential } \ A_\mu(x) \ &\longleftrightarrow \ \text{dynamical metric } \ g_{\mu\nu}(x) \\ \text{charge-current density } \ j_\mu(x) \ &\longleftrightarrow \ \text{mass-energy distribution } \ \mathrm{T}_{??}(x) \end{aligned}$$

The ?? have been put here because at this stage it is isn't obvious what kind of quantity $\mathrm{T}_{??}(x)$ actually is (what type of tensor).

To pursue the analogy further, the Maxwell-equations have the form,

$$\partial^\mu F_{\mu\nu}(x) = \frac{4\pi}{c} j_\nu(x)\,, \quad \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$$

where a general tensor index notation is used – it will be clarified later – and doubly appearing indices are understood as being summed over (Einstein's summation convention); the electromagnetic field tensor $F_{\mu\nu}(x)$ appearing here is given as

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\,.$$

The Maxwell-equations show that there is a dynamical coupling between electromagnetic field tensor and the charge-current-density $j_\mu(x)$, and also – expressed by the second of the Maxwell-equations – that $F_{\mu\nu}$ is, mathematically, a kind of "curvature" deriving from the electrodynamic potential $A_\mu(x)$.

Following this analogy, one may guess that the dynamical metric $g_\mu(x)$ is in a similar way dynamically coupled to the mass-energy distribution by an equation of the form

$$[\text{curvature quantity derived from } g_{\mu\nu}(x)]_{??} = \kappa \cdot \mathrm{T}_{??}(x)$$

This turns out to be a successful step. To discuss it in more detail, the next step will be to introduce basic elements of the mathematical setting in which GR is placed – manifolds, tensor fields, and Lorentzian metrics.