Statistical Physics II
Problem Set 6

Due: Tuesday, May 20, **before** the lecture

8. Fluctuation–dissipation theorem (FDT) (6 points)

a) For a spin $s = \pm 1$ of a paramagnet express the equilibrium correlation function $\langle s(t)s(t')\rangle$ in terms of the conditional expectation values $\langle s(t)\rangle_{s(t')=\pm 1}$.

b) Consider now the effect of a weak magnetic field

$$h(t) = h\theta(t' - t)\,,$$

where θ is the step function. Determine the probability of measuring $\langle s(t')\rangle = \pm 1$. Express the expectation value $\langle s(t)\rangle$ for $t > t'$ in terms of the conditional expectation values $\langle s(t)\rangle_{s(t')=\pm 1}$.

c) For general (weak) $h(t)$, $\langle s\rangle$ follows from the *linear response function $G(t)$*,

$$\langle s(t)\rangle = \int_{-\infty}^{t} \mathrm{d}t''\, G(t - t'')h(t'')\,.$$

Show that for $h(t)$ of the form given in b),

$$\langle s(t)\rangle = h \int_{-\infty}^{t'} \mathrm{d}t''\, \frac{\delta\langle s(t)\rangle}{\delta h(t'')}\,,$$

holds and use this relation to show the FDT

$$\langle s(t)s(t')\rangle = k_B T \int_{-\infty}^{t'} \mathrm{d}t''\, \frac{\delta\langle s(t)\rangle}{\delta h(t'')} \qquad \text{for } t > t'\,.$$

What can be said about the behavior of $\langle s(t)s(t')\rangle$ for $t < t'$?

9. **Debye-Hückel screening (part I) (4 points + 1 extra point)**

Consider a gas mixture of charged particles with densities n_α and charges z_α ($\alpha = \pm$), where $n_\alpha = n/2$ and $z_+ = -z_- = z$.

a) In the standard density functional $\beta F[n_+, n_-]$ identify the second order of the fluctuations $\delta n_\alpha(\vec{r})$ around the reference state of the homogenous ideal gas,

$$\frac{1}{2} \sum_{\alpha,\beta=\pm} \int \mathrm{d}^3 r \int \mathrm{d}^3 r' \, \delta n_\alpha(\vec{r}) G^{-1}_{\alpha\beta}(\vec{r}, \vec{r}') \delta n_\beta(\vec{r}') \,,$$

and compute in RPA $G^{-1}_{\alpha\beta}(\vec{r}, \vec{r}')$, the structure factor $S_{\alpha\beta}(\vec{q}) = [1 - n c_{\alpha\beta}]^{-1}$ and the pair correlation function $h_{\alpha\beta}(r)$.

b*) Using the appropriate generalized virial equation of state to compute the osmotic pressure of the charged gas.

Total score: 10 points + 1 additional point