# RIEMANNIAN METHOD IN QUANTUM FIELD THEORY ABOUT CURVED SPACE-TIME*)

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Starting from a given Lorentz metric $g_{ik}$ on a space-time manifold $M$ and a tube $T$ of world lines along which observers may move, we describe an algorithm to obtain the quantum theory of scalar particles of mass $m$.

Let $e^j$ denote the 4-velocities of the world lines, and let $V_s$, $s$ real, be a family of non-intersecting space-like hypersurfaces. We assume that the submanifolds $V_s$ cover completely the tube $T$. Thus every point of $T$ is crosssed by just one world line and by just one $V_s$.

The essential content of my talk is now in the following statement:

One can reasonably associate to every $V_s$ a Hilbert space $F_s$ of one-particle states which are realized by measurable function concentrated on $V_s$. There is further defined a positive semidefinite operator $H_s$ acting on this Hilbert space $F_s$, and which is interpreted as the one-particle Hamiltonian at the instant $V_s$.

Indeed, having at hand such a procedure one can do the following. One defines the Fock space $\mathscr{F}_s$ to every $F_s$ and defines the appropriate Hamiltonian by second quantizing the operator $H_s$. With the wave operator $\Delta_4$ associated to $g_{ik}$ one considers the wave equation

$$(-\Delta_4 + m^2)\,\varphi = 0\,.$$

If $u \in F_s$ one solves this equation with the initial data

$$\varphi = u \quad \text{and} \quad \mathrm{i}\,(\partial\varphi/\partial n) = H_s u \quad \text{on} \quad V_s\,.$$

This transport the initial data to $V_t$ with $t \neq s$, and this transport extends, obviously, to the associated Fock spaces. Generally, this transport from $V_s$ to $V_t$ will not conserve the particle number, and the one-particle states of $V_s$ will give rise to superpositions of many-particle states attached to $V_t$.

Hence the *very problem* is in constructing $F_s$ and $H_s$. This will be achieved by associating to $M$, or only to the tube $T$, a Riemannian metric $\tilde{g}_{ik}$ given by

$$g_{ik} + \tilde{g}_{ik} = 2e_i e_k$$

(the signature of the Lorentz metric is $+\,-\,-\,-$). Then the construction of $F_s$ and $H_s$ is achieved by solving

$$(-\tilde{\Delta}_4 + m^2) f = u \quad \text{on the set} \quad \bigcup_{t \leq s} V_t$$

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and several other manipulations dictated by experience in the Euclidean formulation of Quantum Field Theory à la Nelson, Guerra, Rosen, Simon, Hegerfeld, and others. They are described in [1] together with some references to previous work.

Applying our procedure to stationary space-times it reproduces what everybody would suppose to hold in that case. But effective calculations are possible too for metrics of the Robertson-Walker type.

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*Reference*

[1] Uhlmann A.: Czech. J. Phys. *11* (1981) 1249.