UNIVERSITÄT LEIPZIG

Institut für Theoretische Physik

Prof. Dr. R. Verch

Dr. J. Zahn

Summer Term 2019

Quantum Field Theory in Curved Spacetime
Problem Sheet 4

Problem 4.1 [12 points]

(a) Let (M, g_{ab}) and $(M', g'_{a'b'})$ be two spacetimes with covariant derivatives ∇ and ∇'. Suppose that ϕ is an isometry between the spacetimes. Show that

$$\phi_\sharp(\nabla_a X)^{c'_1,\dots,c'_r}{}_{b'_1,\dots,b'_s} = \nabla'_{a'}\phi_\sharp(X)^{c'_1,\dots,c'_r}{}_{b'_1,\dots,b'_s}$$

holds for all $\binom{r}{s}$ tensor fields X on M, on appropriately identifying the indices a and a'. In other words, the tensorial push-forward turns ∇ into ∇'.

(b) Is de Sitter spacetime stationary/static/ultrastatic ? What timelike – or other – groups of isometries does it admit?

(c) Give an example of a globally hyperbolic spacetime – different from Minkowski spacetime – which is globally hyperbolic and ultrastatic and where the hypersurfaces Σ_t of equal Killing parameter time t are Cauchy-surfaces.

(d) Consider the "right wedge region" W_R in Minkowski spacetime (see previous problem sheet). Regarding W_R endowed with the Minkowski metric as a spacetime in its own right, it is a stationary spacetime with respect to the 1-parametric group of Lorentz-boosts in x^1-coordinate direction,

$$\Lambda_t = \begin{pmatrix} \cosh(t) & \sinh(t) & 0 & 0 \\ \sinh(t) & \cosh(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (t \in \mathbb{R})$$

Show that W_R is actually static (but not ultrastatic) with respect to the Killing vector field generating the 1-parametric group $\{\Lambda_t\}_{t\in\mathbb{R}}$.

Hint: Introduce *Rindler coordinates* (t, ξ, x^2, x^3) for the right wedge so that $\Lambda_{t'}$ applied to the Minkowski spacetime (inertial) coordinate point (x^0, x^1, x^2, x^3) corresponding to (t, ξ, x^2, x^3) results in the Minkowski spacetime (inertial) coordinate point (y^0, y^1, x^2, x^3) $((y^0, y^1, x^2, x^3)^T = \Lambda_{t'}(x^0, x^1, x^2, x^3)^T)$ corresponding to $(t + t', \xi, x^2, x^3)$.