Prof. Dr. K. Kroy SS 2014

Statistical Physics II
Problem Set 7

Due: Tuesday, May 27, **before** the lecture

9. **Debye-Hückel screening (part II)** **(6 points)**

c) Now consider the following electrostatic potential,

$$\phi_\alpha(\vec{r}) = \sum_{\beta=\pm} \int \mathrm{d}^3 r' \, \frac{z_\alpha z_\beta l_\mathrm{B}}{|\vec{r}-\vec{r}'|} n_\beta(\vec{r}') \,,$$

where $\phi_+(\vec{r}) = -\phi_-(\vec{r}) = \phi(\vec{r})$. Using the extremum principle for the RPA density functional, derive the Poisson-Boltzmann equation $\Delta\phi(\vec{r}) = \kappa^2 \sinh\big(\phi(\vec{r})\big)$. Furthermore, show that the densities are given by $n_\pm(\vec{r}) = n \exp\big(\mp\phi(\vec{r})\big)/2$.

d) Solve the Poisson-Boltzmann equation for a neutral ion mixture in the half space $z > 0$ over an infinitely large charged surface. Compute and plot the resulting densities $n_\pm(z)$.

10. **Phase boundaries** **(5 points)**

a) Calculate the order parameter profile on a plane phase boundary (near the critical point) via minimization of the Landau-Ginzburg functional

$$L_G = n_c k_B T_c \int_V \mathrm{d}\mathbf{r} \, \mathcal{L}_G \qquad \mathcal{L}_G = \frac{\ell^2}{2}(\nabla\psi)^2 + \frac{t}{2}\psi^2 + \frac{g}{4}\psi^4$$

b) Discuss the result from a) [$\psi(z) = \psi_1 \tanh(z/\xi)$], in particular the form and role of the parameters ψ_1 and ξ.

c) Calculate the surface tension by integrating the surface contribution to the free energy L_G resulting from $\mathcal{L}_G$ for the solution found in a) through the phase boundary. Discuss your result.

Total score: 11 points