Cosmology

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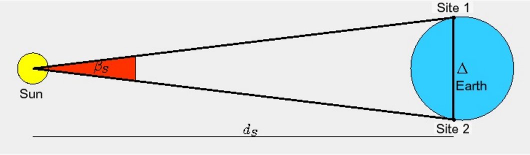
Parallax distance measurement of a sufficiently near stellar object

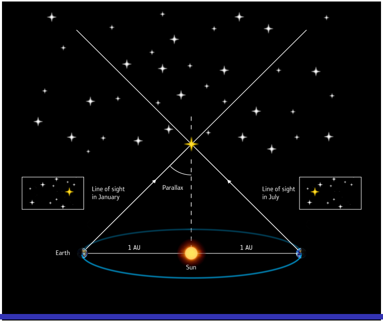
The **parallax distance measurement** is the standard distance measurement method for stellar objects not too far away. It is based on the assumed Euclidean geometry of space and trigonometry to related angles and distances.

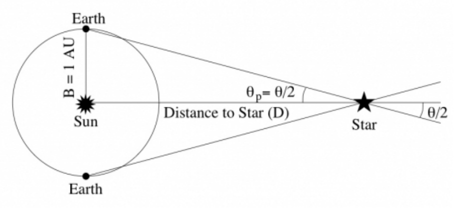
For very close stellar objects (Moon, Sun, planets Venus and Mercury), it is often sufficient to observe the stellar object from two distant sites on Earth. Knowing their relative positions and the angles under which the object is observed relative to the baseline connecting the sites, one can determine the distance of the stellar object, using the known length of the baseline.



The accuracy of the method depends on the determination of the observation angles (or the angular difference, β_s) and the determination of the baseline length.

For stellar objects that are more distant, a longer baseline is needed to compensate the limits of angular resolution. A longer available baseline is given by the diameter of Earth's orbit. A stellar object (e.g. a nearby star) is observed from Earth during the year. Compared to the background of fixed stars which are far more distant, the observed images of the stellar object over one year describe an ellipse — or, in the ideal situation shown in the picture (line of sight from the Sun to the stellar object is perpendicular to Earth's orbital plane), a circle. Using the different positions of the stellar object's images relative to the fixed stars background, one can determine the parallax angle of the stellar object, and hence the distance, knowing the radius of Earth's orbit.





$$D = \frac{1\text{ AU}}{\tan(\theta/2)}$$

where $\theta/2$ is the parallax angle shown, and D is the distance from Sun to the stellar object. Since 1 AU is typically very much smaller than D, one can usually identify D with the distance between Earth and the stellar object.

Note: This applies also if the line of sight between the Sun and the stellar object is not perpendicular to Earth's orbital plane — why?

The parallax distance measurement is the most direct and most accurate to date, but limited due to the limits in angular resolution. Earth-based instruments in the optical spectral range can measure distances by parallax up around 10 kpc. Using very long baseline interferometry in the radio frequency range, one can approximately determine distances up to the nearby galaxies directly, but there is often not a clear source.

The presently most accurate parallax distance measurements are carried out by the **Gaia space probe**. It operates mostly in the optical frequency range and reaches an angular resolution of $\approx$ 20 microarcseconds.

The findings of the Gaia spaceprobe and the measurement techniques used are very interesting in their own right but it would lead too far off track to attempt to discuss them here. See the Wikipedia page: https://en.wikipedia.org/wiki/Gaia_(spacecraft) and references given there for significantly more information.

Luminosity of a stellar object

- **absolute luminosity**:

$$L = \frac{\text{total emitted energy}}{1\ \text{s}}$$

the total energy emitted by the stellar object as electromagnetic radiation per unit of time (1 second), integrated over all frequencies (in the inertial rest system of the stellar object).

- **apparent luminosity**:

$$\ell = \frac{L}{4\pi d^2}\,, \qquad d = \text{distance from Earth}$$

(assuming that the stellar object and the Earth are approximately at rest, and that the energy emission from the stellar object is isotropic). This implies

$$\Delta F \cdot \ell = \frac{\text{sampled energy}}{1\ \text{s}}$$

the sampled energy in a receiver on Earth with detecting surface ΔF perpendicular to the line of sight to the stellar object (again, integrated over all frequencies of electromagnetic radiation).

The corresponding spectral densities $L(\nu)$ and $\ell(\nu)$ are given by

$$L = \int_0^\infty L(\nu)\, d\nu\,, \quad \ell = \int_0^\infty \ell(\nu)\, d\nu$$

so that $\int_{\nu_0}^{\nu_0+\Delta\nu} L(\nu)\, d\nu = L(\nu_0)\Delta\nu + O(\Delta\nu^2)$ is the total electromageetic energy emitted per second by the stellar object in the frequency range between ν_0 and $\nu_0 + \Delta\nu$ (for $\Delta\nu \to 0$; this needs $L(\nu)$ to be sufficiently smooth in ν and may require smoothing of discrete spectra).

Immediated consequence:

$$\ell(\nu) = \frac{L(\nu)}{4\pi d^2}$$

assuming that the emission of electromagnetic energy by the stellar object is isotropic in a frequency range around ν.

Magnitude or brightness of a stellar object

- **apparent brightness (or apparent magnitude)** m of a stellar object is implicitly defined by

$$\ell = 10^{-2m/5} \times 2.52 \times 10^{-5} \frac{\text{erg}}{1\ \text{s} \cdot \text{cm}^2}$$

- **absolute brightness (or absolute magnitude)** M of a stellar object is defined as the apparent brightness that the stellar object would have if observed from a distance of 10 pc. This gives the relation

$$L = 10^{-2M/5} \times 3.02 \times 10^{35} \frac{\text{erg}}{1\ \text{s}}$$

  These are traditional quantities used in astronomy.

- Note: When a stellar object with fixed L is observed from a distance d,

  ⋆ ℓ decreases as d increases

  ⋆ m increases as d increases

Signature of a stellar object

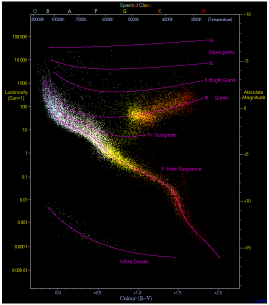
It is very important for astronomy that not only the luminosity of stellar objects can be distinguished, but also other characteristics of their emitted electromagnetic radiation, particularly regarding the resolution of the emitted radiation in time. One refers to the **signature** of a stellar object as a characteristic pattern of its electromagnetic radiation, such as:

- spectral distribution, width of spectral lines
- variation of luminosity and/or spectral distribution in time (e.g. characteristic periodicities)
- angular distribution of luminosity or spectrum

Relations between signature and absolute luminosity allow the determination of the distance from apparent luminosity for certain types of stellar objects. Particularly characteristic types of such stellar objects are called **distance indicators** or **standard candles**.

Some examples of important **distance indicators**, based (mostly) on empirical relations between luminosity and signature

Hertzsprung-Russel diagrams (HRD) The HRD in the picture (taken from Wikipedia) shows the relation between absolute luminosity and spectrum for $\approx 23,000$ stars whose distance, and hence absolute luminosity, has been determined by parallax measurement. Assuming that the relation between absolute luminosity and spectrum represented in the diagram is also valid for stars/star populations which are too far for parallax distance measurement, one can infer their absolute luminosity from their spectrum, in particular for main sequence stars (on the main diagonal band).



Note: The distribution of stars on an HRD, and thereby the relation between luminosity and spectrum, changes in time. As a stellar population grows older, more and more stars leave the main sequence (the main diagonal band). This is observed e.g. in globular stellar clusters. It can also be used to estimate the age of certain stellar populations. See the link *age determination of a stellar cluster* on the webpage for more information on that topic.

Variable stars: Cepheids, RR-Lyrae stars

Such stars show a typical signature in form of a quasi-periodic luminosity variation in time. It is assumed that this is caused by the same physical process and that there is a certain, narrow window of stellar parameters in which the process can take place. Therefore, one may assume that stars of that signature have the same absolute luminosity, which is consistent with parallax distance measurement of that type of stars. Hence, one can calibrate the absolute brightness for such stars by parallax measurement, and conclude the distance of such stars which are much further away from their apparent luminosity. This is particularly useful for Cepheids because they are very bright so that they can be observed in nearby galaxies. That way Hubble has concluded that the Andromeda nebula is a galaxy far remote from our own.